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Linear Parameter-Varying Modeling and Control of Structural Dynamics with Aerothermoelastic Effects

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Structural dynamics can introduce low-frequency modes that must be considered for modeling and control of flexible aircraft. Furthermore, the modal dynamics can be affected by operating conditions such as temperature. A linear parameter-varying (LPV) framework is used to represent structural dynamics. The framework represents the dynamics with dependency on operating parameters as a set of state-space matrices that are affine functions of those parameters. Controllers can then be formulated as state-space matrices that are also affine functions of the parameters. The controller, thus, changes with the operating parameters so that it is inherently gain scheduled. Such a controller is beneficial compared to traditional controllers because of the guaranteed closed-loop properties over time-varying trajectories of the operating conditions. The LPV framework is used to account for the aerothermoelastic effects on the structural dynamics of a generic hypersonic vehicle. A controller that is gain scheduled over temperature is synthesized to damp actively the modal dynamics that vary across a flight profile.

Nomenclature

A = state matrix
B = input matrix
C = output matrix
D = feedthrough matrix
d = disturbance
e = error
K = controller
P = plant model
T = temperature
u = command
W_c = mode shape weighting
W_k = control weighting
W_n = noise weighting
W_p = performance weighting
x = state
y = measurement
$\boldsymbol{\theta}$ = parameter vector
μ = structured singular value

Introduction

AEROSERVOELASTICITY study considers the interaction between aerodynamic, inertial, structural, actuation, and control system dynamics.[1] Flight controllers are usually designed for a rigid-body model, and any aeroservoelastic issues are eliminated by including notch filters to eliminate observability of structural modes. This approach may not be acceptable for future aircraft that have lightweight and flexible components with low natural frequencies. Thus, modeling and control of both rigid-body and structural dynamics must be used.

Structural dynamics has been extensively studied for modeling and control at transonic and high dynamic pressure regimes for limited applications such as flutter and buffet suppression; however, there are several aircraft that must consider aeroservoelastic dynamics within a standard flight envelope. The first-bending mode of the SR-71 fuselage has a natural frequency near 3 Hz that is easily excited by pilot maneuvers.[2] Several uninhabited aerial vehicles such as Theseus and APEX, with lightweight and low-aspect-ratio wings, have several structural modes less than 2 Hz that affect flight characteristics.[3] Also, the proposed high-speed civil transport anticipates a fuselage bending mode near 1.5 Hz that must be actively controlled to attain acceptable ride quality.[4]

Modeling and control of aeroservoelasticity must also be considered for a proposed class of hypersonic aerospacecraft. These vehicles are being investigated for economic competitiveness of access to space missions such as payload delivery to low Earth orbit, reconnaissance, and cruise flight. The main aeroservoelastic feature is a coupling between the wedge-shape body and an airbreathing propulsion system that causes aerodynamic, inertial, structural, control, and even thermal dynamics to interact.

Aeropropulsive and aeroelastic models and associated controllers were extensively developed for several models that generally describe the proposed aerospacecraft.[5] The strong interactions between the coupled dynamics of these models present many challenges for control design to achieve acceptable closed-loop properties.[6] These interactions are a direct result of the scramjet engine, which essentially uses the fuselage as part of the propulsion system. Thus, the aerodynamics and elastic dynamics can affect and respond to the propulsive dynamics. Several control synthesis methodologies have been considered for hypersonic models, including classical control,[7] $\mathcal{H}_\infty$ (Ref. 8), μ (Ref. 9), and a linear parameter-varying (LPV) approach.[10]

The effects of aerothermoelasticity were studied for a hypersonic vehicle known as the National Aerospace Plane (NASP) and shown to affect significantly the open-loop dynamics; however, these effects were generally not considered for controller synthesis. A computational analysis was performed to show a large range of surface temperatures that can be reached during a typical ascent profile.[11] Correspondingly, the structural dynamics show large variations in modal parameters and mode shapes as a result of temperature-induced stiffness changes.

These aerothermoelastic effects were considered in designing controllers for active flutter suppression and ride quality augmentation.[12] The controller synthesis utilized a classical linear quadratic regulator design that directly used information from only the worst-case temperature model with lowest structural damping. The resulting controller was applied over the entire ascent and assumed to be sufficient for any temperature level.

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This paper considers the modeling and control of structural dynamics using a LPV framework. The LPV framework is used to represent systems whose dynamics depend on a set of norm-bounded and time-varying operating parameters. Specifically, the LPV framework considers linear systems with state-space matrices that are affine functions of the operating parameters. One of the benefits to using the LPV framework is the simplicity of representing the dynamics as a single model. Another benefit is the ability to design controllers that include that same affine dependency on the operating parameters. In essence, the dependency ensures a gain-scheduled controller is synthesized that inherently accounts for the time-varying nature of the operating parameters.

The LPV nature of the structural dynamics can be used to account for several types of in-flight variations; however, this paper will particularly focus on modeling and control of aerothermoelastic effects for hypersonic vehicles. The aerothermoelastic effects are described by variations in modal parameters such as natural frequency and damping. The structural model accounts for these variations by formulating the state-space matrices as affine functions of the temperature. An active structural controller is then formulated that is gain scheduled over the temperature.

Note that this paper only considers modeling and control of the effects of aerothermoelasticity, but does not consider the actual computation of aerothermoelastic dynamics. The study of aerothermoelasticity is a complex field and is well beyond the intended scope here. This paper utilizes a set of effects that have been noted in previous aerothermoelasticity research and assumes those effects are generally representative of the variations that may be noted in hypersonic flight.

LPV Systems

Open-Loop Modeling

Aircraft dynamics are typically derived as nonlinear functions of physical parameters such that a single linearized model cannot predict the behavior over a range of flight conditions. For example, rigid-body dynamics are commonly written in terms of stability derivatives that vary with Mach and angle of attack. Traditionally, linear models are computed at distinct flight conditions, and so a single model is unable to represent accurately the dynamics across a flight envelope.

The concept of LPV systems has been developed as a convenient framework to describe a special class of nonlinear systems.[13] This class of systems can be realized as a set of state-space systems whose elements are affine functions of a set of scalar parameters. The general form of a representative system can thus be written using state vector $x \in \mathcal{R}^{n_s}$, exogenous inputs $d \in \mathcal{R}^{n_d}$, controlled input $u \in \mathcal{R}^{n_i}$, regulated outputs $e \in \mathcal{R}^{n_e}$, and measurements $y \in \mathcal{R}^{n_0}$:

$$\dot{x} = A(\boldsymbol{\theta})x + B_1(\boldsymbol{\theta})d + B_2(\boldsymbol{\theta})u$$

$$e = C_1(\boldsymbol{\theta})x + D_{11}(\boldsymbol{\theta})d + D_{12}(\boldsymbol{\theta})u$$

$$y = C_2(\boldsymbol{\theta})x + D_{21}(\boldsymbol{\theta})d + D_{22}(\boldsymbol{\theta})u$$

Such a system is called a parameter-dependent model because the state-space matrices are written directly as functions of the parameter vector $\boldsymbol{\theta} \in \mathcal{R}^{n_\theta}$. Specifically, this paper will consider systems for which the state-space matrices can be written as affine functions of θ. For example, the state matrix $A(\theta) \in \mathcal{R}^{n_s \times n_s}$ can be explicitly written as an affine function of $\boldsymbol{\theta} = \{\theta_1, \ldots, \theta_{n_\theta}\} \in \mathcal{R}^{n_\theta}$ using matrices $A_0, \ldots, A_{n_\theta} \in \mathcal{R}^{n_s \times n_s}$:

$$A(\theta) = A_0 + \boldsymbol{\theta}_1 A_1 + \cdots + \boldsymbol{\theta}_{n_\theta} A_{n_\theta}$$

The general state-space model and the parameter dependency can be separated when describing the LPV system as a block diagram using linear fractional transformation operations.[14] Additional signals, z and w, are introduced to relate the new plant and parameter operator so that the elements of the new plant do not contain a $\boldsymbol{\theta}$ dependence:

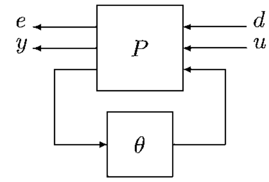

Fig. 1 Open-loop LPV model.

$$\dot{x} = \hat{A}x + \hat{B}_1 d + \hat{B}_2 u + \hat{B}_\theta w$$

$$e = \hat{C}_1 x + \hat{D}_{11} d + \hat{D}_{12} u + \hat{D}_{1\theta} w$$

$$y = \hat{C}_2 x + \hat{D}_{21} d + \hat{D}_{22} u + \hat{D}_{2\theta} w$$

$$z = \hat{C}_\theta + \hat{D}_{\theta 1} d + \hat{D}_{\theta 2} u + \hat{D}_{\theta\theta} w$$

This separation results in a linear time-invariant state-space plant P and a parameter operator $\boldsymbol{\theta}$, associated through the feedback relationship as in Fig. 1. The notation for such a relationship is given by $P(\boldsymbol{\theta}) = F_l(P, \boldsymbol{\theta})$ where the subscript l denotes that the lower loop of P is closed by $\boldsymbol{\theta}$.

LPV models are particularly convenient for representing nonlinear systems with time-varying parameters. The time-varying nature of the dynamics does not violate the nature of the LPV model in that the baseline plant P is still linear and time invariant, and the feedback parameter $\boldsymbol{\theta}(t)$ is the time-varying element.

Aircraft models can sometimes be represented in this LPV framework by noting the dependence of the model on flight condition. Notably, the stability derivatives of the rigid-body dynamics are functions of flight conditions such as Mach and angle of attack, and these flight conditions are changing with time during a maneuver. Thus, the LPV model indicates the structure of the equations does not change with flight condition; rather, the values of the coefficients change as the flight conditions change in time. This approach has been utilized for modeling and control of several aircraft, including the F-14 (Ref. 15) and F-16 (Ref. 16).

Controller Synthesis

The objective of control design for an LPV model is defined as stabilizing the closed-loop system and limiting the size of the regulated outputs in response to exogenous inputs throughout any bounded time-varying parameter trajectory. This concept of performance is determined by bounding the induced $\mathcal{L}_2$ norm of the closed-loop system and is loosely referred to as the $\mathcal{H}_\infty$ norm:

$$\|F_l[P(\boldsymbol{\theta}), K]\|_\infty \doteq \sup_{\theta(t)} \sup_{d \in \mathcal{L}_2} \frac{\|e\|_2}{\|d\|_2} < \gamma$$

Several methods of controller synthesis have been utilized to achieve this control objective. The classical approach of gain scheduling is to design controllers for the plants evaluated at distinct values of $\boldsymbol{\theta}$, and then formulate an interpolation law between these controllers. This approach has often worked well in practice; however, there is no mathematical guarantee of stability or performance along any time-varying parameter trajectory. A robust approach is to design a single controller such that the closed-loop system is robust to any value of $\boldsymbol{\theta}$ within some anticipated set of values. This approach gives guarantees of both stability and performance; however, it may be overly conservative to find such a controller when the values of $\boldsymbol{\theta}$ can have large variations.

A method of control design for LPV systems has been developed that takes advantage of the affine nature of the parameter dependency and assumes those parameters can be measured in real time. This method uses the parameter-varying nature of the plant to suggest an identical parameter-varying nature of the controller. Thus, the controller can be realized with the $\boldsymbol{\theta}$ dependency by introducing the states $x_K \in \mathcal{R}^{n_K}$:

$$\dot{x}_K = A_K(\boldsymbol{\theta})\dot{x}_K + B_K(\boldsymbol{\theta})y$$

$$u = C_K(\boldsymbol{\theta})x_K + D_K(\boldsymbol{\theta})y$$

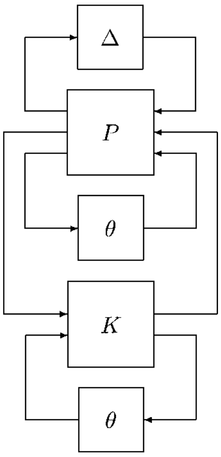

Fig. 2 Closed-loop LPV model.

The state-space matrices of this controller can be written as an affine combination of matrices multiplied by the elements of $\boldsymbol{\theta}$. Like the plant, the controller can be realized as a feedback relationship between a linear, time-invariant K, and a time-varying $\boldsymbol{\theta}$.

The concept of stability is a well-defined concept for such a closed-loop system involving a pair of LPV plants. Notably, stability of general time-varying systems is determined by solutions of Riccati differential equations, whereas the LPV systems can be analyzed using small-gain arguments.[17]

Some level of robustness is determined for the LPV system by associating uncertainties and weightings with the open-loop plant using standard linear fractional operations.[14] The regulated outputs and exogenous inputs are related through an operator Δ that is norm bounded such that all uncertainties are described by $\Delta \in \boldsymbol{\Delta}$ with $\|\boldsymbol{\Delta}\|_\infty < 1$. Thus, the robust performance criterion is a closed-loop $\mathcal{H}_\infty$ norm less than 1.

The resulting closed-loop system with an LPV open-loop plant and an LPV controller is shown in Fig. 2.

The LPV controllers are advantageous over traditional controllers because several properties of the closed-loop system can be guaranteed.[18] One property is that the closed-loop system is stabilized for any value of θ and also for any time-varying trajectory of $\boldsymbol{\theta}$. This is naturally a significant benefit over traditional controllers that guarantee stability only at a set of distinct time-invariant values of $\boldsymbol{\theta}$. Another property is that the closed-loop system satisfies an $\mathcal{H}_\infty$-norm bound on the worst-case gain from disturbances to errors for any θ. Again, traditional controllers can only attempt to provide this guarantee at the distinct values of θ. Thus, this automatic gain-scheduling design is a significant improvement over traditional gain-scheduling approaches.[19]

Controller synthesis can be accomplished by considering a set of linear matrix inequalities (LMI) that may be solved using standard convex optimization algorithms.[20] Essentially, the existence of a stabilizing controller that satisfies a closed-loop performance level is equivalent to a feasibility test based on the bounded real lemma. Similarly, the solutions to a set of LMI expressions result in the LPV controller realization.

Hypersonic Vehicles

Open-Loop Characteristics

The basic configuration of hypersonic vehicle that will be considered is similar to the proposed NASP and X-30 vehicles whose generalization is shown in Fig. 3.

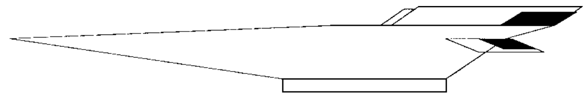

Fig. 3 Simplified model of a generic hypersonic vehicle.

The main characteristic of this vehicle affecting the dynamics is the integrated fuselage and propulsion system. The fuselage is actually designed to be part of the engine system by using the forebody as a compressor and the aftbody as an external nozzle. This design introduces a significant amount of coupling between the aerodynamics and propulsion dynamics. First, the airflow through the compressor introduces a lift force and a nose-up pitch moment, whereas the airflow through the external nozzle introduces a lift force and a nose-down pitch moment, so that variations in propulsion performance alter the aerodynamic characteristics. Conversely, any variation in angle of attack and sideslip affects the engine inlet conditions so that the propulsion performance is altered by variations in aerodynamic characteristics. Also, change in pitch angle results in a change in thrust angle, so that there is an especially strong and fast coupling between pitch and propulsion.

The vehicle can be controlled by commanded responses from the control surfaces and engine. The control surfaces include elevons for longitudinal control and rudders for lateral-directional control, whereas the engine variables include diffuser area ratio and fuel flow rate. The coupling between the aerodynamics and propulsion system introduces some redundancy among control effectors that can be exploited for control design.

A typical mission for this vehicle is to place some payload into low Earth orbit, which requires the vehicle to operate in many flight regimes such as subsonic, transonic, supersonic, hypersonic, and orbital. Each regime introduces control problems that must be alleviated for a successful mission. For example, the control surfaces will probably be small, to minimize heating during hypersonic flight, but this may create difficulties for properly controlling the vehicle at low supersonic speeds. Another potential control problem may arise from the shocks generated by unsteady aerodynamics at transonic flight. Also, the issue of orbit transfers for payload delivery while in space is a control problem for this type of vehicle that introduces issues not usually affecting atmospheric flight.

The control problems in every flight regime are important; however, this paper will limit consideration to the hypersonic regime while the vehicle is still in the atmosphere. One reason for limiting consideration to this regime is simply to concentrate on a smaller set of problems so that a useful solution can be formulated in a short time. Another valid reason to restrict attention to this limited flight regime is because it avoids the issue of choosing a single-stage or two-stage to orbit vehicle. Every airbreathing vehicle must pass through the atmospheric hypersonic regime regardless of whether a booster was used initially or a different onboard propulsion system placed the vehicle at low hypersonic speeds. Thus, this project will assume it is feasible to place the vehicle at hypersonic flight conditions and focus on the difficulties in this regime.

Aerothermoelasticity

The computation of aerothermoelastic effects on an aerospacecraft during hypersonic flight is extremely difficult; therefore, this paper concentrates only on the effects of aerothermoelasticity and not the exact computation. This is achieved by noting the variations in structural dynamics that are identified in previous studies and using those variations as representative results for a general vehicle. This paper demonstrates how to model the representative variations in a system model and compute an associated controller.

The representative effects of aerothermoelasticity are derived from several sources that used a general procedure for determining dynamic changes as a function of temperature.[21] First, the surface temperatures are approximated for various flight conditions. The material properties associated with these temperatures are then estimated and incorporated into the model. Finally, the structural characteristics of the system with temperature-affected materials are computed.

The main effects of temperature are introduced through variations in structural stiffness. These variations arise from material dependencies in parameters such as moduli and also dependencies of the internal stresses on temperature. Finite element analysis can be extended to incorporate these variations.[22]

The aerothermoelastic effects during a typical flight profile were studied for the NASP.[11] This study noted that the surface temperatures could range from cold to nearly 5000°F at certain points and result in large surface gradients. Consequently, the natural frequencies and dampings of the structural modes can vary significantly by up to 30%. These effects will be used as representative effects that may be encountered for the general class of vehicles considered in this paper.

LPV Models of Structural Dynamics

Rigid-body dynamics have received considerable attention for LPV modeling and control; however, structural dynamics can also be considered in this framework. There are several physical parameters with which the structural dynamics show variation, and often this variation can described by an LPV model. Furthermore, models of structural dynamics can be written as LPV systems using several state-space realizations to demonstrate a simple relationship between particular states and parameters.

An LPV model of the structural dynamics can be formulated by noting parameter dependencies of structural matrices. Consider the state matrix of a structural dynamics model that includes a mass matrix M, damping matrix C, stiffness matrix K, and a state-space quadruple, $\{A_Q, B_Q, C_Q, D_Q\}$, to represent the unsteady aerodynamic forces[1]:

$$A(\boldsymbol{\theta}) = \begin{bmatrix} 0 & I & 0 \\ -M^{-1}(K + \bar{q}D_Q) & -M^{-1}C & -\bar{q}M^{-1}C_Q \\ B_Q & 0 & A_Q \end{bmatrix}$$

The $\boldsymbol{\theta}$ dependence is not explicitly written into this generalized formulation of the state matrix, and so it must be developed. This dependency can be derived by analyzing the effects of parameter variations and noting which elements of the state matrix vary as a result of those effects. The LPV of $A(\boldsymbol{\theta})$ is then developed by extracting the variations and replacing them with the parameter $\boldsymbol{\theta}$.

The analysis of aerothermoelastic effects noted a variation in structural stiffness as the temperature varied throughout a flight profile.[11] This variation indicates a strategy for modeling the structural dynamics as an LPV system. Namely, the temperature is represented by the parameter $\boldsymbol{\theta}$, and the stiffness is explicitly modeled as a function of this parameter. Thus, the relationship $K = K_o + \boldsymbol{\theta} K_\theta$ is used to introduce an LPV nature to the state matrix:

$$A(\boldsymbol{\theta}) = A_0 + \theta A_\theta = \begin{bmatrix} 0 & I & 0 \\ -M^{-1}K_o & -M^{-1}C & 0 \\ B_Q & 0 & A_Q \end{bmatrix} + \boldsymbol{\theta} \begin{bmatrix} 0 & 0 & 0 \\ -M^{-1}K_\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The state matrix can also be formulated in a modal form such that information about the stiffness matrix is not explicitly available. An LPV model can be developed from this formulation by noting the relationship between stiffness and modal parameters. Specifically, the natural frequencies of the structural modes are strongly related to the stiffness, so that any decrease in stiffness will result in a corresponding decrease in natural frequencies. Similarly, the structural damping undergoes some variations with temperature, and this will result in a variation in modal damping.

Consider the state matrix of a single-mode system with different realizations. The poles of this matrix are computed as $r \pm i$, where these scalars are related to the natural frequency as $\omega = \sqrt{(r^2 + i^2)}$ and to the damping as $\zeta = -r/\omega$:

$$A(\boldsymbol{\theta}) = \begin{bmatrix} 0 & -1 \\ \omega^2 & -2\zeta\omega \end{bmatrix} = \begin{bmatrix} r & -i \\ i & r \end{bmatrix}$$

These realizations describe the same system; however, one form may be easier to use for LPV modeling. Variations in the modal parameters ω and ζ enter nonlinear because of the multiplication in these terms and, hence, are cumbersome to formulate as a linear fractional transformation. Instead, the realization using r and i elements does not have any nonlinearities arising from multiplications, so that an LPV model can be immediately formulated by noting variations of r and i with temperature. Of course, these parameters do not have a direct physical interpretation, but it is noted that for most structural modes the parameter i is the dominant factor in computing natural frequency. Thus, a parameter dependency can be noted by defining θ again as the temperature and describing the modal elements as $r = r_o + \boldsymbol{\theta} r_\theta$ and $i = i_o + \boldsymbol{\theta} i_\theta$:

$$A(\boldsymbol{\theta}) = A_0 + \boldsymbol{\theta} A_\theta = \begin{bmatrix} r_o & -i_o \\ i_o & r_o \end{bmatrix} + \boldsymbol{\theta} \begin{bmatrix} r_\theta & -i_\theta \\ i_\theta & r_\theta \end{bmatrix}$$

Effects of mode shape variations can also be modeled in the LPV framework. An effect of a mode shape variation is a variation in sensor measurements because of the different magnitude of response. Such variations in mode shape were noted in the aerothermoelastic analysis and so should be included for modeling and control of structural dynamics.[11] The sensor output matrices, C_1 and C_2, can be formulated as affine functions of $\boldsymbol{\theta}$ to account for mode shape dependencies; however, this paper will not consider variations in mode shape.

These LPV models that represent aerothermoelastic effects on stiffness or modal parameters are certainly not exhaustive. Actually, they may be overly simplistic and conservative in that perhaps there should be several parameters on which the model is dependent. The models could be scheduled over Mach and dynamic pressure, for instance, because certain flight regimes will automatically induce large temperature variations after extended flight time. The formulations in this paper merely describe a basic approach to modeling that accounts for aerothermoelastic effects that may be augmented for specific applications.

Also, the parameter variations in the models are noted as dependent on temperature. This dependency is used to describe aerothermoelastic effects; however, several other dependencies could be described in a similar manner. For example, the dynamic pressure variable can be extracted from the system in an LPV manner and used to analyze flutter margins.[23] Similarly, hypersonic aerospacecraft are anticipated to demonstrate significant variations in mass so that the modal form of the LPV model can be considered as dependent on mass instead of temperature.

Example

Flight Control System

A simplified model of a generic vehicle was developed for research into control of hypersonic vehicles.[24] The actual model is based on an overly simplified profile that is visualized with added control surfaces as in Fig. 3. The vehicle is assumed to have an internal scramjet engine integrated into the lower portion of the structure. Also, there is an elevon control surface near the rear of the vehicle. The dynamics of this simplified model are based on NASP concepts. Only longitudinal dynamics are modeled.

This vehicle captures several dynamic properties that are anticipated to be present in full-size hypersonic vehicles with scramjet engines. Most importantly, the fuselage is essentially part of the engine system with the forebody acting as a compressor and the aftbody acting as an external expansion nozzle. This structure introduces the dominant subsystem interactions that couple the aerodynamics and propulsive systems. The angle of attack of the vehicle has a significant effect on the engine inlet conditions and, thus, affects engine performance. Similarly, the airflow through the expansion nozzle introduces a lift force and a nose-down pitching moment, and, thus, it affects aerodynamic orientation.

This vehicle is also affected by an elastic mode in the fuselage. This mode is a bending mode that results in longitudinal flexibility in the fuselage. The entire system is composed of aerodynamics, and engine dynamics and structural dynamics are tightly coupled. Thus, the commands to the engine and elevons can affect the engine performance and aerodynamic characteristics, but these commands can also affect the structural dynamics.

A multiloop control architecture is proposed for a hypersonic vehicle that provides some level of both structure and robustness. This multiloop structure is closely related to the multielement structure in which the plant may be formulated; namely, separated gains based essentially on aerothermoelastic dynamics and rigid-body dynamics. The multiloop controller uses a set of inner-loop gains to achieve objectives associated with the aerothermoelastic dynamics and a set of outer-loop gains to achieve the remaining objectives.

The inner-loop controller is essentially an aerothermoelastic controller whose purpose is to augment actively structural damping in the aeroelastic modes. Thus, it can be viewed as a modal controller. It does not attempt to stabilize the rigid-body dynamics or achieve closed-loop levels of performance; rather, it is merely ensuring that the structural dynamics are highly damped and are not easily excited by the outer-loop controller. This controller will act to minimize structural vibration and eliminate any local angle-of-attack variations resulting from fuselage elasticity that could affect the propulsion system.

The outer-loop controller is essentially a rigid-body controller that resembles the standard flight controllers developed for rigid aircraft. This controller assumes that the inner-loop controller is active and that some desired set of modal parameters are associated with the structural dynamics. It works to stabilize the rigid-body dynamics and maximize the achievable performance in the presence of any modeling uncertainties.

This paper considers the design of only the inner-loop controller that actively affects the structural dynamics. The rigid-body performance in terms of handling qualities and tracking performance will not be evaluated here; however, the complete multiloop control system is presented in Ref. 25.

Open-Loop Dynamics

A reduced-order model was generated to represent this system as a state-space realization. There are five rigid-body states representing height, velocity, angle of attack, pitch angle, pitch rate, and two elastic states for the fuselage first-bending mode.

There are several sensors to provide measurements for evaluating performance metrics; however, the aerothermoelastic controller will only consider the angle of attack as a regulated variable. The angle-of-attack response is strongly dependent on the structural dynamics and provides sufficient information for control design and analysis.

The elevons will be the only commanded variable. The purpose of the controller is to damp actively the structural dynamics without overly effecting the engine and aerodynamic characteristics. The elevons are considered to be most appropriate for this task. The commands issued to engine parameters could also be considered; however, the control surface should have a high enough bandwidth to affect the structural modes, whereas it is doubtful the engine parameters could respond effectively in practice.

The equations of motion are derived for this vehicle using the integrated approach that couples aerodynamic, propulsive, inertial, and elastic forces. The values for the state-space matrices are computed for the model with the dimensions and flight conditions listed in Table 1.

Aerothermoelastic Effects

The open-loop dynamics of the vehicle are represented by the plant P such that $y = Pu$ determines the sensed variables y in response to the controller commands u. It is convenient to introduce a transformation such that the state matrix of P is bidiagonal. Thus, the system can be separated into the components P_r and P_e that loosely represent rigid-body and elastic dynamics. Of course, the rigid-body and elastic dynamics are coupled and cannot be truly separated; however, P_r is dominated by the aeropropulsive dynamics and P_e is dominated by the elasticpropulsive dynamics, and so it is a reasonable approximation to consider them rigid-body and elastic dynamics. The properties of the resulting models demonstrate that this approximation is indeed acceptable for the purposes of this paper:

Table 1 Model dimensions and flight conditions

Parameter	Value
Length	150 ft
Mass	500 slug/ft
Height	100 kft
Mach	8
Dynamic pressure	1017 psf

Table 2 Variation of modal parameters with temperature

T, °F	ω, Hz	ζ
0	2.8	0.026
5000	1.9	0.021

$$P = P_r + P_e = \left[\begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array}\right] + \left[\begin{array}{c|c} A_e & B_e \\ \hline C_e & 0 \end{array}\right]$$

Considering the system in this separated formulation allows the effects of aerothermoelasticity to be associated with a small part of the model. Specifically, the structural dynamics are assumed to be strongly affected by temperature changes, whereas the rigid-body dynamics are currently not modeled as dependent on temperature. Modeling the effects of aerothermoelastic variations on the vehicle can, thus, be reduced to modeling the effects on the P_e subsystem.

The aerothermoelastic effects were noted to cause a decrease in natural frequency and damping of the structural modes. This effect is incorporated by formulating the state matrix of P_e as an affine function of temperature. The range of temperatures considered for this model is chosen as $\boldsymbol{\theta} \in (0°\text{F}, 5000°\text{F})$ to match the range of temperatures noted for hypersonic flight for both the X-30 and HyperX vehicles. The model variations resulting from this temperature range are difficult to predict without a detailed material model; however, some assumptions can be made such that this model gives representative results rather then specific qualitative results. The assumed range of modal dynamics used for modeling and control is given in Table 2.

The state matrix of P_e is chosen to reflect these modal variations through an LPV relationship with θ:

$$A_e(\boldsymbol{\theta}) = \begin{bmatrix} -0.4726 & -17.6449 \\ 17.6449 & -0.4726 \end{bmatrix} + \boldsymbol{\theta} \begin{bmatrix} 4.25 \times 10^{-5} & 1.05 \times 10^{-3} \\ -1.05 \times 10^{-3} & 4.25 \times 10^{-5} \end{bmatrix}$$

Controller Synthesis

An LPV controller is synthesized to damp actively the aerothermoelastic modes for any value of surface temperature along an ascent trajectory. These gains are used as an inner-loop controller for a multi-loop compensator that is designed such that the closed-loop dynamics achieve a level of performance in following pilot commands.[25] The purpose of this inner-loop controller is only to damp actively the structural modes despite aerothermoelastic effects. Thus, this controller is evaluated by considering the closed-loop damping properties with no analysis of handling qualities or tracking performance.

An explicit model-following approach is used for the control design such that the closed-loop dynamics approximate a desired model. This desired model $\hat{P}$ is chosen as a highly damped structural mode with a natural frequency near the open-loop structural frequency for the cold model. Specifically, the desired dynamics have a natural frequency of $\omega = 2.6$ Hz and damping of $\zeta = 0.23$, as shown in Fig. 4.

There are two feedback measurements available to the controller that consist of the measured angle of attack and the desired angle of attack. The output of the controller is the commanded elevon surface deflection.

A performance weighting W_p is used to denote the frequency-varying spectrum of the upper bound on the allowable errors of the closed-loop system. This weighting is essentially a bandpass filter that requires small errors near the structural modal frequency to ensure good model following, but allows larger errors at low and high frequency. The magnitude of the weighting is the inverse of the maximum allowable errors. Thus, the weighting is chosen to have a low- and high-frequency magnitude of 1, which allows an error of 1 rad in this range, whereas the magnitude of 50 near 3 Hz demands that the error in angle of attack be less than 0.02 rad near

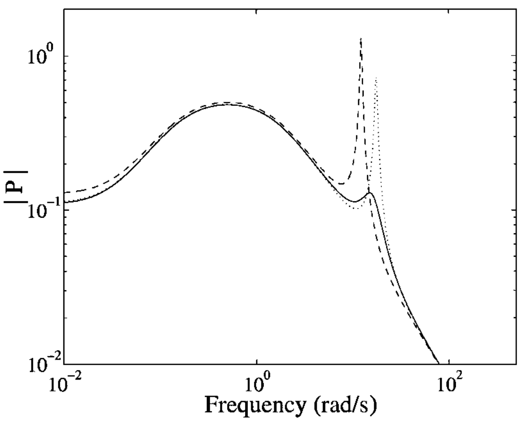

Fig. 4 Transfer functions of the hypersonic vehicle: · · · ·, cold temperature model; – – –, hot temperature model; and ——, desired model.

the structural mode:

$$W_p = \frac{(s+100)(s+0.01)}{(s+1)(s+1)}$$

A penalty weighting W_k must be associated with the control signal to limit the magnitude and rates of the elevon commands. This weighting is essentially a bandstop filter that allows control action near the structural frequency but heavily penalizes any control action at low and high frequency:

$$W_k = 100\frac{(s+25)(s+1)}{(s+1000)(s+0.03)}$$

The controller must robustly satisfy these performance objectives to limit model-following errors and elevon commands, despite modeling errors in the modal parameters of the structural dynamics. These errors are represented by real parametric uncertainty operators affecting the real and imaginary parts of the open-loop poles and correspondingly affecting the natural frequency and damping. These uncertainties are weighted to reflect upper bounds on the error magnitude based on previous error analysis for aeroelastic models.[23] Specifically, the controller must be robust to errors of 20% in the damping and 5% in the natural frequency. These errors are assumed to be representative of the models throughout the temperature ranges indicated by the parameter-varying dependency.

The controller must also be robust to modeling errors in the mode shape of the open-loop aerothermoelastic model. These errors are represented by a complex multiplicative uncertainty on the angle-of-attack measurement. The complex nature of this uncertainty allows uncertainty in both the magnitude and phase of this signal to represent accurately measurement variations that may be encountered during flight. A weighting function W_c is associated with this uncertainty to limit the allowable error as a function of frequency:

$$W_c = 0.05$$

The block diagram used for control synthesis is shown in Fig. 5. The LPV dependency of the open-loop model and the controller is shown by the feedback elements $\boldsymbol{\theta}$.

Closed-Loop Analysis

An LPV controller is synthesized for the inner-loop model using the LMI Control Toolbox.[26] This controller has 27 states and is scheduled over temperature. The frequency response of this controller can be indicated by evaluating the gains at specific values of temperature. This is shown in Fig. 6 for the cold temperature and in Fig. 7 for the hot value.

These transfer function plots demonstrate the controller gains have the desired frequency-varying shape for the inner loop. Specifically, the gain at low and high frequency is quite small, whereas the gain near the structural natural frequency is relatively high. This behavior ensures that the controller is only providing active damping for the structural dynamics and that the closed-loop dynamics are similar to the open-loop dynamics near the natural frequencies of the rigid-body modes.

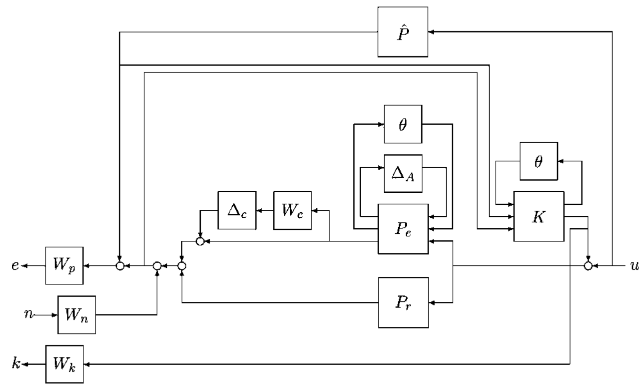

Fig. 5 Block diagram for control synthesis.

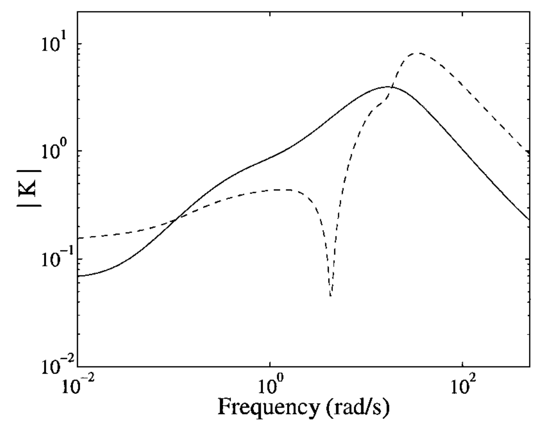

Fig. 6 Controller gains evaluated at cold temperature from elevon command to measured angle of attack (——) and desired angle of attack (– – –).

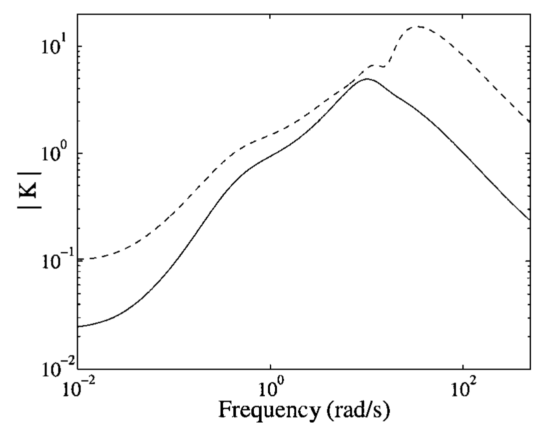

Fig. 7 Controller gains evaluated at hot temperature from elevon command to measured angle of attack (——) and desired angle of attack (– – –).

The ability of the controller to satisfy the linear performance and robustness objectives can be determined by the $\mathcal{H}_\infty$ norm on the closed-loop LPV system. The computed norm of 1.51 is a guaranteed bound on the robust performance that is achieved by the closed-loop dynamics for any value of the parameter trajectory; however, this bound may be overly conservative. The $\mathcal{H}_\infty$ norm does not account for the high degree of structure in the uncertainty description, nor for the real nature of the parametric uncertainty in Fig. 5.

The structured singular value μ can be used to analyze a measure of robust performance that directly accounts for structure in the uncertainty description. A parameter-varying measure of μ has not been formulated in current software packages, and so the robustness must be computed by analyzing the closed-loop dynamics at specific values of the temperature parameter. This approach will not compute robustness of the system with respect to the uncertainty and any time-varying trajectory of the temperature; however, it is a valuable

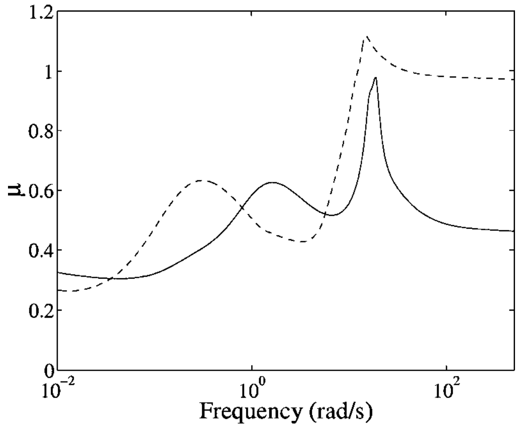

Fig. 8 For robust performance, μ of the closed-loop LPV system evaluated at specific temperatures: – – –, hot and ——, cold.

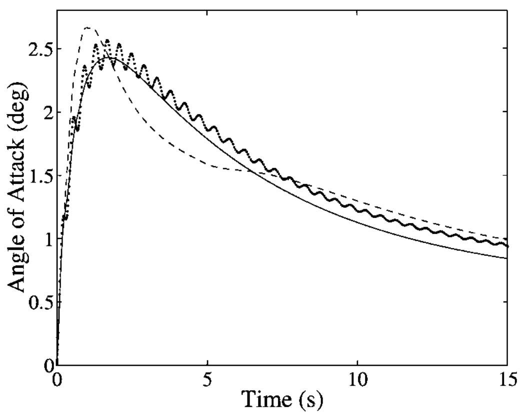

Fig. 9 Responses to a 0.1-rad elevon deflection: · · · ·, open loop; ——, desired closed loop; and – – –, simulated closed loop.

measure because the temperatures are not expected to vary rapidly during an ascent, and so the time-varying nature of this parameter may not be critical for analysis.

A frequency-varying upper bound for μ to analyze robust performance is shown in Fig. 8 This upper bound shows the worst-case robustness levels result from sensitivities near the structural modal frequencies. The cold system has $\mu = 0.95$, and so the controller achieves the desired robust performance goals. The hot system has $\mu = 1.1$, and so this controller is not quite able to achieve the desired robust performance goals. The higher μ values are expected for the hot system because the open-loop dynamics have lower damping than the closed system, so that more control action is required for model following, and, consequently, the performance goals are more difficult to achieve.

The angle-of-attack measurement in response to a command elevon deflection is shown in Fig. 9. The open-loop response clearly shows a large oscillatory component because the deflection excites the low damped structural mode. The closed-loop response does not show this oscillatory component because the controller actively damps the structural mode, and, thus, the elevon deflection does not strongly excite structural motion. Also, this response is computed along a parameter-varying trajectory with temperatures ranging from cold to hot in 10 s. This trajectory is unrealistically fast; however, it shows that the response is damped over the wide range of temperatures with arbitrarily fast time variation.

Conclusions

This paper considered the modeling and control of structural dynamics using the LPV framework. This framework is convenient for representing systems whose state-space matrices can be formulated as affine functions of a parameter vector. Furthermore, robust controllers can be synthesized that are inherently gain scheduled and provide a guaranteed level of stability and $\mathcal{H}_\infty$ performance over any time-varying trajectory of the parameter vector. The aerothermoelastic effects of a hypersonic vehicle were demonstrated as dynamic variations that could be represented in the LPV framework. An example model was utilized to show the modeling principle and generate controllers that were gain scheduled over temperature to provide active damping over a flight profile.

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