

at E

$$N_{CE} \sin 30 = P \quad \underline{N_{CE} = 2P}$$

$$N_{DE} + N_{CE} \cos 30 = 0 \quad \underline{N_{DE} = -\sqrt{3}P}$$

at D

$$N_{BD} = N_{DE} = -\sqrt{3}P$$

$$N_{CD} = P$$

at C

$$N_{AC} + N_{BC} = N_{CE} = 2P$$

$$N_{CE} \cos 30 - N_{AC} \cos 30 - N_{BC} \cos 30 = 0$$

$$N_{AC} + N_{BC} = N_{CE} = 2P$$

$$N_{AC} \sin 30 - N_{BC} \sin 30 - Q - N_{CD} - N_{CE} \sin 30 = 0$$

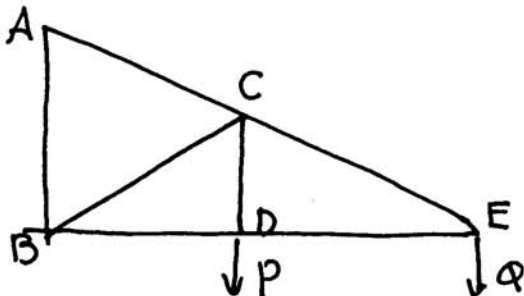
$$\begin{cases} N_{AC} - N_{BC} = 2Q + 2P + 2P \\ N_{AC} + N_{BC} = 2P \end{cases} \Rightarrow \underline{N_{AC} = Q + 3P}$$

$$\underline{N_{BC} = -Q - P}$$

$$\frac{\partial N_{AC}}{\partial Q} = 1 \quad \frac{\partial N_{BC}}{\partial Q} = -1$$

$$\delta_Q = \frac{N_{AC} L_{AC}}{EA_{AC}} \cdot \frac{\partial N_{AC}}{\partial Q} \Big|_{Q=0} + \frac{N_{BC} L_{BC}}{EA} \cdot \frac{\partial N_{BC}}{\partial Q} \Big|_{Q=0} = \frac{3PL}{EA} \cdot 1 + \frac{PL}{EA} \cdot 1 = \frac{4PL}{EA}$$

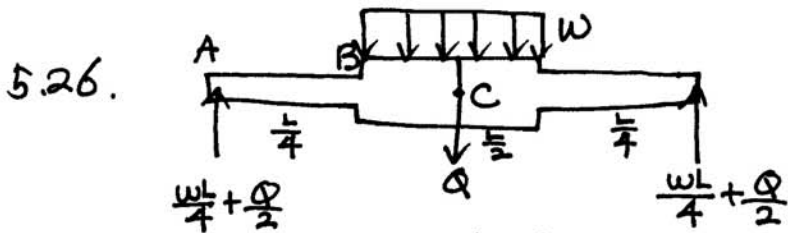
\* Displacement at E ?



derive strain energy  $U$

differentiate  $\frac{\partial U}{\partial Q}$ , then  $Q = P$ .

$$\left( \delta_E = 15.2 \frac{PL}{EA} \right)$$



Consider half of the beam

AB:  $M_{AB} = \left(\frac{wL}{4} + \frac{Q}{2}\right)x$  ;  $\frac{\partial M_{AB}}{\partial Q} = \frac{x}{2}$

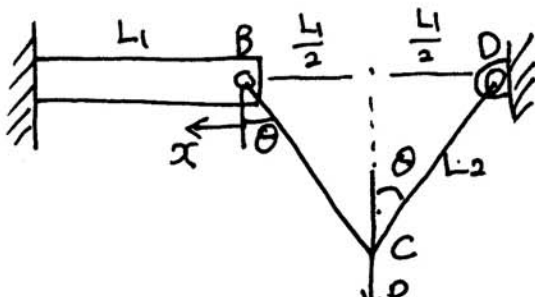
BC:  $M_{BC} = \left(\frac{wL}{4} + \frac{Q}{2}\right)x - \frac{w}{2}\left(x - \frac{L}{4}\right)^2$  ;  $\frac{\partial M_{BC}}{\partial Q} = \frac{x}{2}$

$$\delta_C = 2 \int_0^{L/4} \frac{M_{AB}}{EI_{AB}} \frac{\partial M_{AB}}{\partial Q} \Big|_{Q=0} dx + 2 \int_{L/4}^{L/2} \frac{M_{BC}}{EI_{BC}} \frac{\partial M_{BC}}{\partial Q} dx$$

$$= 2 \int_0^{L/4} \frac{wL}{4EI_{AB}} x \cdot \frac{x}{2} dx + 2 \int_{L/4}^{L/2} \frac{1}{EI_{BC}} \left(\frac{wL}{4}x - \frac{w}{2}\left(x - \frac{L}{4}\right)^2\right) \cdot \frac{x}{2} dx$$

$$= \frac{65}{6144} \frac{wL^4}{EI}$$

5.33.



$$2N \cos \theta = P \quad N = \frac{P}{2 \cos \theta}$$

Beam tip force  $V = N \cos \theta = \frac{P}{2}$

Bending moment  $M = \frac{P}{2} \cdot x$

$$\frac{\partial M}{\partial P} = \frac{x}{2} \quad \frac{\partial N}{\partial P} = \frac{1}{2 \cos \theta}$$

(a)  $\delta_c = \int_0^{L_1} \frac{M}{EI_1} \frac{\partial M}{\partial P} dx + 2 \cdot \frac{N \cdot L_2}{E_2 A_2} \cdot \frac{\partial N}{\partial P}$

$$= \int_0^{L_1} \frac{P}{4EI_1} x^2 dx + \frac{P L_2}{2 \cos^2 \theta E_2 A_2}$$

$$\cos \theta = \frac{\sqrt{L_2^2 - L_1^2/4}}{L_2}$$

$$= \frac{P L_1^3}{12 E_1 I_1} + \frac{P L_2}{2 E_2 A_2} \cdot \frac{L_2^2}{L_2^2 - L_1^2/4}$$

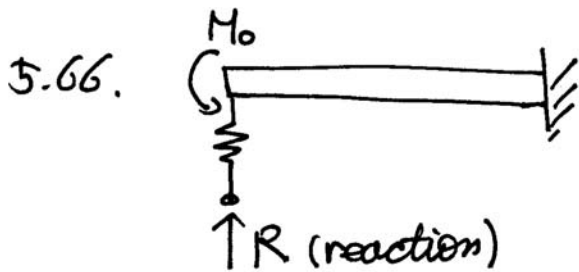
(b)  $\frac{P L_1^3}{12 E_1 I_1} = \frac{2 P L_1^3}{3 E_2 A_2 L_2^2}$  ,  $\frac{r_1}{r_2} = \frac{25}{\sqrt{2}}$

5.51 Shaft  $T = T_0$ ,  $\frac{\partial T}{\partial T_0} = 1$

Beam  $M = T_0$ ,  $\frac{\partial M}{\partial T_0} = 1$ .

$$\theta_c = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial T_0} dx + \int_0^{L_2} \frac{T}{GJ} \frac{\partial T}{\partial T_0} dx$$

$$= \int_0^{1.5} \frac{T_0}{EI} dx + \int_0^{1.2} \frac{T_0}{GJ} dx = 0.02187 + 0.03043 = 0.0523 \text{ rad.}$$



$$R = Ke$$

$$U_{sp} = \frac{1}{2} \cdot R \cdot e = \frac{R^2}{2K}$$

$$M = Rz - M_0 \quad \frac{\partial M}{\partial R} = z$$

$$\delta_R = \frac{\partial U_{sp}}{\partial R} + \frac{\partial U_{bending}}{\partial R} = 0.$$

$$= \frac{R}{K} + \int_0^L \frac{M}{EI} \cdot \frac{\partial M}{\partial R} dx$$

$$= \frac{R}{K} + \frac{R \cdot L^3}{3EI} - \frac{M_0 L^2}{2EI} = 0 \quad \therefore R = \frac{3M_0 K L^2}{6EI + 2KL^3} //$$