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Reducing Conservatism in Flutterometer Predictions Using Volterra Modeling with Modal Parameter Estimation

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Accurate prediction of flutter speeds is essential to efficient and safe flight testing for envelope expansion. Such accuracy is particularly difficult to obtain when analyzing flight data from speeds well below the critical speed at which flutter occurs. The flutterometer was introduced as a tool that could predict the onset of flutter even at low-speed conditions; however, the conservatism in those predictions reduced testing efficiency. A method to augment the flutterometer to decrease the conservatism, and consequently increase the accuracy, of the predicted flutter speeds is presented. The method incorporates a scheme for model updating that ensures consistency between the analytical dynamics and the flight data. The tool is used to compute flutter speeds for the aerostructures test wing. The speeds predicted with the model updating scheme are very close to the actual flutter speed and demonstrate the benefits for improving efficiency of envelope expansion.

Introduction

THE flutterometer is a tool designed to increase the efficiency of flight flutter testing.[1] The concept of this tool is to combine model-based and data-based analysis to predict the onset of flutter during envelope expansion. The procedure is to compute the worst-case stability margins for a theoretical model with respect to uncertainty as determined by flight data. The development for such a procedure is based on μ-method analysis.[2]

The tool has been used to predict flutter speeds for a variety of aeroelastic systems. The flutterometer was tested, along with a variety of techniques, for a simple system that will be addressed again in this paper.[3] Also, the technique has been successfully applied to the prediction of flutter for systems such as a wind-tunnel model[4] and F-16 with stores.[5]

Flutter speeds predicted by the flutterometer have an inherent level of conservatism resulting from the worst-case nature of the predictions. This conservatism provides an inherent level of safety; however, that conservatism can also adversely impact the efficiency of the flight test. Some aircraft have relatively small flutter margins, and so the envelope must be determined as accurately as possible. Any conservatism in the prediction of this envelope could result in time-consuming flight testing.

The conservatism in the flutterometer is directly related to the model and its associated uncertainty. The only way to remove the conservatism is to improve the accuracy of the model. The formulation of a theoretically perfect model would guarantee the flutterometer has minimal conservatism; however, such a formulation is not yet feasible. The issue of model updating is, therefore, an important objective for aeroelasticity research.[6]

This paper augments the flutterometer with a simple approach for model updating. This approach actually builds on previous research that used Volterra kernels to analyze flight data and reduce uncertainty.[7] The Volterra kernels are now coupled with a parameter estimation algorithm to derive modal properties that are more indicative of the true dynamics of the system. These properties are used to update the model and associated uncertainty description. The resulting formulation is an updated model that can represent the system dynamics with more accuracy than the original model. Consequently, the updated flutterometer predictions are more accurate and have less conservatism than original flutterometer predictions.

The flutterometer is used to predict flutter speeds for the aerostructures test wing (ATW). The inclusion of model updating allows the tool to account for a phase shift that was not indicated by preflight computational analysis. The resulting predictions are highly accurate and, thus, indicate a reduction in conservatism.

Parameter Estimation

Formulation

The basic problem of parameter estimation is to analyze data and identify coefficients of an equation of motion. This problem is, in essence, a structured form of the more general problem of system identification.[8] Parameter estimation has been especially successful when applied to rigid-body equations of motion for an aircraft[9]; however, it has had only limited success when applied to the flexible-body equations of motion associated with aeroelasticity.

The difficulty with direct application of parameter estimation for aeroelasticity arises from both the data and the equations of motion. Flight systems have inherent difficulties generating aeroelastic data because of issues such as poor excitation at high frequencies, poor measurement of excitation force, and poor observability in dense modal spaces. Furthermore, the equations of motion contain terms, such as lags, that have only small contributions to the response at most flight conditions. The problem is particularly difficult when an attempt is made to identify unsteady aerodynamics separately from structural dynamics. Thus, the estimation of all parameters in the model from flight data is questionable.

This paper adopts a simple approach to parameter estimation that considers the aeroelastic dynamics. The concept is not to estimate specific parameters in either the structural dynamics or unsteady aerodynamics; rather, estimates are computed for the combined aeroelastic dynamics. The correct parameters in the separated dynamics are obviously of interest, but they are not necessarily required for the computation of flutter. The prediction of aeroelastic instability can be accomplished by the use of the correct parameters for the coupled dynamics.

The general formulation of the approach uses a state-space representation of the model. This representation uses two sets of matrices, $\{A, B, C, D\}$, and $\{\hat{A}, \hat{B}, \hat{C}, \hat{D}\}$, as the realization of the theoretical model. This model, P, relates the predicted value of a measurement

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Y in response to excitation given as u:

$$Y = Pu \tag{1}$$

$$= \left[\begin{array}{c|c} A+\hat{A} & B+\hat{B} \\ \hline C+\hat{C} & D+\hat{D} \end{array}\right] u \tag{2}$$

Assume flight data have been recorded as the measurements of the aircraft in response to the excitation. A model $\bar{P}$ can be identified that relates these actual measurements $\bar{Y}$ to the excitation u by the use of another state-space realization

$$\bar{Y} = \bar{P}u \tag{3}$$

$$= \left[\begin{array}{c|c} \bar{A} & \bar{B} \\ \hline \bar{C} & \bar{D} \end{array}\right] u \tag{4}$$

The objective of parameter estimation, as described in this paper, is to choose optimal values of $\{\hat{A}, \hat{B}, \hat{C}, \hat{D}\}$ so that the predicted value of Y is similar to $\bar{Y}$. This application is somewhat different from traditional applications of parameter estimation that attempt to identify specific parameters. In a sense, the approach discussed here is a type of model updating that attempts to identify errors in specific parameters.

Modal Parameter Estimation

The aeroelastic system can always be written in modal form as a set of one-state convergences and two-state modes. The one-state convergences are associated with the lag terms, and these will not be updated because of observability difficulties.[10] The two-state modes are usually easy to observe, and so the parameter estimation will concentrate on these modal dynamics. In this case, the two-state modes are neither pure structural or aerodynamic modes because they actually represent a model of the coupled aeroelastic system.

Consider the model, including theoretical dynamics and update matrices, formulated in this specific modal form. In this formulation, R represents the real part of the eigenvalues and I represents the imaginary part of the eigenvalues associated with modal parameters:

$$Y = \left[\begin{array}{cc|c} R+\hat{R} & I+\hat{I} & B_1 \\ -I-\hat{I} & R+\hat{R} & B_2 \\ \hline C_1+\hat{C}_1 & C_2+\hat{C}_2 & D+\hat{D} \end{array}\right] u \tag{5}$$

$$= \left(D+\hat{D}+\left\{[C_1+\hat{C}_1 \quad C_2+\hat{C}_2] \times \left(sI - \begin{bmatrix} R+\hat{R} & I+\hat{I} \\ -I-\hat{I} & R+\hat{R} \end{bmatrix}\right)^{-1} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}\right\}\right) u \tag{6}$$

$$= \left[D+\hat{D}+\frac{(C_1B_1+C_2B_2+\hat{C}_1B_1+\hat{C}_2B_2)(s-R-\hat{R})}{(s-R-\hat{R})^2+(I+\hat{I})^2}\right. \tag{7}$$

$$\left.+\frac{(C_1B_2-C_2B_1+\hat{C}_1B_2-\hat{C}_2B_1)(I+\hat{I})}{(s-R-\hat{R})^2+(I+\hat{I})^2}\right] u \tag{8}$$

Similarly, consider the identified model in this modal form:

$$\bar{Y} = \left[\begin{array}{cc|c} \bar{R} & \bar{I} & \bar{B}_1 \\ -\bar{I} & \bar{R} & \bar{B}_2 \\ \hline \bar{C}_1 & \bar{C}_2 & \bar{D} \end{array}\right] u$$

$$= \left\{\bar{D}+[\bar{C}_1 \quad \bar{C}_2]\left(sI - \begin{bmatrix} \bar{R} & \bar{I} \\ -\bar{I} & \bar{R} \end{bmatrix}\right)^{-1} \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}\right\} u$$

$$= \left[\bar{D}+(\bar{C}_1\bar{B}_1+\bar{C}_2\bar{B}_2)\frac{s-\bar{R}}{(s-\bar{R})^2+\bar{I}^2} + (\bar{C}_1\bar{B}_2-\bar{C}_2\bar{B}_1)\frac{\bar{I}}{(s-\bar{R})^2+\bar{I}^2}\right] u \tag{9}$$

The use of this modal form allows direct formulation of the updates needed to the theoretical model. Actually, the values of the updates can be chosen to match the models exactly and, consequently, ensure that $Y = \bar{Y}$ so that the predicted data match the measured data.

$$\hat{R} = \bar{R} - R$$
$$\hat{I} = \bar{I} - I$$
$$\hat{D} = \bar{D} - D$$
$$\hat{C}_1 = \frac{(\bar{C}_1\bar{B}_1+\bar{C}_2\bar{B}_2-C_1B_1)B_1+(\bar{C}_1\bar{B}_2-\bar{C}_2\bar{B}_1-C_1B_2)B_2}{B_1^2+B_2^2}$$
$$\hat{C}_2 = \frac{(\bar{C}_2\bar{B}_1-\bar{C}_1\bar{B}_2-C_2B_1)B_1+(\bar{C}_1\bar{B}_1+\bar{C}_2\bar{B}_2-C_2B_2)B_2}{B_1^2+B_2^2} \tag{10}$$

Model Updating

The flutterometer is a tool to be used during flight testing for envelope expansion. Thus, the issue of parameter estimation for model updating must be addressed in that context. The application of parameter estimation for model updating is considered for the aircraft at the ith test point during an envelope expansion.

The procedure at the ith test point is to record flight data and identify a state-space model $\bar{P}$ that relates the measurements and excitations. The theoretical model P is compared to this identified model, and the optimal values of modal estimates are computed as $\{\hat{R}^i, \hat{I}^i, \hat{D}^i, \hat{C}_1^i, \hat{C}_2^i\}$. The objective is to find the values of $\{\hat{A}, \hat{B}, \hat{C}, \hat{D}\}$ for model updating. Most importantly, the models P and $\bar{P}$ are expressed in the modal form described earlier, so that the model updating is straightforward.

The easier, and most direct, approach to model updating is to choose the model updates such that the theoretical model exactly matches the model identified from flight data. Correspondingly, the model updates are chosen as the optimal values from the modal parameter estimation made with the ith data set. Note that $\hat{B}$ can be set to null because a single update to the observability matrix, given as $\hat{C}$, is sufficient to account for the effects of errors in both input and output:

$$\hat{A} = \begin{bmatrix} \hat{R}^i & \hat{I}^i \\ -\hat{I}^i & \hat{R}^i \end{bmatrix} \tag{11}$$

$$\hat{B} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{12}$$

$$\hat{C} = \begin{bmatrix} \hat{C}_1^i & \hat{C}_2^i \end{bmatrix} \tag{13}$$

$$\hat{D} = \hat{D}^i \tag{14}$$

Such an approach has safety implications that must be considered for flight testing. Essentially, this simple approach completely replaces the theoretical dynamics with the identified dynamics. The safety concerns arise given that the identified model may have significant errors because of properties associated with the flight data. Obviously such an erroneous model should not be used for flutter prediction, and so the model updating must try to avoid this situation.

A reasonable approach for model updating can be chosen that uses the parameter estimates while still retaining some level of safety. The concept would be to update the model to correct errors that show some consistency or pattern as the envelope is expanded.

Consider a possible situation for the natural frequency of a mode. The theoretical model shows an error of 1.0 Hz at a test point, but shows an error of -1.0 Hz at another test point. The correct update to the model is not clear. The model may indeed have an error that is strongly dependent on the flight condition or, perhaps, the variations in the data may indicate random errors that are not realistic.

Conversely, consider a different situation. The theoretical model shows an error of 1.0 Hz at a test point and shows an error of 1.1 Hz at another test point. The estimate of this error may be incorrect at each test point; however, the consistency of the error indicates the model probably has an inherent error of roughly 1.0 Hz in the natural frequency.

The approach for model updating used in this paper is to adopt a scheme based on running averages of errors. The errors at each test point, indicated by the optimal values of modal parameter estimation, are accumulated in a vector. The actual updates applied to the model are then the average values of these vectors from the first to the ith test point.

$$\hat{A} = \text{mean}\left\{\begin{bmatrix} \hat{R}^1 & \hat{I}^1 \\ -\hat{I}^1 & \hat{R}^1 \end{bmatrix}, \ldots, \begin{bmatrix} \hat{R}^i & \hat{I}^i \\ -\hat{I}^i & \hat{R}^i \end{bmatrix}\right\} \tag{15}$$

$$\hat{B} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{16}$$

$$\hat{C} = \text{mean}\left\{ \begin{bmatrix} \hat{C}_1^1 & \hat{C}_2^1 \end{bmatrix}, \ldots, \begin{bmatrix} \hat{C}_1^i & \hat{C}_2^i \end{bmatrix} \right\} \tag{17}$$

$$\hat{D} = \text{mean}\{\hat{D}^1, \ldots, \hat{D}^i\} \tag{18}$$

In this way, the updating will only reflect consistent errors. Random errors, which are assumed to have an average near zero, will not be reflected in the model update. Consistent errors will have a nonzero average that will be reflected in the model update. Also, smoothly time-varying errors will be reflected in the averages, and, thus, these errors will result in model updates. The safety level is not overly sacrificed with this approach because the model is only updated by the use of errors whose consistency provides confidence that the update is indeed valid.

This approach may seem simplistic in comparison to the advanced techniques being investigated for the general problem of parameter estimation; however, the simplicity presents distinct advantages for this application. As mentioned earlier, the direct estimation of structural and aerodynamic parameters from flight data is not feasible, but the simplistic estimation of aeroelastic modal parameters is straightforward. Also as mentioned earlier, the direct utilization of errors estimated at a single test point may lead to incorrect predictions of flutter speeds, but a simplistic average will generate updates that reflect consistent errors without sacrificing safety.

Flutterometer

Model Formulation

The initial process in the flutterometer is the formulation of a model. This model must be a state-space description of the aeroelastic dynamics. In particular, the model must be parametrized around flight condition and associated uncertainties.

The model updating, both for theoretical dynamics and uncertainties, is performed by comparison of theoretical and measured transfer functions at a flight condition. Thus, the theoretical model must be formulated at a reference condition matching the test point. This formulation is easily done by the use of one of several methods, including the approaches used standard software packages like NASTRAN and ZAERO; however, the model must retain its parameterization around flight condition and uncertainty.

The procedure used for model formulation in the flutterometer is actually straightforward. The preflight theoretical dynamics are generated at a reference condition V^0 with standard approaches. A state-space model $P(V^0)$ is generated and parametrized around flight condition δ_V and uncertainty Δ. This parameterization makes it trivial to formulate the model at any reference condition. Thus, the model associated with the velocity V^i at the ith test point is generated by simple replacement of δ_V with $\delta_V + V^i - V^0$. The resulting model can be expressed as $P(V^i)$ with feedback signals relating these dynamics to δ_V and Δ.

Also, the model needs to be expressed in modal form to facilitate the parameter estimation. This modal form is generated by standard routines for state transformations.[10] The final model describes the aeroelastic dynamics at V^i with 2×2 and 1×1 modal blocks that are parametrized around flight condition and uncertainty.

Data Filtering

The next process in the flutterometer is the filtering of flight data. This step is vital because aeroelastic flight data are typically of very poor quality. In general, these data do not satisfy linearity requirements due to nonlinearities in the aircraft dynamics and high levels of noise in the measurements. The process of model updating requires data that accurately reflect the linear dynamics of the aeroelastic system. Thus, the data must be filtered to provide the required information.

The filtering process used in this paper is based on Volterra modeling. The concept of Volterra modeling is to represent data as a set of signals that describe different orders. In other words, the data can be described by a linear component, plus a quadratic component, plus higher-order components. The dynamics of each component are given by a Volterra kernel.[11,12]

Volterra kernels are identified whose input–output characteristics match the flight data.[7] These kernels are identified by the use of a wavelet-based approach that makes use of multiresolution decomposition.[13] Volterra kernels have been used to represent many aeroelastic systems and demonstrate that the linear, quadratic, and third-order terms often provide sufficient accuracy.[14,15] Thus, this paper will only consider the identification of low-order terms to represent the aeroelastic dynamics.

The data are filtered by the extraction of the linear component. This component is found by compution of optimal values of the Volterra kernels from the flight data. The first-order kernel represents the linear dynamics. The desired linear component of the data is, thus, generated by a simple convolution of the excitation signal with the kernel.

This type of filtering is only one approach from a multitude of possibilities. Any filtering can be used; however, previous experience has shown that Volterra-based filtering works particularly well. Standard approaches, such as low-pass filtering, or advanced approaches, such as time-frequency feature filtering, may be used, but they are not necessarily optimal for this type of application. The Volterra filtering is appropriate because it inherently considers transfer functions for data representation, which are exactly what is required by the flutterometer.

Model Updating

The process of model updating makes direct use of the products of model formulation and data filtering. Essentially, the parameters of the formulated model are altered based on information from the filtered data. An important issue to note is that model updating actually refers to both the nominal dynamics and the associated uncertainty. These elements are both part of the uncertain model used for μ-method analysis, and so they should both be updated.

The first step of model updating is to alter the theoretical dynamics. This alteration is done by perturbation of the state-space matrices by the use of the modal parameter updates. More precisely, the updates are the running averages of the differences between the theoretical and estimated dynamics at each test point. The altered dynamics are now considered the nominal dynamics.

The other step of model updating is to compute the uncertainty associated with the nominal dynamics. This step is done via the test of a condition for model validation. Essentially, the effect of uncertainty is to allow the analytical dynamics to vary within a bounded range. The model updating occurs by an increase in the size of uncertainty until the flight data lie within the range of dynamics. The actual data used for this process may vary such that consideration of a single test point will reduce conservatism, but also safety margins, whereas consideration of many test points will increase conservatism, but also safety margins, for the resulting flutter predictions.[1]

In this way, the data filtered via Volterra kernels are actually used multiple times. This data are used for parameter estimation because they contain only the desired linear component. These data are also used for model validation because they relate a realistic measure of linear uncertainty.[7] Essentially, the filtered data contain only information about the linear dynamics, and so they are optimal for use for parameter estimation and model validation of linear models.

Robust Flutter Speeds

The final process in the flutterometer is the computation of a robust flutter speed. This process is a direct application of μ-method analysis to the model with optimal parameter and uncertainty updates. The resulting analysis is a robust stability computation that considers the worst-case flutter speeds of the model with respect to the uncertainty.

Flight Test

ATW

The ATW was developed at NASA Dryden Flight Research Center.[16] This testbed was specifically designed for testing methods to predict the onset of flutter. The ATW was essentially a wing and boom assembly that was flown by using an F-15 aircraft and associated flight-test fixture. The ATW was mounted horizontally to the fixture and the resulting system attached to the undercarriage of the F-15, as shown in Fig. 1.

The ATW was flown to several test points during envelope expansion. These points included altitudes of 20000 (6096 m); 15000 (4572 m); and 10000 ft (3048 m) and Mach numbers of 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, and 0.825. The final flight ended with destruction of the ATW due to flutter at flight conditions of 10,000 ft and 0.83 for Mach number. Equivalently, the flight conditions at which flutter occured correspond to 460 kn of equivalent airspeed (KEAS).

Envelope Expansion

Estimates of modal parameters were computed from the flight data. At each test point, a standard frequency-domain method of system identification was used to formulate a model whose magnitude and phase characteristics were similar to the transfer function. The modal parameters of that model were then extracted and used as representative of the ATW parameters.

The modal dampings that were extracted at each test point are given in Fig. 2. The flutter instability affecting the bending mode is clearly evident in the data trends.

The modal frequencies for the ATW are given in Fig. 3.

The actual mechanism of flutter is somewhat difficult to discern from the flight data. The dampings in Fig. 2 seem to indicate the mechanism is a classical type of flutter such that one mode is becoming less stable, whereas the other mode is becoming more stable. Conversely, the frequencies in Fig. 3 do not appear to coalesce, as expected for classical flutter, until a possible coalescence at the airspeed very close to the onset of flutter.

Also note in Figs. 2 and 3 is that only 15 estimates are shown, even though the flights operated at 21 test points. The responses from several of the test points were unable to present sufficient information about the bending mode. The response levels were quite low at these test points, and so accurate modal estimates could not be obtained. The reason for the poor data quality at some points was unclear but possibilities included high levels of turbulence and noise or unexplained high damping.

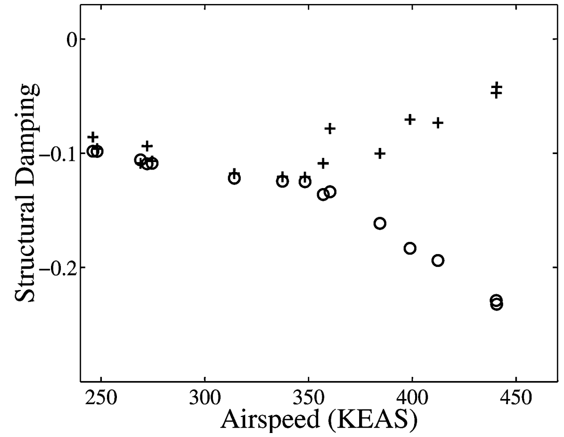

Fig. 2 Measured modal dampings for bending mode, + and torsion mode, ○.

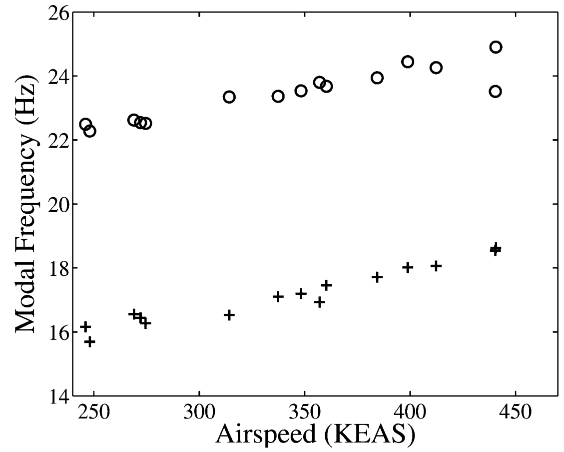

Fig. 3 Measured modal frequencies for bending mode, + and torsion mode, ○.

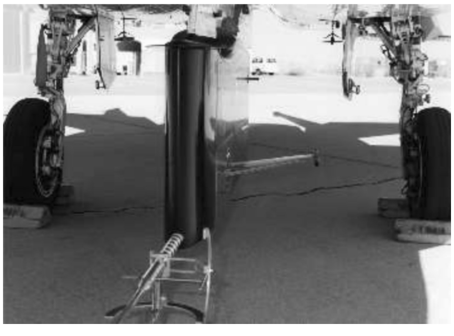

Fig. 1 Mounting of the ATW.

Data Filtering

The flight data are filtered to extract the linear component by the use of Volterra kernels. Previous analysis of the ATW has indicated that its dynamics are relatively linear; therefore, only first- and second-order terms in the Volterra expansion were used to reflect the linear and quadratic dynamics.

The actual computation of kernels required several parameters, such as memory length and resolution,[7] to be chosen. Several sets of parameters were tested, and the results indicated a range within which the results were fairly consistent and reasonable. Consequently, the kernels were identified with a 1-s duration and 256 points. These parameters indicate the kernel decayed within 1-s and comprised 256 wavelet coefficients.

Figure 4 shows a measurement from the flight data that is representative of the measurements taken throughout the flight. These data show the response to a chirp excitation for the accelerometer located in the boom near the trailing-edge endcap. This chirp is generated by variation of the sinusoids, commanded to the excitation mechanism on the system, between 5 and 35 Hz (Ref. 16).

The corresponding simulated data, as computed by the first-order Volterra kernel, are shown in Fig. 5. Clearly, the data filtering has reduced a significant portion of the energy in the response. Some of this energy was random noise, whereas some of this energy resulted from excitation of nonlinearities. Either way, the second-order

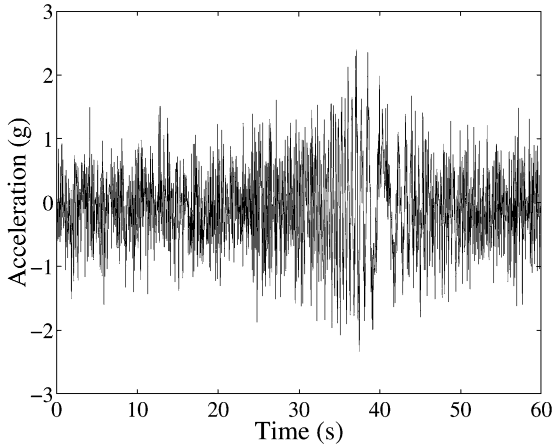

Fig. 4 Response of the trailing-edge boom accelerometer to a chirp excitation.

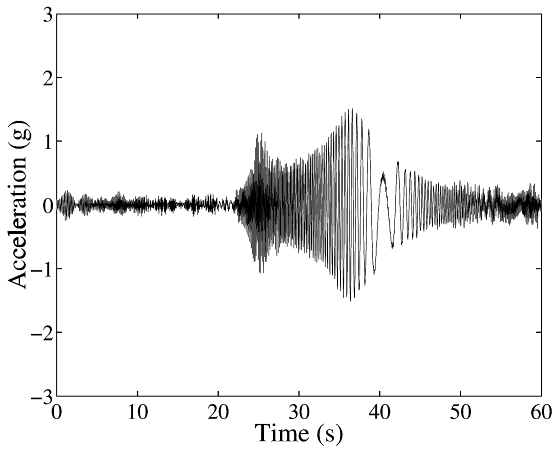

Fig. 5 Simulated response to chirp input from first-order Volterra kernel.

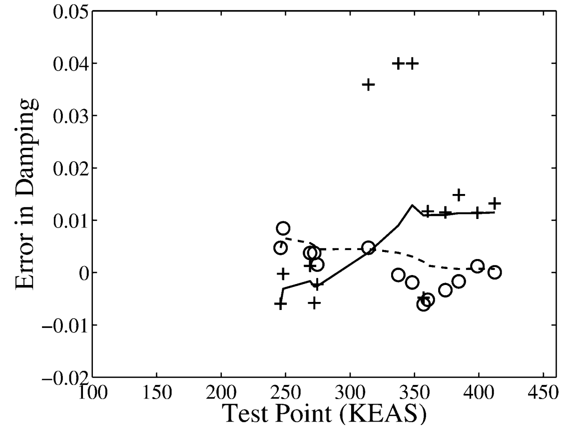

Fig. 6 Error, + and running average of the error, —— in modal damping of the theoretical model for bending mode and error, ○ and running average of the error, – – – for torsion mode.

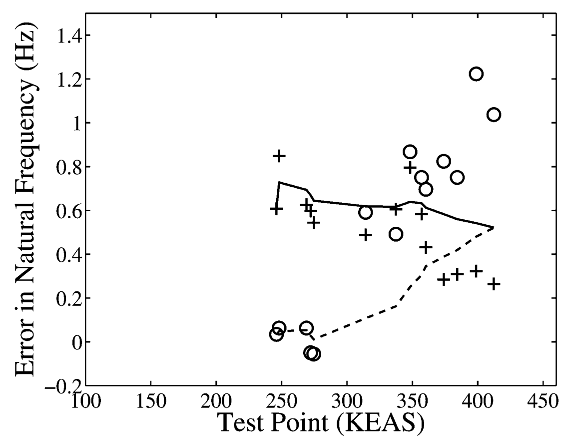

Fig. 7 Error, + and running average of the error, —— in natural frequency of the theoretical model for bending mode and error, ○ and running average of the error, – – – for torsion mode.

kernel was negligible, and so it is assumed that nonlinearities do not play a prevalent role in the dynamics and, thus, can be ignored for in the prediction of the onset of flutter.

Clearly, the data filtering has reduced a significant portion of the noise. Some of this noise may have actually resulted from nonlinearities that were not represented by the first-order kernel; however, the second-order kernel is negligible, and so nonlinearities can be ignored.

Model Updating

The filtered data were used for model updating. This updating altered the theoretical dynamics to match properties estimated from the data. Specifically, the damping and natural frequencies, along with observability, are updated for each mode.

The updates to the damping for each mode are of particular interest because damping is traditionally used to indicate flutter. The differences, and running average of the differences, between the theoretical and measured damping are given in Fig. 6 for each test point.

The errors in damping for the bending and torsion modes shown in Fig. 6 seem to show different trends. The error in damping for the bending mode is small at the low-speed test points but generally increases as the envelope is expanded. Conversely, the error in damping for the torsion mode is small at the low-speed test points and actually decreases to even smaller levels as the envelope is expanded. These trends suggest that the theoretical model of the torsion mode is more accurate, at least in terms of damping, than the theoretical model of the bending mode.

Such behavior is extremely important for flutter prediction. The onset of flutter results in an unstable bending mode that is characterized by zero damping. The magnitude of damping for the bending, as seen in Fig. 2, is decreasing as the envelope is expanded. Thus, the increasing error indicates that the theoretical model does not correctly predict the flight conditions at which flutter occurs and the bending mode becomes unstable.

Also, the running averages in Fig. 6 show the updates used to alter the theoretical model. The running average is seen to be an excellent indicator of the error in the torsion mode and a reasonable indicator of the error in the bending mode. In particular, the running average represents the error well for low- and high-speed test points and smooths out the effects of the inconsistently large errors at some of the test points.

The updates to the natural frequency for each mode are of related interest because natural frequency provides additional information that may be used to predict flutter. The differences between the theoretical and measured natural frequencies for each mode are shown in Fig. 7 along with the corresponding running averages.

The errors in natural frequency, like the errors in damping, show differing trends between the torsion and bending modes; however, the trends in Figs. 6 and 7 are actually opposite. Specifically, the bending mode shows an increasing error in damping and a decreasing error in natural frequency as the envelope is expanded. Conversely, the torsion mode shows a decreasing error in damping and an increasing error in natural frequency as the envelope is expanded. These trends are quite interesting and suggest frequency and damping were modeled with very different levels of accuracy.

The updates to observability of each mode are shown in Fig. 8. The differing trends between errors in bending and torsion dynamics is further exemplified by these results.

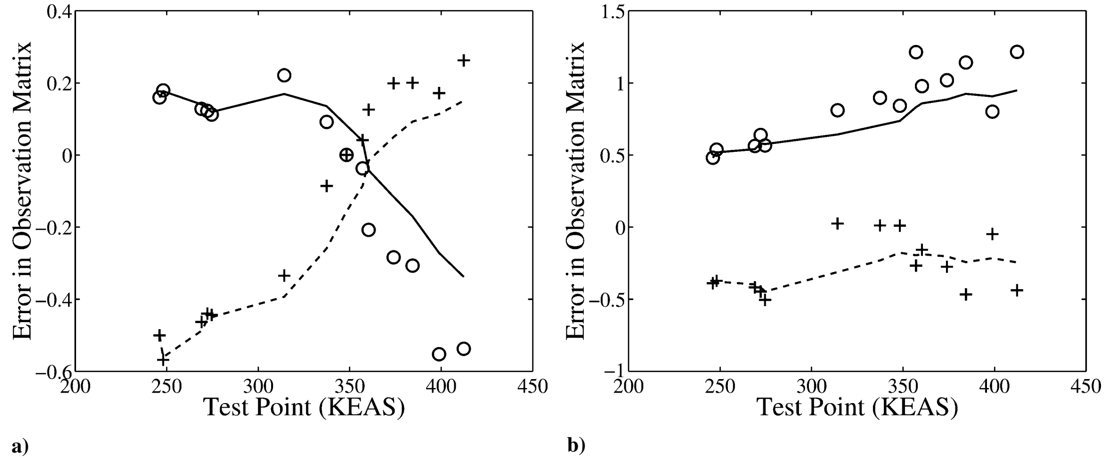

Fig. 8 Error, ○ and average error, —— in observation of first state and error, + and average error, – – – in observation of second state of the theoretical model for a) bending mode and b) torsion mode.

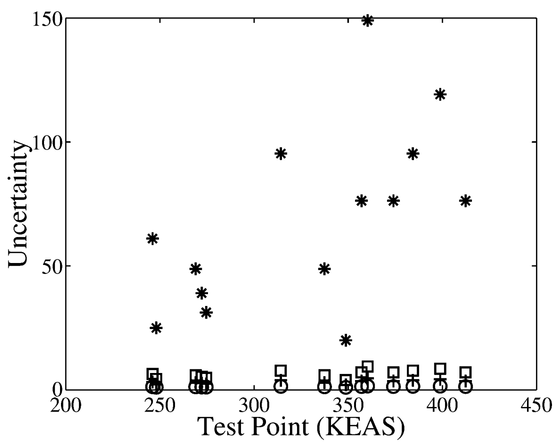

Fig. 9 Uncertainty in updated model for structural stiffness and damping, ○; aerodynamic forces, *; excitation, +; and measurement, □.

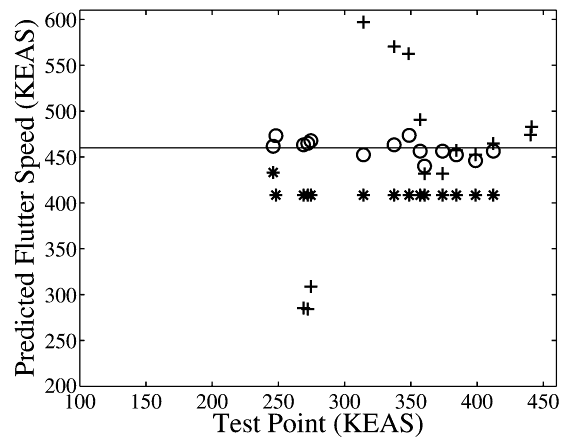

Fig. 10 Flutter speeds predicted by damping extrapolation, +; original flutterometer, *; and updated flutterometer, ○.

Figure 8 shows another significant difference between the modeling of the bending and torsion modes. The error in observability of the two states associated with the bending mode are changing signs. Conversely, the error in observability of the two states associated with the torsion mode remain the same sign. Such a difference in sign for the bending mode implies a change of phase.

Note the updates to observability. The observability is directly related to mode shape and, consequently, the flutter mechanism. The data in Fig. 8 indicate that the theoretical predictions of mode shape are distinctly different than the data indicate. Such an error is probably the reason that computational analysis was unable to predict the onset of flutter to within 50 KEAS. The ATW experienced a phase change in the aeroelastic dynamics that was not predicted by preflight modeling.

Uncertainty Estimation

The theoretical model is generated with an uncertainty description. This description allows for variation in the parameters of the structural stiffness and structural damping matrices. Also, the description includes terms for magnitude and phase variations of the aerodynamic forces. Finally, the uncertainty description accounts for errors in magnitude and phase of the excitation energy and data measurements due to unmeasured quantities such as wind gusts and noise.

The uncertainty levels required to ensure the updated models are not invalidated by the flight data are shown in Fig. 9. These uncertainty levels are computed automatically by the flutterometer. The range of transfer functions from the model with these levels of uncertainty are sufficient to bound the transfer functions of the flight data.

The maximum amount of uncertainty needed at any test point was 9.5% for the structural stiffness and damping, 149% for the aerodynamic forces, 4.7% for the excitation, and 1.5% for the measurement.[16] The updated models did not require much uncertainty for most parameters; however, the uncertainty associated with the aerodynamic forces was quite large. This situation was acceptable because the flutter speeds were considerably more sensitive to structural parameters than to the variations in aerodynamic forces. Thus, these uncertainty levels do not correspond to excessive conservatism in the model.

Note that these uncertainty levels correspond to the model with updating of the average perturbations. If the models are updated with the exact perturbation needed to match theoretical and experimental transfer functions, then no uncertainty is needed for the model, but the results would be accepted with less confidence. As stated earlier, the averaging approach for model updating reduces local effects at a single test and, thus, increases confidence in the results by identifying errors trends at many test points.

Flutter Prediction

Flutter speeds are predicted for the ATW with several techniques. One set of predictions results from standard extrapolation of damping parameters. Another set of predictions results from the original implementation of the flutterometer that only updates uncertainty descriptions. The final set of predictions results from the new implementation of the flutterometer that uses Volterra modeling of the data and includes modal parameter estimation. These predictions are shown in Fig. 10.

The predictions from the new implementation of the flutterometer are clearly superior to either of the other sets of predictions. Specifically, the flutterometer now estimates a flutter speed that is within 10 KEAS of the true flutter speed when flight data from any test point of the envelope expansion are used.

The flutterometer with model updating has clearly removed most of the conservatism in the predictions that were associated with the original flutterometer. The predictions are now able to account for the flight data by estimation of parameters for the model for which less uncertainty is required to validate the flight data. The result of this updating is a flutterometer that computes accurate predictions of flutter speed.

Conversely, the flutterometer with model updating has not removed too much conservatism or caused the predictions to be overly optimistic. The predictions in Fig. 10 show that the predictions remain very close to the true flutter speed. The inclusion of model updating has increased the accuracy of the flutterometer without sacrificing the safety provided by considering a worst-case flutter mechanism.

The extensions to the flutterometer presented in this paper make the tool especially valuable in comparison to the traditional method of extrapolation of damping trends. The predictions from damping are seen in Fig. 10 to be widely scattered and not very accurate until the envelope expansion has reached test points above 350 KEAS. The flutterometer now accurately predicts the flutter speed at both low- and high-speed test points.

Conclusions

This paper has introduced an approach to improve the accuracy of the flutter speeds predicted by the flutterometer. Such accuracy results from augmention of the tool to include a scheme for model updating based on modal parameter estimation by the use of Volterra kernels. The updates are optimal in the sense that they reflect consistent errors between the theoretical dynamics and the linear dynamics associated with the first-order Volterra kernels that represent the flight data. Flutter speeds are computed for the ATW by the use of the new version of the flutterometer. The model updates are able to correct a phase shift that was not predicted by computational analysis. Consequently, the flutterometer predicted highly accurate flutter speeds.

References

[1]Lind, R., and Brenner, M., "Flutterometer: An On-Line Tool to Predict Robust Flutter Margins," *Journal of Aircraft*, Vol. 37, No. 6, 2000, pp. 1105–1112.

[2]Lind, R., and Brenner, M., *Robust Aeroservoelastic Stability Analysis*, Springer-Verlag, London, 1999, pp. 111–116.

[3]Lind, R., "Flight Testing with the Flutterometer," *Journal of Aircraft*, Vol. 40, No. 3, 2003, pp. 574–579.

[4]Borglund, D., "Robust Aeroelastic Stability Analysis Considering Frequency-Domain Aerodynamic Uncertainty," *Journal of Aircraft*, Vol. 40, No. 1, 2003, pp. 189–193.

[5]Beuker, B., Bennani, S., van Stavern, J. W. and Meijer, J. J., "Robust Flutter Analysis of an F-16A/B in Heavy Store Configuration," AAIA Paper 2002-3967, Aug. 2002.

[6]Brenner, M.J., "Aeroservoelastic Model Uncertainty Bound Estimation from Flight Data," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 4, 2002, pp. 748–754.

[7]Prazenica, R. J., Lind, R., and Kurdila, A. J., "Uncertainty Estimation from Volterra Kernels for Robust Flutter Analysis," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 2, 2003, pp. 331–339.

[8]Ljung, L., *System Identification*, Prentice–Hall, Englewood Cliffs, NJ, 1987, pp. 169–207.

[9]Iliff, K. W., "Parameter Estimation for Flight Vehicles," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 5, 1989, pp. 609–622.

[10]Kailath, T., *Linear Systems*, Prentice–Hall, Englewood Cliffs, NJ, 1980, pp. 49–58.

[11]Rugh, W. J., *Nonlinear System Theory: The Volterra-Wiener Approach*, Wiley, New York, 1980, pp. 50–90.

[12]Schetzen, M., *The Volterra and Wiener Theories of Nonlinear Systems*, Wiley, New York, 1980, pp. 11–49.

[13]Kurdila, A. J., Prazenica, R. J., Rediniotis, O., and Strganac, T., "Multiresolution Methods for Reduced-Order Models for Dynamical Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 2, 2001, pp. 193–200.

[14]Marzocca, P., Librescu, L., and Silva, W. A., "Aeroelastic Response of Nonlinear Wing Sections Using a Functional Series Technique," *AIAA Journal*, Vol. 40, No. 5, 2002, pp. 813–822.

[15]Marzocca, P., Librescu, L., and Silva, W. A., "Volterra Series Approach for Nonlinear Aeroelastic Response of 2-D Lifting Surfaces," AAIA Paper 2001-2459, April 2001.

[16]Lind, R., Voracek, D. F., Truax, R., Doyle, T., Potter, S., and Brenner, M., "A Flight Test to Demonstrate Flutter and Evaluate the Flutterometer," *Aeronautical Journal*, Vol. 107, No. 1076, 2003, pp. 577–588.