Non Linear Programming
EML 6934

Class Periods:  T, Period 4 (10:40 AM - 11:30 AM), Th, Periods 4-5 (10:40 AM - 12:35 PM,
Location:  BEN 0328 (Benton Hall)
Academic Term:  Spring 2022

Instructor:
Prabir Barooah
Email Address: please use canvas message for electronic communication, not email
Office Phone Number: 352 294 0411
Office Hours:  M,W,Th, hours available: 2-3 pm, or by appointment, office location: MAE-B 324

Teaching Assistants:
None

Course Description
The goal of the course is to provide a firm foundation on solution methods for nonlinear programming problems (NLPs). A nonlinear program (NLP) is the following optimization problem:

\[ \begin{align*}
\min_{x} & \quad f(x) \\
\text{s.t.} & \quad g_i(x) \leq 0, \ i = 1, \ldots, m, \ \text{and} \ h_j(x) = 0, \ j = 0, \ldots, k
\end{align*} \]

Where the “cost” \( f \), and \( f_i \)'s and \( g_i \) are functions mapping real vectors to real numbers (\( \mathbb{R}^n \to \mathbb{R} \)). To solve the NLP is to find \( x^* \), if it exists, that solves the NLP. Here \( x \) (an \( n \)-dimensional real vector) is called the decision variable.

We will discuss (i) algorithms to solve the NLP when solutions exist, and how to determine if that is the case, (ii) analysis of these algorithms, and finally - since we love control systems - (iii) applications to approximate optimal control, such as model predictive control and reinforcement learning. We will not cover “mixed integer programs”, where some of the entries in the decision vector \( x \) can only take integer values. But existing methods of solving mixed integer programs are based on NLPs, so the course will help even if you are interested in that.

See the schedule below for a more detailed description of the topics.

Course Pre-Requisites / Co-Requisites
Linear algebra, and experience with Matlab/Python. It is strongly recommended that you take EML 5311.

Course Objectives
The goal of the course is to provide a firm foundation on solution methods for the NLP described above. Meaning, we will discuss (i) algorithms to solve the NLP when solutions exist, and how to determine if that is the case, (ii) analysis of these algorithms, and finally - since we love control systems - (iii) applications to approximate optimal control, such as model predictive control and reinforcement learning. We will not cover “mixed integer programs”, where some of the entries in the decision vector \( x \) can only take integer values.

Materials and Supply Fees:
None

Required Textbooks and Software

Textbook
No required textbooks. The following books are strongly recommended, almost all lecture material will come from these two:

Non Linear Programming, EML 6934
Prabir Barooah, Spring 2022
2. Boyd and Vandenberghe's cvx (Convex Optimization by Stephen Boyd & Lieven Vandenberghe)

I'll also make some typed/handwritten notes available by canvas from time to time.

Software:
A computer with python/matlab installed. Preferably a laptop so you can bring it to the class on Thursdays. We will jump into computation from the very beginning of the course, so you need to have a working copy of matlab/python in your computer from the beginning.

Also, please install the following software in your computer:
   1. Cvx ([http://cvxr.com/cvx/](http://cvxr.com/cvx/)). If you plan to use python, you will have to use the python version of cvx, ([https://cvxopt.org/](https://cvxopt.org/))
   2. Casadi ([https://web.casadi.org/](https://web.casadi.org/))

I have not used cvx and casadi with python, so cannot help you much if you run into trouble. I have only used them with matlab.

If you plan to use python, I strongly encourage use Anaconda python.

Course Schedule

Subject to change, and not in chronological order:

Acronym used: (DL): Linear and Nonlinear programming, David Luenberger, (BnV): Convex Optimization, Stephen Boyd and Lieven Vandenberghe

Unconstrained Optimization

Some Mathematical review will be interspersed with the topics listed below.

DL: Section 6.0: what the problem is
   Section 6.1: first order necessary conditions, and example 4 of 6.2,
   Section 6.3: second order necessary conditions, sufficient conditions
   Section 6.4: convex functions.
   Section 6.5: minimization of convex functions

BnV, Section 2.1.4 - convex sets

BnV, strong and strict convexity, Section 9.1.2

Initial foray into algorithms for solving unconstrained minimization problems

BnV,
   section 9.2: Descent methods, descent directions for convex functions
   Section 9.3: Gradient descent method
      Section 9.3.1: convergence analysis with exact and inexact line search
      Section 9.3.2: examples
      Section 9.4: Steepest descent
Section 9.4.1
Section 9.4.4
Section 9.5: Newton's method
  Upto section 9.5.2 - to be done in great detail
  Section 9.5.3: convergence analysis: brief overview only, unto eq. 9.37

Constrained minimization

DL,
  Section 10.1: constraints
  Section 10.2: Tangent plane
  Section 10.3: First order necessary conditions with equality constraints
  Section 10.4: Examples (do ex. 2 and 4)

  (the results in these sections are extremely important, and DL's proofs of them are sometimes compact. A lot of work is required to expand the proofs to understand them completely)
  Section 10.8: Inequality constraints (skip from second order necessary conditions onwards), KKT conditions

Algorithms for equality constrained optimization problems

  1. Gradient descent on null space,...

  Chapter 14.0, Section 14.1: Lagrange methods for solving equality constrained minimization, and in particular, equality constrained quadratic programming problems.

  Chapter 14.2: Newton's method (skip the rest)

Algorithms for general constrained minimization, part one, Penalty and Barrier methods

BnV,
  Section 11.1: Interior point methods for solving inequality and equality constrained optimization problems
  Section 11.2: log barrier function and central path
  Section 11.3: Barrier methods...

DL
  Section 11.0 Primal methods
    Section 11.1: Advantage of primal methods
    Section 11.2: feasible direction methods (brief discussion, skip details on non-closure of algorithmic maps)
Section 11.3: active set methods
Section 11.4: the gradient projection method

Duality
BnV, chapter 5

Section 5.1: The Lagrange dual function
Section 5.2: The Lagrange dual problem, strong duality
Section 5.5: Optimality conditions (upto 5.5.3, KKT conditions)

Applications: 1. Approximating (infinite-dimensional) optimal control problems as (finite-dimensional) NLPs, 2. Training NNs
Tools: cvx and Casadi to set up and solve NLPs without the drudgery.

**Attendance Policy, Class Expectations, and Make-Up Policy**
State whether attendance is required and if so, how will it be monitored? What are the penalties for absence, tardiness, cell phone policy, laptop policy, etc. What are the arrangements for missed homework, missed quizzes, and missed exams? This statement is required: Excused absences must be consistent with university policies in the Graduate Catalog and require appropriate documentation. Additional information can be found in Attendance Policies.

**Evaluation of Grades**

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<th>Assignment</th>
<th>Total Points</th>
<th>Percentage of Final Grade</th>
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<tr>
<td>Homework Sets (5)</td>
<td>100 each</td>
<td>60%</td>
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<tr>
<td>Class participation</td>
<td>100 each</td>
<td>15%</td>
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<tr>
<td>Final project</td>
<td>100</td>
<td>25%</td>
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**Grading Policy**
(subject to change)

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<td>87.0 - 89.9</td>
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<td>84.0 - 86.9</td>
<td>B+</td>
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<tr>
<td>81.0 – 83.9</td>
<td>B</td>
<td>3.00</td>
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<tr>
<td>Percent</td>
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<td>75.0 - 79.9</td>
<td>C+</td>
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<td>72.0 – 74.9</td>
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<td>66.0 - 68.9</td>
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More information on UF grading policy may be found at:

UF Graduate Catalog
Grades and Grading Policies

Students Requiring Accommodations
Students with disabilities who experience learning barriers and would like to request academic accommodations should connect with the Disability Resource Center. It is important for students to share their accommodation letter with their instructor and discuss their access needs, as early as possible in the semester.

Course Evaluation
Students are expected to provide professional and respectful feedback on the quality of instruction in this course by completing course evaluations online via GatorEvals. Click here for guidance on how to give feedback in a professional and respectful manner. Students will be notified when the evaluation period opens, and can complete evaluations through the email they receive from GatorEvals, in their Canvas course menu under GatorEvals, or via ufl.bluera.com/ufl/. Summaries of course evaluation results are available to students here.

University Honesty Policy
UF students are bound by The Honor Pledge which states, "We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honor and integrity by abiding by the Honor Code. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: "On my honor, I have neither given nor received unauthorized aid in doing this assignment." The Honor Code specifies a number of behaviors that are in violation of this code and the possible sanctions. Furthermore, you are obligated to report any condition that facilitates academic misconduct to appropriate personnel. If you have any questions or concerns, please consult with the instructor or TAs in this class.

Software Use
All faculty, staff, and students of the University are required and expected to obey the laws and legal agreements governing software use. Failure to do so can lead to monetary damages and/or criminal penalties for the individual violator. Because such violations are also against University policies and rules, disciplinary action will be taken as appropriate. We, the members of the University of Florida community, pledge to uphold ourselves and our peers to the highest standards of honesty and integrity.

Student Privacy
There are federal laws protecting your privacy with regards to grades earned in courses and on individual assignments. For more information, please see the Notification to Students of FERPA Rights.

Campus Resources:

Health and Wellness

U Matter, We Care:
If you or a friend is in distress, please contact umatter@ufl.edu or 352 392-1575 so that a team member can reach out to the student.
Counseling and Wellness Center: counseling.ufl.edu/cwc, and 392-1575; and the University Police Department: 392-1111 or 9-1-1 for emergencies.

Sexual Assault Recovery Services (SARS)
Student Health Care Center, 392-1161.

University Police Department at 392-1111 (or 9-1-1 for emergencies), or police.ufl.edu.

Academic Resources

E-learning technical support, 352-392-4357 (select option 2) or e-mail to Learning-support@ufl.edu.

Career Resource Center, Reitz Union, 392-1601. Career assistance and counseling.

Library Support, Various ways to receive assistance with respect to using the libraries or finding resources.

Teaching Center, Broward Hall, 392-2010 or 392-6420. General study skills and tutoring.

Writing Studio, 302 Tigert Hall, 846-1138. Help brainstorming, formatting, and writing papers.

Student Complaints Campus

On-Line Students Complaints