# **EML 6934: Nonlinear Programming**

Spring 2023 M W F, 1:55 PM - 2:45 PM Benton Hall 0328 (BEN 328)

This syllabus and course contents are subject to change at the discretion of the instructor.

#### **Professor**

• Matthew Hale

• Email: <u>matthewhale@ufl.edu</u>

#### **Office Hours**

• Thursday 1-3 PM in MAE-B 321

## **Teaching Assistants**

None

# **Course Description and Objectives**

The goal of this course is to provide a firm mathematical foundation for solution methods for nonlinear programs (NLPs). A nonlinear program is an optimization problem of the form

$$\begin{aligned} & & \text{minimize}_x \ f(x) \\ & \text{subject to} \ g_i(x) \leq 0, \quad i = 1, \dots, m \\ & h_j(x) = 0, \ j = 1, \dots, k \end{aligned}$$

Here the cost is f, and the functions  $g_i$  and  $h_j$  are constraints. All of them map vectors to real numbers (i.e., they are maps  $\mathbb{R}^n \to \mathbb{R}$ ). We wish to solve such NLPs, which means computing a vector  $x^*$  that minimizes f among all points that satisfy the constraints given by  $g_i$  and  $h_j$ .

The goal of the course is to provide a firm foundation on solution methods for nonlinear programs. We will discuss (i) methods to determine when solutions to NLPs exist, (ii) algorithms to solve NLPs when solutions exist, (iii) analysis of these algorithms, and (iv) applications to approximate optimal control problems, such as model predictive control and reinforcement learning. We will not cover mixed integer programs in which some of the entries in the decision vector x can only take integer values.

# **Course Pre-Requisites / Co-Requisites**

- A strong background in linear algebra
- Experience with MATLAB and/or Python
- It is strongly recommended that you take EML 5311 Control System Theory before this class

# **Textbooks and Software**

- There is not a required textbook for this class. The books below are optional:
  - o *Linear and Nonlinear Programming*, D. Luenberger, 2<sup>nd</sup> edition.
  - Convex Optimization, by Boyd and Vandenberghe. Free here: https://web.stanford.edu/~boyd/cvxbook/
- This class will require access to either MATLAB or Python. Install the following on your computer:
  - o CVX (http://cvxr.com/cvx/)
  - o Casadi (<a href="https://web.casadi.org/">https://web.casadi.org/</a>)

• These software packages have large online communities that can help with problems.

### **Important Dates**

• Homework dates will be announced in class. There will be five total.

# **Materials and Supply Fees**

• None

## **Evaluation of Grades and Grading Policy**

Evaluation Mechanism on a Percent Basis

Homework 33.33% Midterm 33.33% Final 33.33%

#### **Course Contents**

These contents are subject to change and are not necessarily listed in chronological order.

I. Unconstrained optimization problems

Some mathematical review will be interspersed with these topics.

- A. Luenberger, Chapter 6
  - 1. Section 6.1: First-order necessary conditions, Example 4 of 6.2
  - 2. Section 6.3: Second-order necessary conditions, sufficient conditions
  - 3. Section 6.4: Convex functions
  - 4. Section 6.5: Minimization of convex functions
- B. Boyd & Vandenberghe, Section 2.1: Convex sets
- C. Boyd & Vandenberghe, Section 9.1: Strict and strong convexity
- II. Algorithms for solving unconstrained minimization problems
  - A. Boyd & Vandenberghe, Chapter 9
    - 1. Section 9.2: Descent methods, descent directions for convex functions
    - 2. Section 9.3: The gradient descent method
    - 3. Section 9.4: Steepest descent
    - 4. Section 9.5: Newton's method
- III. Constrained minimization
  - A. Luenberger, Chapter 10
    - 1. Section 10.1: Constraints
    - 2. Section 10.2: Tangent plane
    - 3. Section 10.3: First-order necessary conditions with equality constraints
    - 4. Section 10.4: Examples
    - 5. Section 10.8: Inequality constraints, KKT conditions
  - B. Algorithms for equality constrained optimization problems
    - 1. Gradient descent on null space
    - 2. Luenberger, Section 14.1: Lagrange methods for equality constrained minimization
    - 3. Luenberger, Section 14.2: Newton's method
- IV. Algorithms for general constrained minimization, part I: Penalty and barrier methods
  - A. Boyd & Vandenberghe, Chapter 11
    - 1. Section 11.1: Interior point methods for constrained problems
    - 2. Section 11.2: Log barrier functions and central paths

- 3. Section 11.3: Barrier methods
- B. Luenberger, Chapter 11
  - 1. Section 11.1: Advantage of primal methods
  - 2. Section 11.2: Feasible direction methods
  - 3. Section 11.3: Active set methods
  - 4. Section 11.4: The gradient projection method

# V. Duality

- A. Boyd & Vandenberghe, Chapter 5
  - 1. Section 5.1: The Lagrange dual function
  - 2. Section 5.2: The Lagrange dual problem, strong duality
  - 3. Section 5.5: Optimality conditions
- VI. Algorithms for general constrained minimization, part II: Dual and primal-dual methods
  - A. Luenberger, Chapter 13

## VII. Applications

- A. Approximating optimal control problems as NLPs
- B. Training neural networks

## Grading

Homework will be graded on a scale from 1-5. A grade of 5 indicates that all problems have been solved without any mistakes (of any size). A grade of 4 indicates that solutions show a correct understanding on a conceptual level and are generally mathematically correct, modulo a small number of minor mistakes. A grade of 3 indicates numerous mistakes and/or a key conceptual misunderstanding. A grade of 2 indicates multiple significant conceptual misunderstandings. A grade of 1 reflects that very little has been done correctly. Homework solutions will be posted to Canvas after the due date.

### **Grade Corrections**

Grade corrections must be submitted via email within 3 business days of the grade posting.

### Final Grade

Percentage Range		Grade Point
$93.33 \le \%$ GE <	$100.00 \Longrightarrow A$	4.00
$90.00 \le \%$ GE <	$93.33 \Longrightarrow A$ -	3.67
$86.67 \le \%$ GE <	$90.00 \Longrightarrow B+$	3.33
$83.33 \le \%GE <$	$86.67 \Longrightarrow B$	3.00
$80.00 \le \%GE <$	$83.33 \Longrightarrow B$ -	2.67
$76.67 \le \%$ GE <	$80.00 \Longrightarrow C+$	2.33
$73.33 \le \%$ GE <	$76.67 \Longrightarrow C$	2.00
$70.00 \le \%$ GE <	$73.33 \Longrightarrow C$ -	1.67
$66.67 \le \%$ GE <	$70.00 \Longrightarrow D+$	1.33
$63.33 \le \%$ GE <	$66.67 \Longrightarrow D$	1.00
$60.00 \le \%$ GE <	$63.33 \Longrightarrow D$ -	0.67
$00.00 \le \%GE <$	$60.00 \Longrightarrow E$	0.00

#### Students Requiring Accommodations

Students with disabilities requesting accommodations should first register with the Disability Resource Center (352-392-8565, <a href="https://www.dso.ufl.edu/drc">https://www.dso.ufl.edu/drc</a>) by providing appropriate documentation. Once registered, students will receive an accommodation letter which must be presented to the instructor when

requesting accommodation. Students with disabilities should follow this procedure as early as possible in the semester.

#### Course Evaluation

Students are expected to provide feedback on the quality of instruction in this course by completing online evaluations at <a href="https://evaluations.ufl.edu/evals">https://evaluations.ufl.edu/evals</a>. Evaluations are typically open during the last two or three weeks of the semester, but students will be given specific times when they are open. Summary results of these assessments are available to students at <a href="https://evaluations.ufl.edu/results/">https://evaluations.ufl.edu/results/</a>.

#### University Honesty Policy

UF students are bound by The Honor Pledge which states, "We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honor and integrity by abiding by the Honor Code. On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: "On my honor, I have neither given nor received unauthorized aid in doing this assignment." The Honor Code (<a href="https://sccr.dso.ufl.edu/policies/student-honor-code-student-conduct-code/">https://sccr.dso.ufl.edu/policies/student-honor-code-student-conduct-code/</a>) specifies a number of behaviors that are in violation of this code and the possible sanctions. Furthermore, you are obligated to report any condition that facilitates academic misconduct to appropriate personnel. If you have any questions or concerns, please consult with the instructor or TAs in this class.

# Commitment to a Safe and Inclusive Learning Environment

The Herbert Wertheim College of Engineering values broad diversity within our community and is committed to individual and group empowerment, inclusion, and the elimination of discrimination. It is expected that every person in this class will treat one another with dignity and respect regardless of gender, sexuality, disability, age, socioeconomic status, ethnicity, race, and culture.

If you feel like your performance in class is being impacted by discrimination or harassment of any kind, please contact your instructor or any of the following:

- Your academic advisor or Graduate Program Coordinator
- Robin Bielling, Director of Human Resources, 352-392-0903, rbielling@eng.ufl.edu
- Curtis Taylor, Associate Dean of Student Affairs, 352-392-2177, taylor@eng.ufl.edu
- Toshikazu Nishida, Associate Dean of Academic Affairs, 352-392-0943, nishida@eng.ufl.edu

#### Software Use

All faculty, staff, and students of the University are required and expected to obey the laws and legal agreements governing software use. Failure to do so can lead to monetary damages and/or criminal penalties for the individual violator. Because such violations are also against University policies and rules, disciplinary action will be taken as appropriate. We, the members of the University of Florida community, pledge to uphold ourselves and our peers to the highest standards of honesty and integrity.

#### Student Privacy

There are federal laws protecting your privacy with regards to grades earned in courses and on individual assignments. For more information, please see: https://registrar.ufl.edu/ferpa.html

# Campus Resources:

## Health and Wellness

# U Matter, We Care:

Your well-being is important to the University of Florida. The U Matter, We Care initiative is committed to creating a culture of care on our campus by encouraging members of our community to look out for one another and to reach out for help if a member of our community is in need. If you or a friend is in distress, please contact umatter@ufl.edu so that the U Matter, We Care Team can reach

out to the student in distress. A nighttime and weekend crisis counselor is available by phone at 352-392-1575. The U Matter, We Care Team can help connect students to the many other helping resources available including, but not limited to, Victim Advocates, Housing staff, and the Counseling and Wellness Center. Please remember that asking for help is a sign of strength. In case of emergency, call 9-1-1.

**Counseling and Wellness Center:** <a href="http://www.counseling.ufl.edu/cwc">http://www.counseling.ufl.edu/cwc</a>, and 392-1575; and the University Police Department: 392-1111 or 9-1-1 for emergencies.

#### Sexual Discrimination, Harassment, Assault, or Violence

If you or a friend has been subjected to sexual discrimination, sexual harassment, sexual assault, or violence contact the <u>Office of Title IX Compliance</u>, located at Yon Hall Room 427, 1908 Stadium Road, (352) 273-1094, <u>title-ix@ufl.edu</u>

### **Sexual Assault Recovery Services (SARS)**

Student Health Care Center, 392-1161.

**University Police Department** at 392-1111 (or 9-1-1 for emergencies), or <a href="http://www.police.ufl.edu/">http://www.police.ufl.edu/</a>.

### Academic Resources

**E-learning technical support**, 352-392-4357 (select option 2) or e-mail to Learning-support@ufl.edu. <a href="https://lss.at.ufl.edu/help.shtml">https://lss.at.ufl.edu/help.shtml</a>.

**Career Resource Center**, Reitz Union, 392-1601. Career assistance and counseling. https://www.crc.ufl.edu/.

**Library Support**, <a href="http://cms.uflib.ufl.edu/ask">http://cms.uflib.ufl.edu/ask</a>. Various ways to receive assistance with respect to using the libraries or finding resources.

**Teaching Center**, Broward Hall, 392-2010 or 392-6420. General study skills and tutoring. https://teachingcenter.ufl.edu/.

**Writing Studio, 302 Tigert Hall**, 846-1138. Help brainstorming, formatting, and writing papers. <a href="https://writing.ufl.edu/writing-studio/">https://writing.ufl.edu/writing-studio/</a>.

Student Complaints Campus: https://www.dso.ufl.edu/documents/UF Complaints policy.pdf.

On-Line Students Complaints: http://www.distance.ufl.edu/student-complaint-process.