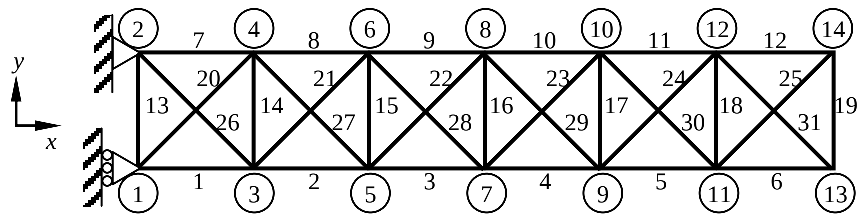
Project 1. Finite Element Analysis and Design of a Plane Truss



Consider a plane truss in the figure. The horizontal and vertical members have length L, while inclined members have length $\sqrt{2}L$. Assume the Young's modulus E = 100 GPa, density ρ = 7,830 kg/m^3, cross-sectional area A = 1.0 cm^2, and L = 0.3 m.

PART1

1. Use a finite element program to determine the deflections and element forces for the following three load cases. Presents you results in the form of a table.

   Load Case A) $F_{x13} = F_{x14}$ = 10,000 N
   Load Case B) $F_{y13} = F_{y14}$ = 1,000 N
   Load Case C) F_{x13} = 10,000 N and F_{x14} = –10,000 N

2. Assuming that the truss behaves like a cantilever beam, one can determine the equivalent cross sectional properties of the beam from the results for Cases A through C above. The three beam properties are: axial rigidity $(EA)_{eq}$ (this is different from the AE of the truss member), flexural rigidity $(EI)_{eq}$ and shear rigidity $(GA)_{eq}$. Let the beam length be equal to l. (l = 6×0.3 = 1.8 m)

   The axial deflection of a beam due to an axial force F is given by:

$$u_{tip} = \frac{Fl}{EA_{eq}} \tag{1}$$

   The transverse deflection due to a transverse force F at the tip is:

$$v_{tip} = \frac{Fl^3}{3\ EI_{eq}} + \frac{Fl}{GA_{eq}} \tag{2}$$

   In Eq. (2) the first term on the RHS represents the deflection due to flexure and the second term due to shear deformation. In the elementary beam theory (Euler-Bernoulli beam theory) we neglect the shear deformation, as it is usually much smaller than the flexural deflection.

   The transverse deflection due to an end couple C is given by:

$$v_{tip} = \frac{Cl^2}{2\ EI_{eq}} \tag{3}$$

   Substitute the average tip deflections obtained in Part 1 in Eqs. (1)–(3) to compute the equivalent section properties: $(EA)_{eq}$, $(EI)_{eq}$, and $(GA)_{eq}$. You may use the average of deflections at Nodes 13 and 14 to determine the equivalent beam deflections.

3. Verify the beam model by adding two more bays to the truss (l = 8×0.3 = 2.4 m). Compute the tip deflections of the extended truss for the three load cases A–C using the FE program. Compare the FE results with deflections obtained from the equivalent beam model (Eqs. (1)–(3))

PART 2

When structures are subject only to stress and minimum gage constraints, the *fully-stressed design* (FSD) can be very useful to find the best design. The basic concept is as follows: For the best design, each member of the structure that is not at its minimum gage is fully stressed under at least one of the design load conditions. This basic concept implies that we should remove material from members that are not fully stressed unless prevented by minimum gage constraints. The FSD technique is usually complemented by a resizing algorithm based on the assumption that the load distribution in the structure is independent of member sizes. That is, the stress in each member is calculated, and then the member is resized to bring the stresses to their allowable values assuming that the loads carried by members remained constant (this is logical since the FSD criterion is based on a similar assumption). For example, for truss structures, where the design variables are often cross-sectional areas, the force in any member is $\sigma \cdot A$ where σ is the axial stress and A the cross-sectional area. Assuming that $\sigma \cdot A$ is constant leads to the following stress ratio resizing technique:

$$A_{\text{new}} = A_{\text{old}} \left| \frac{\sigma}{\sigma_{\text{allowable}}} \right| \tag{4}$$

which gives the resized area A_{new} in terms of the current area A_{old}, the current stress σ, and the allowable stress $\sigma_{\text{allowable}}$. If A_{new} obtained by Eq. (4) is smaller than the minimum gage, the minimum gage should be selected rather than the value given by Eq. (4).

Perform FSD design for the original truss structure under three loading conditions (tension, shear and bending). The maximum allowable stress after considering safety factor is $\sigma_{\text{allowable}}$ = 100MPa. The minimum gage area is 0.1 cm^2. Provide a table of stresses and cross-sectional areas of all elements, along with the final weight of the structure. Identify which load case is critical to determine the cross-sectional area of each element.