Non Linear Programming EML 6934 Class Periods: T, Period 4 (10:40 AM - 11:30 AM), Th, Periods 4-5 (10:40 AM - 12:35 PM, Location: BEN 0328 (Benton Hall) Academic Term: Spring 2022

Instructor:

Prabir Barooah <u>Email Address: please use canvas message for electronic communication, not email</u> Office Phone Number: 352 294 0411 Office Hours: M,W,Th, hours available: 2-3 pm, or by appointment, office location: MAE-B 324

Teaching Assistants:

None

Course Description

The goal of the course is to provide a firm foundation on solution methods for nonlinear programming problems (NLPs). A nonlinear program (NLP) is the following optimization problem:

 $\min_x f(x)$

S.t. $g_i(x) \setminus leq 0, I=1,...m, and h_j(x)=0, j=0,....k$

Where the "cost" f, and f_i's and g_i are functions mapping real vectors to real numbers ($R^n \ B$). To solve the NLP is to find x*, if it exists, that solves the NLP. Here x (an n-dimensional real vector) is called the decision variable.

We will discuss (i) algorithms to solve the NLP when solutions exist, and how to determine if that is the case, (ii) analysis of these algorithms, and finally - since we love control systems - (iii) applications to approximate optimal control, such as model predictive control and reinforcement learning. We will not cover "mixed integer programs", where some of the entries in the decision vector x can only take integer values. But existing methods of solving mixed integer programs are based on NLPs, so the course will help even if you are interested in that.

See the schedule below for a more detailed description of the topics.

Course Pre-Requisites / Co-Requisites

Linear algebra, and experience with Matlab/Python. It is strongly recommended that you take EML 5311.

Course Objectives

The goal of the course is to provide a firm foundation on solution methods for the NLP described above. Meaning, we will discuss (i) algorithms to solve the NLP when solutions exist, and how to determine if that is the case, (ii) analysis of these algorithms, and finally - since we love control systems - (iii) applications to approximate optimal control, such as model predictive control and reinforcement learning. We will not cover "mixed integer programs", where some of the entries in the decision vector x can only take integer values.

Materials and Supply Fees:

None

Required Textbooks and Software

<u>Textbook</u>

No required textbooks. The following books are strongly recommended, almost all lecture material will come from these two:

1. Luenberger's blue book ("Linear and Nonlinear Programming", by Luenberger, 2nd edition)

2. Boyd and Vandenberghe's cvx (Convex Optimization by Stephen Boyd & Lieven Vandenberghe)

I'll also make some typed/handwriten notes available by canvas from time to time.

Software:

A computer with python/matlab installed. Preferably a laptop so you can bring it to the class on Thursdays. We will jump into computation from the very beginning of the course, so you need to have a working copy of matlab/python in your computer from the beginning.

Also, please install the following software in your computer:

- 1. Cvx (<u>http://cvxr.com/cvx/</u>). If you plan to use python, you will have to use the python version of cvx, (<u>https://cvxopt.org/</u>.)
- 2. Casadi (<u>https://web.casadi.org/</u>)

I have not used cvx and casadi with python, so cannot help you much if you run into trouble. I have only used them with matlab.

If you plan to use python, I strongly encourage use Anaconda python.

Course Schedule

Subject to change, and not in chronological order:

Acronysm used: (DL): Linear and Nonlinear programming, David Luenberger, (BnV): Convex Optimization, Stephen Boyd and Lieven Vandengerghe

Unconstrained Optimization

Some Mathematical review will be interspersed with the topics listed below.

DL: Section 6.0: what the problem is

Section 6.1: first order necessary conditions, and example 4 of 6.2,

Section 6.3: second order necessary conditions, sufficient conditions

Section 6.4: convex functions.

Section 6.5: minimization of convex functions

BnV, Section 2.1.4 - convex sets

BnV, strong and strict convexity, Section 9.1.2

Initial foray into algorithms for solving unconstrained minimization problems

BnV,

section 9.2: Descent methods, descent directions for convex functions

Section 9.3: Gradient descent method

Section 9.3.1: convergence analysis with exact and inexact line search

Section 9.3.2: examples

Section 9.4: Steepest descent

Section 9.4.1 Section 9.4.4 Section 9.5: Newton's method Upto section 9.5.2 - to be done in great detail Section 9.5.3: convergence analysis: brief overview only, unto eq. 9.37

Constrained minimization

DL,

Section 10.1: constraints

Section 10.2: Tangent plane

Section 10.3: First order necessary conditions with equality constraints

Section 10.4: Examples (do ex. 2 and 4)

(the results in these sections are extremely important, and DL's proofs of them are sometimes compact. A lot of work is required to expand the proofs to understand them completely)

Section 10.8: Inequality constraints (skip from second order necessary conditions onwards), KKT conditions

Algorithms for equality constrained optimization problems

1. Gradient descent on null space,...

Chapter 14.0, Section 14.1: Lagrange methods for solving equality constrained minimization, and in particular, equality constrained quadratic programming problems.

Chapter 14.2: Newton's method (skip the rest)

Algorithms for general constrained minimization, part one, Penalty and Barrier methods

BnV,

Section 11.1: Interior point methods for solving inequality and equality constrained optimization problems Section 11.2: log barrier function and central path

Section 11.3: Barrier methods...

DL

Section 11.0 Primal methods

Section 11.1: Advantage of primal methods

Section 11.2: feasible direction methods (brief discussion, skip details on non-closure of algorithmic maps)

Section 11.3: active set methods

Section 11.4: the gradient projection method

Duality

BnV, chapter 5

Section 5.1: The Lagrange dual function

Section 5.2: The Lagrange dual problem, strong duality

Section 5.5: Optimality conditions (upto 5.5.3, KKT conditions)

Algorithms for constrained minimization, part two, Dual and primal dual methods

DL

Parts of Chapter 13

Applications: 1. Approximating (infinite-dimensional) optimal control problems as (finite-dimensional) NLPs, 2. Training NNs

Tools: cvx and Casadi to set up and solve NLPs without the drudgery.

Attendance Policy, Class Expectations, and Make-Up Policy

State whether attendance is required and if so, how will it be monitored? What are the penalties for absence, tardiness, cell phone policy, laptop policy, etc. What are the arrangements for missed homework, missed quizzes, and missed exams? This statement is required: Excused absences must be consistent with university policies in the <u>Graduate Catalog</u> and require appropriate documentation. Additional information can be found in <u>Attendance Policies</u>.

Evaluation of Grades

Assignment	Total Points	Percentage of Final Grade
Homework Sets (5)	100 each	60%
Class participation	100 each	15%
Final project	100	25%
		100%

Grading Policy

(subject to change)

Percent	Grade	Grade Points
90.0 - 100.0	Α	4.00
87.0 - 89.9	A-	3.67
84.0 - 86.9	B+	3.33
81.0 - 83.9	В	3.00

Non Linear Programming, EML 6934 Prabir Barooah, Spring 2022

Percent	Grade	Grade Points
78.0 - 80.9	B-	2.67
75.0 - 79.9	C+	2.33
72.0 - 74.9	С	2.00
69.0 - 71.9	C-	1.67
66.0 - 68.9	D+	1.33
63.0 - 65.9	D	1.00
60.0 - 62.9	D-	0.67
0 - 59.9	E	0.00

More information on UF grading policy may be found at: <u>UF Graduate Catalog</u> <u>Grades and Grading Policies</u>

Students Requiring Accommodations

Students with disabilities who experience learning barriers and would like to request academic accommodations should connect with the <u>Disability Resource Center</u>. It is important for students to share their accommodation letter with their instructor and discuss their access needs, as early as possible in the semester.

Course Evaluation

Students are expected to provide professional and respectful feedback on the quality of instruction in this course by completing course evaluations online via GatorEvals. <u>Click here for guidance on how to give feedback in a professional and respectful manner</u>. Students will be notified when the evaluation period opens, and can complete evaluations through the email they receive from GatorEvals, in their Canvas course menu under GatorEvals, or via <u>ufl.bluera.com/ufl/</u>. <u>Summaries of course evaluation results are available to students here</u>.

University Honesty Policy

UF students are bound by The Honor Pledge which states, "We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honor and integrity by abiding by the Honor Code. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: "On my honor, I have neither given nor received unauthorized aid in doing this assignment." <u>The Honor Code</u> specifies a number of behaviors that are in violation of this code and the possible sanctions. Furthermore, you are obligated to report any condition that facilitates academic misconduct to appropriate personnel. If you have any questions or concerns, please consult with the instructor or TAs in this class.

Software Use

All faculty, staff, and students of the University are required and expected to obey the laws and legal agreements governing software use. Failure to do so can lead to monetary damages and/or criminal penalties for the individual violator. Because such violations are also against University policies and rules, disciplinary action will be taken as appropriate. We, the members of the University of Florida community, pledge to uphold ourselves and our peers to the highest standards of honesty and integrity.

Student Privacy

There are federal laws protecting your privacy with regards to grades earned in courses and on individual assignments. For more information, please see the <u>Notification to Students of FERPA Rights</u>.

Campus Resources:

Health and Wellness

U Matter, We Care:

If you or a friend is in distress, please contact <u>umatter@ufl.edu</u> or 352 392-1575 so that a team member can reach out to the student.

Counseling and Wellness Center: <u>counseling.ufl.edu/cwc</u>, and 392-1575; and the University Police Department: 392-1111 or 9-1-1 for emergencies.

Sexual Assault Recovery Services (SARS) Student Health Care Center, 392-1161.

University Police Department at 392-1111 (or 9-1-1 for emergencies), or police.ufl.edu.

Academic Resources

<u>E-learning technical support</u>, 352-392-4357 (select option 2) or e-mail to Learning-support@ufl.edu.

<u>Career Resource Center</u>, Reitz Union, 392-1601. Career assistance and counseling.

<u>Library Support</u>, Various ways to receive assistance with respect to using the libraries or finding resources.

Teaching Center, Broward Hall, 392-2010 or 392-6420. General study skills and tutoring.

Writing Studio, 302 Tigert Hall, 846-1138. Help brainstorming, formatting, and writing papers.

Student Complaints Campus

On-Line Students Complaints