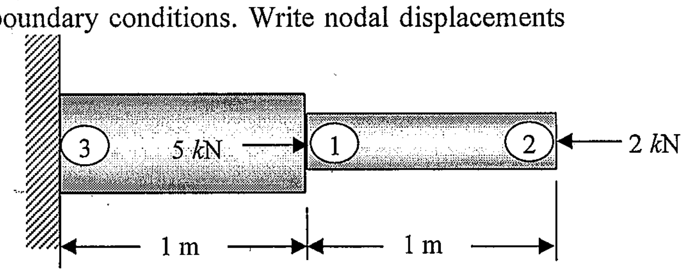
Name: EXAM 1 KEY UFID:________

EML5526 Finite Element Analysis and Applications Exam 1, Feb. 24th, 4"X6" formula sheet and calculator allowed

1. A stepped bar is clamped at one end, and subjected to concentrated forces as shown. **Note: the node numbers are not in usual order! Do not change node numbers!** Assume: E=100 GPa, Small area of cross section =1 cm^2, Large area of cross section =2 cm^2 (a) Write the global FE equation [K]{Q} = {F} where {Q} = $\{u_1, u_2, u_3\}^T$ and {F} = $\{F_1, F_2, F_3\}$. **Do not change the orders** in {Q} and {F}. (b) Solve the above matrix equation after applying boundary conditions. Write nodal displacements and element stresses.



Solution:

(a) Element stiffness matrices

Element 1:
$$10^7\begin{array}{cc} \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} & \begin{matrix} u_3 \\ u_1 \end{matrix} \end{array}$$

Element 2:
$$10^7\begin{array}{cc} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & \begin{matrix} u_1 \\ u_2 \end{matrix} \end{array}$$

5

(b) Assembly

$$10^7\begin{bmatrix} 3 & -1 & -2 \\ -1 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 = 5000\,\text{N} \\ F_2 = -2000\,\text{N} \\ R_3 \end{Bmatrix}$$

10

(c) Apply displacement boundary condition by deleting the third row and column:

$$10^7\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 5000 \\ -2000 \end{Bmatrix}$$

5

(d) By solving the above equation, u_1 = 0.15 mm, u_2 = −0.05 mm. Element stresses can be obtained by

$$s^{(1)} = \frac{P^{(1)}}{A^{(1)}} = \frac{E}{L}(u_1 - u_3) = 15\text{MPa}$$ 5

$$s^{(2)} = \frac{P^{(2)}}{A^{(2)}} = \frac{E}{L}(u_2 - u_1) = -20\text{MPa}$$ 5

Change node numbers but correct results 15.

" " incorrect " 10.

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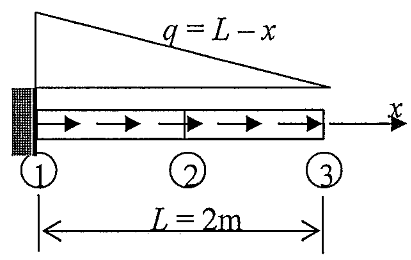
2. A bar of length L = 2m carries linearly varying axial load, as shown in the figure. Consider two equal-length bar elements, using axial displacements as nodal DOFs. Calculate work-equivalent nodal loads {**F**} = {F_1, F_2, F_3}.



(1) For Element 1, $N_1 = 1 - x$, $N_2 = x$, $q = 2 - x$

$$F_1^{(1)} = \int_0^1 (2-x)(1-x)\mathrm{d}x = \frac{6}{5}$$

$$F_2^{(1)} = \int_0^1 (2-x)x\mathrm{d}x = \frac{2}{3}$$

(2) For Element 2, $N_1 = 2 - x$, $N_2 = x - 1$, $q = 2 - x$

$$F_2^{(2)} = \int_1^2 (2-x)(2-x)\mathrm{d}x = \frac{1}{3}$$

$$F_3^{(2)} = \int_1^2 (2-x)(x-1)\mathrm{d}x = \frac{1}{6}$$

Thus, work-equivalent nodal loads are

$$\{F_1, F_2, F_3\} = \{F_1^{(1)}, F_2^{(1)} + F_2^{(2)}, F_3^{(2)}\} = \left\{\frac{5}{6}, 1, \frac{1}{6}\right\}$$

Getting the general idea of work-equivalent load 5

~~Having~~

Shape function correctly. : 10 pts.

Integral $\int q N_i$: +10 pts.

Correct integration : + 5 pts.

Correct assembly : + 5 pts.

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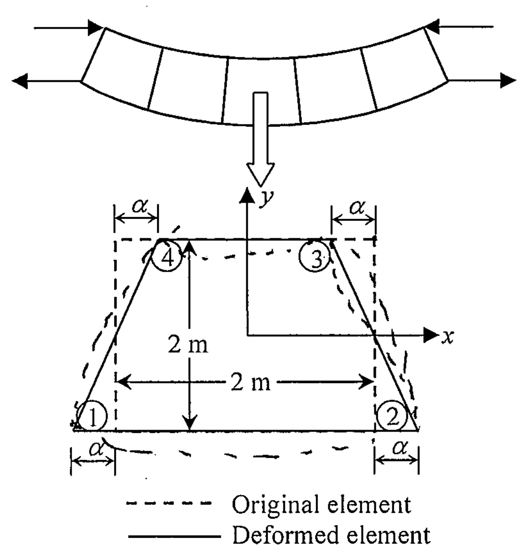
3. A beam under pure bending is modeled using Q6 elements. For a Q6 element shown, nodal displacements are given as $v_1 = v_2 = v_3 = v_4 = 0$, $u_1 = u_3 = -\alpha$, $u_2 = u_4 = \alpha$. Determine internal DOFs a_1, a_2, a_3, and a_4 such that the element satisfies the pure bending conditions; i.e., ε_{xx} is a function of y only, $\varepsilon_{yy} = \gamma_{xy} = 0$.



$$N_1 = \tfrac{1}{4}(1-x)(1-y) \qquad N_2 = \tfrac{1}{4}(1+x)(1-y) \qquad N_3 = \tfrac{1}{4}(1+x)(1+y)$$
$$N_4 = \tfrac{1}{4}(1-x)(1+y) \qquad N_5 = 1-x^2 \qquad N_6 = 1-y^2$$
$$u = \sum_{I=1}^{4} N_I u_I + N_5 a_1 + N_6 a_2 \qquad v = \sum_{I=1}^{4} N_I v_I + N_5 a_3 + N_6 a_4$$

Strains from interpolation:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\alpha}{4}\left[(1-y)+(1-y)-(1+y)-(1+y)\right] - 2xa_1 = -\alpha y - 2xa_1$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = -2ya_4$$

$-\alpha$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\alpha}{4}\left[(\cancel{1}-x)-(\cancel{1}+x)-(\cancel{1}+x)+(\cancel{1}-x)\right] - 2ya_2 - 2xa_3 = \cancel{(}\alpha - 2a_3)x - 2ya_2$$

From the condition of ε_{xx} being a function of y only, $a_1 = 0$

From the condition of $\varepsilon_{yy} = 0$, $a_4 = 0$

From the condition of $\gamma_{xy} = 0$, $a_2 = 0, a_3 = -\frac{\alpha}{2}$

Getting the idea of $\varepsilon_{xx} = f(y)$, $\varepsilon_{yy} = \gamma_{xy} = 0$: 20.

Correct formula but wrong answer : 25

Incomplet at the end. : 25 -

$$u = -\frac{\alpha}{4}\left[\cancel{1}-\cancel{x}-\cancel{y}+xy-\cancel{1}-\cancel{x}+\cancel{y}+xy+\cancel{1}+\cancel{x}+\cancel{y}+xy-\cancel{1}+\cancel{x}-\cancel{y}+xy\right] + (1-x^2)a_1 + (1-y^2)a_2$$

$$\therefore u = -\frac{\alpha}{4}xy + (1-x^2)a_1 + (1-y^2)a_2$$

$$v = (1-x^2)a_3 + (1-y^2)a_4$$

Name: EDGE Exam Key UFID:________

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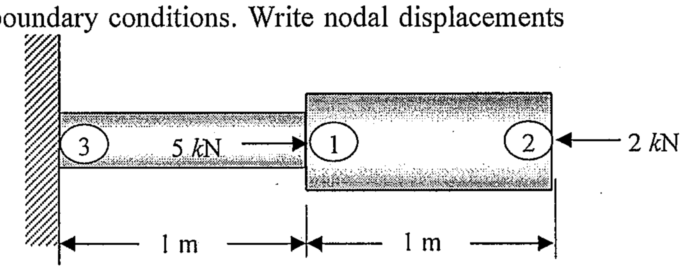

Solution:

(a) Element stiffness matrices

Element 1:

$$10^7 \begin{array}{c} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} u_3 \\ u_1 \end{array}$$

Element 2:

$$10^7 \begin{array}{c} \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \end{array} \begin{array}{c} u_1 \\ u_2 \end{array}$$

(b) Assembly

$$10^7 \begin{bmatrix} 3 & -2 & -1 \\ -2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 = 5000\,\mathrm{N} \\ F_2 = -2000\,\mathrm{N} \\ R_3 \end{Bmatrix}$$

(c) Apply displacement boundary condition by deleting the third row and column:

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(d) By solving the above equation, u_1 = 0.3 mm, u_2 = 0.2 mm. Element stresses can be obtained by

$$S^{(1)} = \frac{P^{(1)}}{A^{(1)}} = \frac{E}{L}(u_1 - u_3) = 30\mathrm{MPa}$$

$$S^{(1)} = \frac{P^{(2)}}{A^{(2)}} = \frac{E}{L}(u_2 - u_1) = -10\mathrm{MPa}$$

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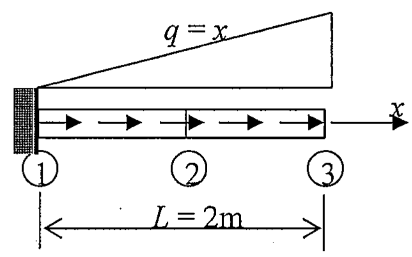

(1) For Element 1, $N_1 = 1 - x, \qquad N_2 = x, \qquad q = x$

$$F_1^{(1)} = \int_0^1 x(1-x)\mathrm{d}x = \frac{1}{6}$$

$$F_2^{(1)} = \int_0^1 x^2 \mathrm{d}x = \frac{1}{3}$$

(2) For Element 2, $N_1 = 2 - x, \qquad N_2 = x - 1, \qquad q = x$

$$F_2^{(2)} = \int_1^2 x(2-x)\mathrm{d}x = \frac{2}{3}$$

$$F_3^{(2)} = \int_1^2 x(x-1)\mathrm{d}x = \frac{5}{6}$$

Thus, work-equivalent nodal loads are

$$\{F_1, F_2, F_3\} = \{F_1^{(1)}, F_2^{(1)} + F_2^{(2)}, F_3^{(2)}\} = \left\{\frac{1}{6}, 1, \frac{5}{6}\right\}$$

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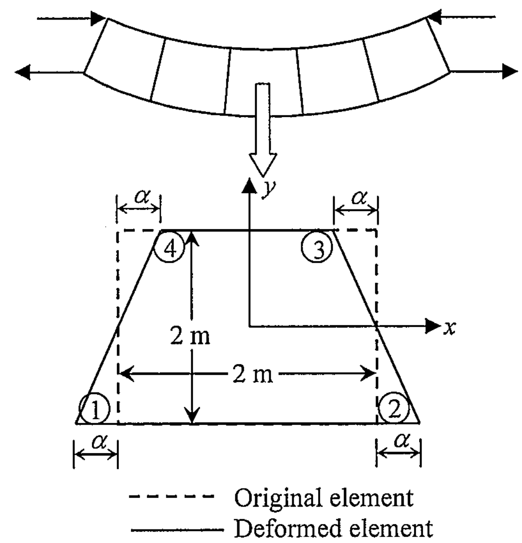

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From the condition of $\varepsilon_{yy} = 0$, $a_4 = 0$

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