EGM 5533 HW 3 Jin, Yu 1924-9011

3.5. Sol.

For $\sigma_{xy}=\sigma_{xz}=\sigma_{yz}=0$; $\sigma_{xx}=\sigma_{yy}=\sigma_{zz}=-p$

$\Rightarrow \varepsilon_{xy}=\frac{1+\nu}{E}\sigma_{xy}=0$, $\varepsilon_{xz}=\frac{1+\nu}{E}\sigma_{xz}=0$, $\varepsilon_{yz}=\frac{1+\nu}{E}\sigma_{yz}=0$

$\therefore \varepsilon_{xx}=\frac{1}{E}(\sigma_{xx}-\nu\sigma_{yy}-\nu\sigma_{zz})=\frac{1}{E}(1-2\nu)(-p)$

$\varepsilon_{yy}=\frac{1}{E}(\sigma_{yy}-\nu\sigma_{xx}-\nu\sigma_{zz})=\frac{1}{E}(1-2\nu)(-p)$

$\varepsilon_{zz}=\frac{1}{E}(\sigma_{zz}-\nu\sigma_{xx}-\nu\sigma_{yy})=\frac{1}{E}(1-2\nu)(-p)$

$\Rightarrow e=\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}=\frac{3}{E}(1-2\nu)(-p)$

$\therefore -ke=-\frac{E}{3(1-2\nu)}\cdot\frac{3}{E}(1-2\nu)(-p)=p$

So for this state of stress $p=-ke$ where $k=\frac{E}{3(1-2\nu)}$ and $e=\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$

3.15. Sol:

(a) $E=72GPa$, $\nu=0.33$, $\sigma_{xx}=200MPa$, $\sigma_{zz}=0$

$\varepsilon_{yy}=\frac{1}{E}(\sigma_{yy}-\nu\sigma_{xx}-\nu\sigma_{zz})=\frac{1}{E}(\sigma_{yy}-\nu\sigma_{xx})=0$

$\Rightarrow \sigma_{yy}=\nu\sigma_{xx}=0.33\times200=66MPa$

$\therefore \sigma_{yy}=66MPa$

(b) $\varepsilon_{xx}=\frac{1}{E}(\sigma_{xx}-\nu\sigma_{yy}-\nu\sigma_{zz})=\frac{1}{E}(\sigma_{xx}-\nu\sigma_{yy})$

$=\frac{1}{72\times10^9}(200\times10^6-0.33\times66\times10^6)=0.0025$

$\therefore \varepsilon_{xx}=0.0025$

(c) $\varepsilon_{zz}=\frac{1}{E}(\sigma_{zz}-\nu\sigma_{xx}-\nu\sigma_{yy})=\frac{1}{E}(-\nu\sigma_{xx}-\nu\sigma_{yy})=-\frac{\nu}{E}(\sigma_{xx}+\sigma_{yy})$

$=-\frac{0.33}{72\times10^9}(200\times10^6+66\times10^6)=-0.0012$

$\therefore \varepsilon_{zz}=-0.0012$

$\Delta A=b(b+b\varepsilon_{zz})-b^2=b^2\varepsilon_{zz}=(20)^2\times(-0.0012)=-0.48mm^2$ $\therefore \Delta A=-0.48mm^2$

3.21. Sol:

(a) $\varepsilon_{xx}=0.0003$, $\varepsilon_{yy}=0.0002$, $\varepsilon_{zz}=0.0001$, $\varepsilon_{xy}=0.00005$, $\varepsilon_{xz}=\varepsilon_{yz}=0$

$$\Rightarrow \begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz}\\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz}\end{bmatrix}=\begin{bmatrix}0.0003 & 0.00005 & 0\\ 0.00005 & 0.0002 & 0\\ 0 & 0 & 0.0001\end{bmatrix}$$

$$\det|\underline{\varepsilon}-\lambda\underline{I}|=0 \Rightarrow \begin{vmatrix} 0.0003-\lambda & 0.00005 & 0 \\ 0.00005 & 0.0002-\lambda & 0 \\ 0 & 0 & 0.0001-\lambda \end{vmatrix}=0$$

$(0.0003-\lambda)(0.0002-\lambda)(0.0001-\lambda)-0.00005^2(0.0001-\lambda)=0$

$(5.75\times10^{-8}-5\times10^{-4}\lambda+\lambda^2)(0.0001-\lambda)=0$

$\Rightarrow \lambda_1=0.0001,\ \lambda_2=0.0001793,\ \lambda_3=0.0003207$

$\therefore \underline{r}_1=[0\ 0\ 1]^T,\ \underline{r}_2=[0.3827\ \ -0.9239\ \ 0]^T,\ \underline{r}_3=[-0.9239\ \ -0.3827\ \ 0]^T$

(b). With $C_1=103GPa,\ C_2=55GP,\ C_3=27.6GPa$

$\Rightarrow \sigma_{xx}=C_1\varepsilon_{xx}+C_2\varepsilon_{yy}+C_2\varepsilon_{zz}=(103\times10^9)\times(0.0003)+(55\times10^9)\times(0.0002)+(55\times10^9)\times(0.0001)=47.4\,MPa$

$\sigma_{yy}=C_2\varepsilon_{xx}+C_1\varepsilon_{yy}+C_2\varepsilon_{zz}=(55\times10^9)\times(0.0003)+(103\times10^9)\times(0.0002)+(55\times10^9)\times(0.0001)=42.6\,MPa$

$\sigma_{zz}=C_2\varepsilon_{xx}+C_2\varepsilon_{yy}+C_1\varepsilon_{zz}=(55\times10^9)\times(0.0003)+(55\times10^9)\times(0.0002)+(103\times10^9)\times(0.0001)=37.8\,MPa$

$\sigma_{xy}=C_3\gamma_{xy}=2\varepsilon_{xy}C_3=2\times0.00005\times27.6\times10^9=2.76\,MPa,\ \ \sigma_{xz}=\sigma_{yz}=0$

(c). $$\det|\underline{\sigma}-\lambda\underline{I}|=0 \Rightarrow \begin{vmatrix} 47.4-\lambda & 2.76 & 0 \\ 2.76 & 42.6-\lambda & 0 \\ 0 & 0 & 37.8-\lambda \end{vmatrix}=0$$

$\Rightarrow \lambda_1=37.8\,MPa,\ \lambda_2=41.3425\,MPa,\ \lambda_3=48.6575\,MPa$

$\therefore \underline{r}_1=[0\ 0\ 1]^T,\ \underline{r}_2=[0.4146\ -0.91\ \ 0]^T,\ \underline{r}_3=[-0.91\ \ -0.4146\ \ 0]^T$

(d). With $E=72GPa,\ \nu=0.33 \Rightarrow \frac{E}{(1+\nu)(1-2\nu)}=1.5922\times10^{11},\ \frac{E}{1+\nu}=5.4135\times10^{10},\ 1-\nu=0.67,\ \nu=0.33$

$\Rightarrow \sigma_{xx}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{xx}+\nu(\varepsilon_{yy}+\varepsilon_{zz})]=47.77\,MPa,\ \sigma_{yy}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{yy}+\nu(\varepsilon_{xx}+\varepsilon_{zz})]=42.35\,MPa$

$\sigma_{zz}=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{zz}+\nu(\varepsilon_{xx}+\varepsilon_{yy})]=36.94\,MPa,\ \sigma_{xy}=\frac{E}{1+\nu}\varepsilon_{xy}=2.71\,MPa,\ \sigma_{xz}=\sigma_{yz}=0$

$$\det|\underline{\sigma}-\lambda\underline{I}|=0 \Rightarrow \begin{vmatrix} 47.77-\lambda & 2.71 & 0 \\ 2.71 & 42.35-\lambda & 0 \\ 0 & 0 & 36.94-\lambda \end{vmatrix}=0$$

$\Rightarrow \lambda_1=36.94,\ \lambda_2=41.2275,\ \lambda_3=48.8925$

$\therefore \underline{r}_1=[0\ \ 0\ \ 1]^T,\ \underline{r}_2=[0.3827\ -0.9239\ \ 0]^T,\ \underline{r}_3=[-0.9239\ -0.3827\ \ 0]^T$

For isotropic material, the principal axes of strain are the same as the principal axes of stress.