Mechanical properties of hybrid composites using finite element method based micromechanics

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A micromechanical analysis of the representative volume element of a unidirectional hybrid composite is performed using finite element method. The fibers are assumed to be circular and packed in a hexagonal array. The effects of volume fractions of the two different fibers used and also their relative locations within the unit cell are studied. Analytical results are obtained for all the elastic constants. Modified Halpin–Tsai equations are proposed for predicting the transverse and shear moduli of hybrid composites. Variability in mechanical properties due to different locations of the two fibers for the same volume fractions was studied. It is found that the variability in elastic constants and longitudinal strength properties was negligible. However, there was significant variability in the transverse strength properties. The results for hybrid composites are compared with single fiber composites.

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1. Introduction

Hybrid composites contain more than one type of fiber in a single matrix material. In principle, several different fiber types may be incorporated into a hybrid, but it is more likely that a combination of only two types of fibers would be most beneficial [1]. They have been developed as a logical sequel to conventional composites containing one fiber. Hybrid composites have unique features that can be used to meet various design requirements in a more economical way than conventional composites. This is because expensive fibers like graphite and boron can be partially replaced by less expensive fibers such as glass and Kevlar [2]. Some of the specific advantages of hybrid composites over conventional composites include balanced effective properties, reduced weight and/or cost, with improvement in fatigue and impact properties [1].

Experimental techniques can be employed to understand the effects of various fibers, their volume fractions and matrix properties in hybrid composites. However, these experiments require fabrication of various composites which are time consuming and cost prohibitive. Advances in computational micromechanics allow us to study the various hybrid systems by using finite element simulations and it is the goal of this paper.

Hybrid composites have been studied for more than 30 years. Numerous experimental works have been conducted to study the effect of hybridization on the effective properties of the composite [3–11]. The mechanical properties of hybrid short fiber composites can be evaluated using the rule of hybrid mixtures (RoHM) equation, which is widely used to predict the strength and modulus of hybrid composites [3]. It is shown however, that RoHM works best for longitudinal modulus of the hybrid composites. Since, elastic constants of a composite are volume averaged over the constituent microphases, the overall stiffness for a given fiber volume fraction is not affected much by the variability in fiber location. The strength values on the other hand are not only functions of strength of the constituents; they are also very much dependent on the fiber/matrix interaction and interface quality. In tensile test, any minor (microscopic) imperfection on the specimen may lead to stress build-up and failure could not be predicted directly by RoHM equations [12].

The computational model presented in this paper considers random fiber location inside a representative volume element for a given volume fraction ratio of fibers, in this case, carbon and glass. The variability in fiber location seems to have considerable effect on the transverse strength of the hybrid composites. For the transverse stiffness and shear moduli, a semi-empirical relation similar to Halpin–Tsai equations has been derived. Direct Micromechanics Method (DMM) is used for predicting strength, which is based on first element failure method; although conservative, it provides a good estimate for failure initiation [13].

1.1. Model for hybrid composite

The fiber orientation depends on processing conditions and may vary from random in-plane and partially aligned to approximately...
uniaxial [1]. The fiber packing arrangement, for most composites, is random in nature, so that the properties are approximately same in any direction perpendicular to the fiber (i.e. properties along the 2-direction and 3-direction are same, and is invariant with rotations about the 1-axis), resulting in transverse isotropy [14]. For this paper, it is assumed that the fibers are arranged in a hexagonal pattern and the epoxy matrix fills up the remaining space in the representative volume element (RVE). Hexagonal pattern was selected because it can more accurately represent transverse isotropy as compared to a square arrangement. The RVE consists of 50 fibers. Multiple fibers were selected to allow randomization of fiber location. Hybrid composites are created by varying the number of fibers of carbon and glass to obtain hybrid composites of different volume fractions.

A cross section of a hybrid composite of polypropylene reinforced with short glass and carbon fibers is shown in Fig. 1 [3]. The black circles represent glass fibers ($V_g = 6.25\%$) and the white circles represent carbon fibers ($V_c = 18.75\%$). In order to represent such an arrangement, we consider the schematic of the RVE as shown in Fig. 2. Green and red represent two different fiber materials, while the matrix is shown in white. Also, it is assumed that the radii of the fibers are the same and only the count of carbon and glass fibers vary. This gives us much more flexibility in creating the finite element mesh. Although, this RVE architecture is a lot simplistic and entails some basic assumptions like same size and location of the fibers and absence of voids but there is still a lot to earn from the parameters that have been used.

The properties of the composite are independent of the 1-direction, hence a 2D analysis is performed. We have assumed here that the fibers remain unidirectional with no fiber undulation and waviness. An overall fiber volume fraction of 60\% is assumed for all the composites analyzed in this paper. The proportions of the reinforcements have been varied to obtain five hybrid composites, keeping the total volume fraction of reinforcement phases constant. The volume fraction of any particular reinforcement, say A, was determined by the relation

$$V_{fA} = 0.6 \frac{N_A}{N_T} \quad (1)$$

where $N$ is the number of fibers of reinforcement A and $N_T$ is the total number of fibers. (see Table 1).

2. Analysis for elastic constants

The RVE of the composite is analyzed using commercially available finite element software (ABAQUS/CAE 6.9-2). The composite is assumed to be under a state of uniform strain at the macroscopic level called macroscale strains or macrostrains, and the corresponding stresses are called macrostresses. However, the microstresses, which are the actual stresses inside the RVE can have a spatial variation. The macrostresses are average stress required to produce a given state of macro-deformations, and they can be computed using finite element method. The macrostresses and macrostrain follow the relation

$$\{\sigma^M\} = [C] \{\varepsilon^M\} \quad (2)$$

where $[C]$ is the elastic constant of the homogenized composite, also known as the stiffness matrix. In this method, the RVE is subjected to six independent macrostrains. For each applied non-zero macrostress, it is also subjected to periodic boundary conditions such that all other macrostrains are zero. The six cases are: Case 1: $\gamma_{11}^M = 1$; Case 2: $\gamma_{12}^M = 1$; Case 3: $\gamma_{13}^M = 1$; Case 4: $\gamma_{13}^M = 1$; Case 5: $\gamma_{15}^M = 1$; Case 6: $\gamma_{13}^M = 1$ [15], where the subscripts 1, 2, 3 are parallel to the material principal directions, as shown in Fig. 3, and the superscript M stands for macrostress or macrostrain.

2.1. Finite element analysis

For case 1, 2 and 4, a mixture of three and four-node plane strain elements, CPE3/CPE4 and for case 3, a mixture of three and four node generalized plane strain elements, CPEG3/CPEG4 were used. For cases 5 and 6 (longitudinal shear), three and four node shell elements were used, because out of plane displacements have to be applied for this case. Periodic boundary conditions (PBC) were applied on opposite faces of the RVE which are described in Table 2. Appropriate constraints on the RVE depend on the loading condition and have been determined by symmetry and periodicity conditions in [16]. For each strain case, six macrostresses were calculated, three normal and three shear stresses in the 1-2-3 directions, in each element in the finite element model and volume averaged to find the macrostress for the RVE. The finite element model used is shown in Fig. 3, which contains 27,000 elements. The $[C]$ matrix can be inverted to obtain the compliance matrix or $[S]$ matrix, from which the elastic constants can be computed using the following relation

$$[C]^{-1} = [S] = \begin{bmatrix}
\frac{1}{E_1} & \frac{-v_{12}}{E_2} & \frac{-v_{13}}{E_3} \\
\frac{-v_{12}}{E_2} & \frac{1}{E_2} & \frac{-v_{23}}{E_3} \\
\frac{-v_{13}}{E_3} & \frac{-v_{23}}{E_3} & \frac{1}{E_3}
\end{bmatrix} \begin{bmatrix}
0 & 0 & 0 \\
0 & \frac{1}{G_{12}} & 0 \\
0 & 0 & \frac{1}{G_{13}}
\end{bmatrix} \begin{bmatrix}
0 & 0 & 0 \\
0 & \frac{1}{G_{12}} & 0 \\
0 & 0 & \frac{1}{G_{13}}
\end{bmatrix} \begin{bmatrix}
0 & 0 & 0 \\
0 & \frac{1}{G_{12}} & 0 \\
0 & 0 & \frac{1}{G_{13}}
\end{bmatrix}
\quad (3)$$

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Specimen & $V_g$ & $V_c$ \\
\hline
H1 & 0.54 & 0.06 & 0.6 \\
H2 & 0.42 & 0.18 & 0.6 \\
H3 & 0.3 & 0.3 & 0.6 \\
H4 & 0.18 & 0.42 & 0.6 \\
H5 & 0.06 & 0.54 & 0.6 \\
\hline
\end{tabular}
\caption{Specimen numbering for the Hybrid Composites.}
\end{table}

Fig. 1. Cross sectional area of a composite with $V_g = 18.75\%$ and $V_c = 6.25\%$ [3].
Effect of hybridization. As will it has to follow the relation

\[ G_{13} = \frac{E_2}{2(1 + \nu_{23})} \]  

(4)

As shown later in Table 8, all the composites including the hybrid composites studied in this paper closely follow transverse isotropic behavior. One reason for this may be the hexagonal packing of the fiber, which represents better isotropy in the 2–3 plane. As for the hybrid composites, 10 samples of each volume fraction ratio were considered, with the fiber locations randomly selected for each sample. The mean and standard deviation of the results were studied.

Rule of hybrid mixtures was used to predict the longitudinal modulus \( E_1 \) and the longitudinal Poisson’s ratios, \( v_{12} \) and \( v_{13} \) for all the composites. For the transverse modulus \( E_2 \), and the shear moduli \( G_{12}, G_{13} \) and \( G_{23} \), the Halpin–Tsai equation was modified to predict the results obtained from finite element method. As will be shown later the modified Halpin–Tsai equations agree with reasonable accuracy for the hybrid composites. For all the elastic constants, samples were generated with random fiber locations and results obtained for all the samples were studied to evaluate the effect of hybridization.

### Table 3

<table>
<thead>
<tr>
<th>Property</th>
<th>E-glass fiber</th>
<th>Carbon fiber (IM7)</th>
<th>Epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_1 ) (GPa)</td>
<td>72.4</td>
<td>263</td>
<td>3.5</td>
</tr>
<tr>
<td>( E_2 ), ( E_3 ) (GPa)</td>
<td>72.4</td>
<td>19</td>
<td>3.5</td>
</tr>
<tr>
<td>( G_{12}, G_{13} ) (GPa)</td>
<td>30.2</td>
<td>27.6</td>
<td>1.29</td>
</tr>
<tr>
<td>( G_{23} ) (GPa)</td>
<td>30.2</td>
<td>7.04</td>
<td>1.29</td>
</tr>
<tr>
<td>( v_{12} ), ( v_{13} )</td>
<td>0.2</td>
<td>0.2</td>
<td>0.35</td>
</tr>
<tr>
<td>( v_{23} )</td>
<td>0.2</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>

3. Evaluation of strength properties

Failure is predicted using micromechanical failure analysis, which inspects every element in the finite element model for failure, also known as the Direct Micromechanics (DMM) approach to failure prediction. A flowchart that describes DMM can be found in [15]. Thus for a given state of macrostress, we need to calculate microstresses in every element in the RVE. The macrostrain for a given state of macrostress can be obtained from the constitutive relation for that composite using

\[ \sigma^M = [C^{-1}] \{ \epsilon^M \} \]  

(5)

From the unit cell analysis as discussed before, we have the microstresses in every element for six independent unit macrostrain cases. Thus, the microstresses for a given macrostress state can be obtained from principle of superposition as follows:

\[ \{ \sigma^{(e)} \} = [F^{(e)}] \{ \epsilon^M \} \]  

(6)

where \( \{ \sigma^{(e)} \} \) is the microstress in Element \( e \), and the matrix \( [F^{(e)}] \) represents the microstresses in Element \( e \) for various states of unit macrostresses. For example, the first column in \( F_{ij} \) contains the six microstresses in Element \( e \) caused by unit macrostrain \( \epsilon_{ij}^{(e)} \). However, in the present method it is assumed that there exist no thermal residual stresses in the material. Also, it is assumed that when the first element fails, the composite has failed.

It is assumed, that failure criteria for fibers and matrix phases are known. We have considered quadratic interaction failure criteria for carbon fiber which is the one proposed by Hashin for unidirectional fiber composites [18] and maximum principal stress failure criteria for glass fiber and epoxy.

4. Results and discussions

We have divided this section into two parts; one for the elastic constants and the other for the strength properties. Results obtained from analytical formulations, wherever applicable, have been compared with FEA results.
4.1. Elastic properties

The longitudinal modulus $E_1$ was calculated for the composites by varying the volume fraction of the reinforcements. It was observed that $E_1$ varies linearly with the variation of volume fraction. $E_1$ is plotted in Fig. 4 with the volume fraction of carbon varying from 0 to 0.6 as we move from left to right. $E_1$ for the composites are also tabulated in Table 4. Results obtained from the RoHM are also presented in the same table. The RoHM can be stated as

$$E_1 = E_{1c}V_{fc} + E_{1g}V_{fg} + E_mV_m$$

(7)

where $E_{1c}$, $E_{1g}$ and $E_m$ refers to the modulus values for carbon, glass and matrix respectively, and $V_{fc}$, $V_{fg}$ and $V_m$ refer to the volume fraction carbon, glass and matrix respectively. It can be seen that RoHM predicts the longitudinal moduli with very high accuracy.

The transverse modulus $E_2$ however, cannot be predicted accurately using equations of the form (7). A general method to estimate $E_2$ involves the use of semi-empirical equations such as the Halpin–Tsai equation that are adjusted to match experimental results. The Halpin–Tsai equation for single fiber composite is [14]

$$E_2 = \frac{1 + \eta_f V_f}{E_m + \eta_f V_f}$$

(8)

where

$$\eta = \frac{(E_f/E_m) + 1}{(E_f/E_m) + \zeta}$$

In the equations above, $\zeta$ is a curve-fitting parameter, which is dependent on the fiber packing arrangement. For the hybrid composites, we propose a modification to the Halpin–Tsai Eq. (8), which incorporates the volume fractions of all the reinforcements as follows:

$$E_2 = \frac{1 + \xi (\eta_c V_{fc} + \eta_g V_{fg})}{E_m + \xi (\eta_c V_{fc} + \eta_g V_{fg})}$$

(9)

where,

$$\eta_c = \frac{(E_{fc}/E_m) - 1}{(E_{fc}/E_m) + \zeta}$$

and

$$\eta_g = \frac{(E_{fg}/E_m) - 1}{(E_{fg}/E_m) + \zeta}$$

Here the subscripts ‘c’ and ‘g’ refer to carbon and glass respectively. The optimum value of $\zeta$ was determined using a least square error procedure. It was found that $\zeta = 1.165$ yielded the best results for $E_2$ including single fiber composites.

In Table 5 we have the $E_2$ values computed from both the finite element analysis and modified Halpin–Tsai equation. We see that (9) does a good job of predicting the transverse modulus of the composites. The variation of $E_2$ with increasing volume fraction of carbon is shown in Fig. 5.

The poisons’ ratio $v_{12}$ and $v_{13}$ were computed for all composites and they were nearly equal for all cases. It was found that these two Poisson’s ratios had a linear variation when volume fraction of carbon was gradually increased, as seen in Fig. 6. The RoHM for Poisson’s ratios can be stated as

$$v_{12} = v_{12c}V_{fc} + v_{12g}V_{fg} + v_mV_m$$

(10)

Once again RoHM provides a good prediction of Poisson’s ratio, where the Poisson’s ratio of the composite, $v_{12}$ can be found out using (10), where $v_{12c}$, $v_{12g}$, and $v_m$ refers to the Poisson’s ratio of carbon, glass, and matrix respectively.

A approach similar to the transverse modulus was considered for predicting the shear moduli, $G_{12}$, $G_{13}$ and $G_{23}$. The modified Halpin–Tsai relation for predicting the shear moduli is as shown below:

$$G = \frac{1 + \xi (\eta_c V_{fc} + \eta_g V_{fg})}{G_m - \xi (\eta_c V_{fc} + \eta_g V_{fg})}$$

(11)

where, $\eta_c = (G_{fc}/G_m) - 1)/(G_{fc}/G_m + \zeta)$ and $\eta_g = (G_{fg}/G_m) - 1)/(G_{fg}/G_m + \zeta)$

In the above equation $G$ refers to composite shear modulus ($G_{12}$, $G_{13}$ or $G_{23}$). For each case, the corresponding fiber shear moduli have to be considered in calculating the parameter $\eta$. The optimal value of $\zeta$ was found out to be 1.01 for $G_{12}$ and $G_{13}$, and 0.9 for $G_{23}$. The corresponding plots for variation of the three shear moduli with volume fraction of carbon are shown in Figs. 7 and 8. The moduli values calculated using modified Halpin–Tsai equation and the finite element analysis are also presented in Tables 6 and 7.

Poisson’s ratio $v_{23}$ is calculated and variation with changes in reinforcement volume fraction is studied. An analytical expression for $v_{23}$ is not required, since for transverse isotropic composites $v_{23}$ can be calculated from $G_{23}$ and $E_2$. Since, we have an analytical expression for predicting $E_2$ and $G_{23}$, we can predict $v_{23}$ once we have the other two material properties. The variation of $v_{23}$ with volume fraction of carbon is as shown in Fig. 9.

As mentioned before, 10 random fiber locations inside the RVE were selected for each volume fraction for the hybrid composites. It was observed that none of the elastic constants showed significant variability with fiber location. The Poisson’s ratio $v_{23}$ had some variability, as shown in Fig. 9 but it was observed that the coefficient of variation for all the elastic constants were negligibly small. This can be attributed to the fact that elastic constants were calculated by volume averaging the microstresses for all the elements. Hence, the spatial variation of the microstresses does not have significant effect on the elastic constants. Table 8 shows that all the composites in the present study follow transverse isotropic behavior.

4.2. Strength properties

The material properties are as per Table 9. Composite failure can be characterized as fiber failure or matrix failure, considering our assumption that the interface does not fail. First we will consider loading in the longitudinal or fiber direction. In this case, since fiber failure strain, $e_{23}^{(1)}$ is higher than matrix failure strain, $e_{23}^{(1)}$, we can conclude that matrix will govern the failure. So, composite failure will occur at the strain level corresponding to the matrix failure strain, $e_{23}^{(1)}$. Hence, longitudinal strength of the composite can be predicted from the following relation

$$S_2^{(1)} = E_1 e_{23}^{(1)}$$

(12)

where $E_1$ is the longitudinal moduli of the composite. This equation gives a good measure of the failure strength for initiation of failure.
The results from finite element analysis are plotted against the volume fraction of carbon in Fig. 10. \( S_{\text{L}} \) has been calculated from (12) and presented in Table 10. As can be observed, the longitudinal strength varies linearly with volume fraction and can be predicted with reasonable accuracy.

Longitudinal compressive strength of the composites were calculated and plotted with volume fraction of carbon as shown in Fig. 11. Both strengths in the longitudinal direction show a linear dependence with volume fraction. It must be noted, however, that for the compressive strength, no microbuckling of the fiber or instability analysis was performed, and failure was due only to stresses. Detailed micromechanical analysis of failure modes such as microbuckling or kinking can be found in [20,21]. Furthermore, the longitudinal strengths show no variability with fiber location. The linear nature of the plot can be explained from observing (12) which depends on \( E_1 \) of the composite. As seen before, \( E_1 \) Table 4

<table>
<thead>
<tr>
<th>Type of composite</th>
<th>Carbon/epoxy (GPa)</th>
<th>Hybrid composites</th>
<th>Glass/epoxy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE analysis</td>
<td>159</td>
<td>148</td>
<td>654</td>
</tr>
<tr>
<td>RoHM</td>
<td>159.2</td>
<td>147.7</td>
<td>656</td>
</tr>
<tr>
<td>% Diff (absolute)</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table 5

<table>
<thead>
<tr>
<th>Type of composite</th>
<th>Carbon/epoxy (GPa)</th>
<th>Hybrid</th>
<th>Glass/epoxy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE analysis</td>
<td>8.77</td>
<td>9.05</td>
<td>12.11</td>
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<tr>
<td>Modified Halpin Tsai</td>
<td>8.59</td>
<td>8.88</td>
<td>12.11</td>
</tr>
<tr>
<td>% Diff (absolute)</td>
<td>2.07</td>
<td>1.84</td>
<td>0.13</td>
</tr>
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</table>

Fig. 5. Variation of \( E_2 \) with volume fraction of carbon.

Fig. 6. Variation of \( \nu_{12} \) and \( \nu_{13} \) with volume fraction of carbon.

Fig. 7. Variation of \( G_{12} \) and \( G_{13} \) with volume fraction of carbon.

Fig. 8. Variation of \( G_{23} \) with volume fraction of carbon.
has a linear variation with volume fraction of the fiber, hence the strengths follow similar pattern since $e_{th}$ is a constant for all the samples.

A theory of elasticity analysis for a transverse normal loading of a doubly periodic rectangular array of elastic filaments can be found in [22]. However, a simple analytical model for transverse strength of one reinforcement composite can be found in [14]. The relation has been used to predict the strength of the two phase composites, pure carbon/epoxy and glass/epoxy. When compared with the DMM strength, it has a difference of 15%.

Transverse tensile and compressive strength obtained from DMM approach are plotted in Figs. 12 and 13 with volume fraction of carbon. As observed, transverse strength of the composite reduces when a second reinforcement is added. Hybridization therefore results in lowering the transverse strength. Another very

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Transverse moduli $G_{12} (G_{13})$ (GPa) for composites.</th>
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</thead>
<tbody>
<tr>
<td>Type of composite</td>
<td>Carbon/epoxy</td>
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<tr>
<td>FE analysis</td>
<td>4.41</td>
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<tr>
<td>Modified Halpin–Tsai</td>
<td>4.41</td>
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<td>% Diff (absolute)</td>
<td>0.05</td>
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<th>Table 7</th>
<th>Transverse moduli $G_{23}$ (GPa) for composites.</th>
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<tbody>
<tr>
<td>Type of composite</td>
<td>Carbon/epoxy</td>
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<tr>
<td>FE analysis</td>
<td>3.04</td>
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<tr>
<td>Modified Halpin–Tsai</td>
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<td>% Diff (absolute)</td>
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<thead>
<tr>
<th>Table 8</th>
<th>Comparison of $G_{23}$ to test transverse isotropy.</th>
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<tr>
<td>Specimen</td>
<td>$G_{23}$ (FEA)</td>
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<td>Carbon/epoxy</td>
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<td>Hybrid composites</td>
<td>3.14</td>
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<td></td>
<td>3.36</td>
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<td></td>
<td>3.60</td>
</tr>
<tr>
<td></td>
<td>4.19</td>
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<tr>
<td>Glass/epoxy</td>
<td>4.35</td>
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<table>
<thead>
<tr>
<th>Table 9</th>
<th>Strengths of constituent materials [17,19].</th>
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<tbody>
<tr>
<td>Carbon</td>
<td>Glass</td>
</tr>
<tr>
<td>Longitudinal tensile strengths (MPa)</td>
<td>4120</td>
</tr>
<tr>
<td>Longitudinal compressive strength (MPa)</td>
<td>2990</td>
</tr>
<tr>
<td>Transverse tensile and compressive strengths (MPa)</td>
<td>298</td>
</tr>
<tr>
<td>Shear strength (MPa)</td>
<td>1760</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>–</td>
</tr>
<tr>
<td>Compressive strength (MPa)</td>
<td>–</td>
</tr>
</tbody>
</table>

Fig. 9. Variation of $\nu_{23}$ with volume fraction of carbon.

Fig. 10. Variation of $S_{L}^{(+)}$ with volume fraction of carbon.

Fig. 11. Variation of $S_{L}^{(-)}$ with volume fraction of carbon.
A very similar variation is observed for the transverse compressive strength of the composites. Once again the drop in strength when glass fiber is introduced in carbon/epoxy, is significantly higher than when 6% carbon fiber is introduced in glass/epoxy composite.

Elasticity analysis of an array of elastic filaments subjected to longitudinal shear has been studies by Adam and Doner [23]. Transverse shear strengths $S_{12}$ and $S_{13}$ have been calculated for hybrid composites and presented in Figs. 14 and 15. It is observed that there is negligible variation of transverse shear strengths, owing to the fact that shear strengths are controlled by matrix strength and are essentially same for all the composites.

It should be however noticed that $S_{12}$ and $S_{13}$ are not the same for any composite. This is mostly due to the asymmetric hexagonal arrangement of the fibers in the 1 and 2 directions. Variation of the shear strength $S_{23}$ is also presented for hybrid composites in Fig. 16. Since, $S_{23}$ is an in-plane property in the transverse directions, follows a very similar trend as the transverse tensile and compressive strengths as shown before in Figs. 13 and 14.

It is important to note here that strengths unlike the elastic constants are not equal in 2 and 3 directions. This is owing to the fact that, the RVE is not symmetric about 2 and 3 directions. Although
average stresses across the RVE were equal, the microstresses inside the RVE are not. A comparison of the variation of $S_{T2}^{(+)}$ and $S_{T3}^{(+)}$ are made for 2 and 3 directions and presented in Figs. 17 and 18 respectively. Here $S_{T2}^{(+)}$, $S_{T2}^{(-)}$ and $S_{T3}^{(+)}$, $S_{T3}^{(-)}$ represent transverse tensile and compressive stresses in the 2 and 3 directions respectively.

In order to explain the effect of hybridization on the transverse strength in more detail, failed elements have been identified in the Figs. 19 and 20. The DMM method of failure is continued till 1% of the volume of the unit cell was failed. Glass fiber is red and carbon green in the above figures. It can be observed that the microstress concentration is always near the region surrounding the glass fibers. This shows that, introduction of glass causes a high local stress concentration owing to its high transverse modulus and thus strength of the composites drop on addition of glass.

All the strength values computed using the present model are presented in Table 11. For the hybrid composite, results for specimen H3 have been provided in this table. As can be observed, both transverse tensile and compressive strengths for hybrid composites are lower than the binary composites. Arithmetic mean, standard deviation and coefficients of variation for strength properties are presented in Table 12. Longitudinal tensile and compressive strengths had no dependence on the fiber location. On the other
Table 10
Longitudinal tensile strength, $S_{l,l}$ (MPa) for composites.

<table>
<thead>
<tr>
<th>Composite</th>
<th>Specimen</th>
<th>FEA</th>
<th>Analytical</th>
<th>% Difference</th>
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</thead>
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<td>Carbon/epoxy</td>
<td></td>
<td>2128</td>
<td>598</td>
<td>1360</td>
</tr>
<tr>
<td>Hybrid</td>
<td>H1</td>
<td>1972</td>
<td>2069</td>
<td>4.67</td>
</tr>
<tr>
<td>Hybrid</td>
<td>H2</td>
<td>1665</td>
<td>1748</td>
<td>4.77</td>
</tr>
<tr>
<td>Hybrid</td>
<td>H3</td>
<td>1360</td>
<td>1428</td>
<td>4.78</td>
</tr>
<tr>
<td>Hybrid</td>
<td>H4</td>
<td>1055</td>
<td>1108</td>
<td>4.79</td>
</tr>
<tr>
<td>Hybrid</td>
<td>H5</td>
<td>750</td>
<td>788</td>
<td>4.81</td>
</tr>
<tr>
<td>Glass/epoxy</td>
<td></td>
<td>598</td>
<td>628</td>
<td>4.74</td>
</tr>
</tbody>
</table>

Table 11
Summary of Strength properties for Composites.

<table>
<thead>
<tr>
<th>Strength</th>
<th>Carbon/epoxy</th>
<th>Glass/epoxy</th>
<th>Hybrid/epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal tensile (MPa)</td>
<td>2128</td>
<td>598</td>
<td>1360</td>
</tr>
<tr>
<td>Longitudinal compressive (MPa)</td>
<td>1807</td>
<td>684</td>
<td>1158</td>
</tr>
<tr>
<td>Transverse tensile (MPa)</td>
<td>41</td>
<td>38</td>
<td>27</td>
</tr>
<tr>
<td>Transverse compressive (MPa)</td>
<td>101</td>
<td>86</td>
<td>59</td>
</tr>
<tr>
<td>Longitudinal shear, $S_{l,s}$ (MPa)</td>
<td>42</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>Longitudinal shear, $S_{l,t}$ (MPa)</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>Transverse shear, $S_{l,t}$ (MPa)</td>
<td>28</td>
<td>24</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 12
Standard deviation and coefficient of variation for strengths ($\mu, \sigma, \sigma/\mu$ stands for mean, standard deviation and coefficient of variance (%), respectively).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$S_{l,l}$ (MPa)</th>
<th>$S_{l,l}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$ $\sigma$ $\sigma/\mu$</td>
<td>$\mu$ $\sigma$ $\sigma/\mu$</td>
</tr>
<tr>
<td>1</td>
<td>1972 0.45 0.022 1677 0.01 0</td>
<td>1666 0.20 0.012 1418 0 0</td>
</tr>
<tr>
<td>2</td>
<td>1360 0.14 0.010 1158 0 0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1055 0.04 0.003 899 0 0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>750 0.05 0.006 639 0 0</td>
<td></td>
</tr>
</tbody>
</table>

Longitudinal modulus, longitudinal poisson's ratios and longitudinal shear modulus, with very good accuracy. For predicting the transverse moduli, transverse poisson's ratio and the transverse shear moduli, modified Halpin–Tsi equation has been proposed, that matches the finite element results with reasonable accuracy.

Longitudinal tensile and compressive strengths vary linearly with the volume fraction of the reinforcement, and are dependent on the longitudinal modulus and the least strain to failure of the constituent. All other strengths show variability with the fiber location inside the RVE. This is attributed to the transverse modulus of the introduced fibers to form the hybrid composite, which causes a local stress concentration, resulting in the failure of the neighboring matrix elements.

Experimental data for hybrids has been reviewed by many researchers. It is observed that the rule of mixtures can approximately predict the longitudinal and transverse mechanical properties of unidirectional interply hybrids [1,3,5,6]. This is consistent with the results presented in this paper.

Overall, the objective of the present work was to develop a computational model that is compatible to test hybrid composites with varying volume fraction of reinforcements and study the effect of hybridization on mechanical properties of the composite. Future work in this area would be using similar models to model progressive damage in hybrid composites.

Acknowledgements

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References