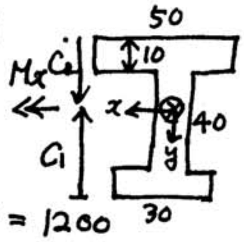
7.3 Max. comp. stress $= 50\,MPa$

$$M_x = \frac{|\sigma_{zz}|\, I_x}{C_2}$$



$$A = 50\times10 + 40\times10 + 30\times10 = 1200$$

$$A\cdot C_1 = 30\times10\times5 + 40\times10\times30 + 50\times10\times55$$

$$\therefore C_1 = 34.167,\quad C_2 = 25.833 \quad mm$$

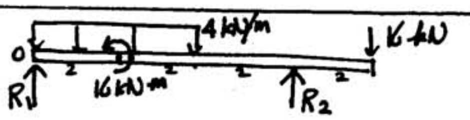
$$I_x = \tfrac{1}{12}\,50\times10^3 + 50\times10\times20.833^2$$
$$+ \tfrac{1}{12}\,10\times40^3 + 40\times10\times4.167^2 +$$
$$+ \tfrac{1}{12}\,30\times10^3 + 30\times10\times29.167^2$$
$$= 5.3917\times10^5 \; mm^4$$

$$\therefore M_x = \frac{50\times10^6\cdot 5.3917\times10^{-7}}{0.025833} = 1043 \; N\cdot m$$

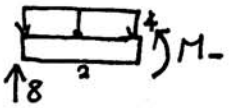
Max. tensile stress

$$(\sigma_{zz})_{tensile} = \frac{M_x\cdot C_1}{I_x} = 66.16\,MPa$$

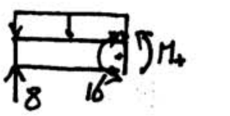
7.11



$$\Sigma M_{R_1} = 16 - 4\times4\times2 + R_2\cdot6 - 16\cdot8 = 0$$
$$R_2 = 24 \; kN$$
$$\Sigma M_{R_2} = R_1\cdot6 - 4\times4\times4 - 16 + 16\cdot2 = 0$$
$$R_1 = 8 \; kN$$



$$M_- - 8\cdot2 + 4\cdot2\cdot1 = 0$$
$$M_- = 8 \; kN\cdot m$$



$$M_+ - 8\cdot2 + 4\cdot2\cdot1 + 16 = 0$$
$$M_+ = -8\,kN\cdot m$$

$$I_x = \left(\tfrac{1}{12}\cdot100\cdot20^3 + 100\times20\cdot60^2\right)\times2 + \tfrac{1}{12}\cdot20\times100^3$$
$$= 1.63\times10^7\; mm^4 = 1.63\times10^{-5}\,m^4$$

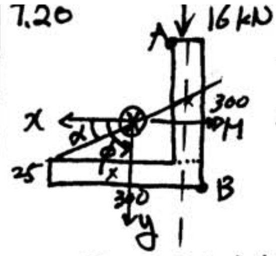
Left : σ_{max} at bottom

$$\sigma_{max} = \frac{M\cdot y}{I_x} = 34.57\,MPa$$

Right : σ_{max} at top

$$\sigma_{max} = \frac{M\cdot y}{I_x} = 34.57\,MPa.$$

7.20



$$A = 300\times25 + 275\times25 = 14{,}375$$

$$A\bar{x} = 300\cdot25\cdot150 + 275\cdot25\cdot12.5$$

$$\bar{x} = 84.24 = \bar{y}$$

$$I_x = I_y = 1.244\times10^8 \; mm^4$$

$$I_{xy} = 300\times25\times65.76\times71.74$$
$$+ 275\times25\times(-71.74)\cdot(-78.26) = 7.3981\times10^7$$

$$\phi = 90^\circ$$

$$\tan\alpha = \frac{M_x I_{xy} + \cancel{M_y I_x}}{M_x I_y + \cancel{M_y I_{xy}}} = \frac{I_{xy}}{I_y} = 0.5947$$

$$\alpha = 30.74^\circ$$

$$M = PL = 16\times25 = 40\,kN\cdot m = -M_x$$

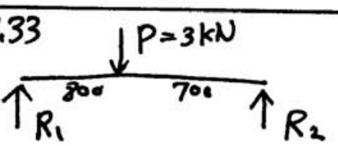
max. tensile at A $(-59, -216)$

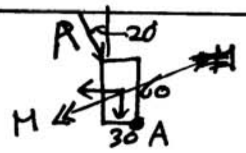
$$\sigma_A = \frac{1}{I_x I_y - I_{xy}^2}\left[-M_x I_{xy} x + M_x I_y y\right] = 90\,MPa$$

max. comp. at B $(-84.24,\ 84.24)$

$$\sigma_B = -67\,MPa.$$

7.33





$I_x = 5.4\times10^5\; mm^4 \qquad I_y = 1.35\times10^5$

$\phi = 110^\circ \qquad R_1 = 1.4\,kN \qquad R_2 = 1.6\,kN$

$M = 1.12\,kN\cdot m$

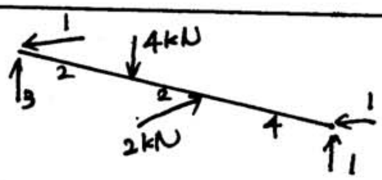
$M_x = M\sin\phi = 1.052$

$M_y = M\cos\phi = 0.383$

Max. tensile at A $(-15, 30)$

$$\sigma_A = 100.99\,MPa$$

7.40



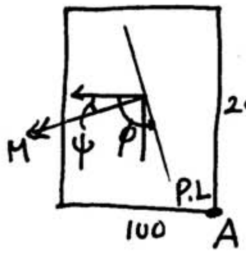
$M_x = 8 - z$

$M_y = z$

$I_x = 66.67\times10^6 \quad I_y = 16.67\times10^6 \ mm^4$

$\tan\psi = \dfrac{M_y}{M_x} = \dfrac{z}{8-z}$

$\phi = 90+\psi$



$\dfrac{M_y}{M_x} = -\cot\phi = -\dfrac{z}{8-z}$

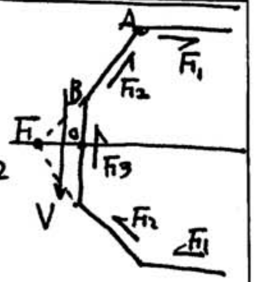
$\tan\alpha = \dfrac{\cancel{I_{xy}} - I_x\cot\phi}{I_y - \cancel{I_{xy}}\cot\phi} = -\dfrac{I_x}{I_y}\cot\phi = \dfrac{4z}{8-z}$

$\sigma_{zz}\Big|_A = \dfrac{M_x(y - x\tan\alpha)}{I_x - \cancel{I_{xy}}\tan\alpha} = 12 + 1.5z$

$\sigma_{max} = \sigma_A\Big|_{z=4m} = 12+1.5\times4 =$

$= 18MPa$

8.12 Let $V = I_x$



$I = (\frac{1}{12}\cdot b_f t^3 + b_f\cdot t\cdot 75^2)\cdot 2$

$+ (\frac{1}{12}\cdot 4\sqrt{2}\cdot 25^3 + 4\sqrt{2}\cdot 25\cdot 62.5^2)\cdot 2$

$+ \frac{1}{12}\cdot 4\cdot 100^2 = 3.7\times10^6 \ mm^4$

$q_A = 50\cdot4\cdot75 = 15000 \ N/mm$

$q_B = q_A + 25\cdot4\sqrt{2}\cdot62.5 = 23{,}840$

$q_0 = q_B + 50\cdot4\cdot25 = 28{,}840$

$F_1 = \frac{q_A}{2}(50) = 375{,}000 = 0.375\times10^6$

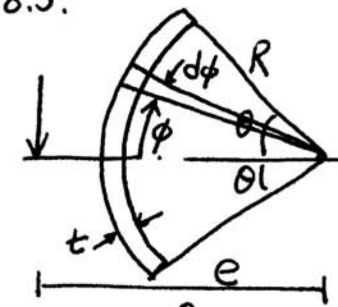
$F_3 = [q_B + \frac{2}{3}(q_0 - q_B)]\times100$

$= 2.7173\times10^6$

$\Sigma M_{F_1} = V(50-e) + 150F_1 - 50F_3 = 0$

$e = 28.5 \ mm.$

8.5.



Let $V = I_x$.

$q = \frac{VQ}{I_x} = Q$

moment equilibrium

$I_x = 2\int_0^\theta (R\sin\phi)^2 tR\,d\phi = tR^3(\theta - \sin\theta\cos\theta)$

$\sin^2\phi = \frac{1}{2}(1-\cos2\phi)$

$\sin2\phi = 2\sin\phi\cos\phi$

$q = \int_\phi^\theta (R\sin\alpha)\,tR\,d\alpha = tR^2(\cos\phi - \cos\theta)$

$dM = R\,dF = R\,q\,R\,d\phi$

$\therefore M = 2\int_0^\theta R^2 q\,d\phi = 2R^4 t\int_0^\theta(\cos\phi - \cos\theta)d\phi$

$= 2R^4 t(\sin\theta - \theta\cos\theta)$

$= Ve = I_x e$

$\therefore e = \dfrac{M}{I_x} = \dfrac{2tR^4(\sin\theta - \theta\cos\theta)}{tR^3(\theta - \sin\theta\cos\theta)}$

$= \dfrac{2R(\sin\theta - \theta\cos\theta)}{\theta - \sin\theta\cos\theta}$