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ScienceDirect
Composites: Part B 38 (2007) 924–932
composites
Part B: engineering
www.elsevier.com/locate/compositesb

Strength prediction of multi-layer plain weave textile composites using the direct micromechanics method

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Received 19 May 2006; accepted 17 July 2006
Available online 24 January 2007

Abstract

Previously developed micromechanical methods for stiffness and strength prediction are adapted for analysis of multi-layer plain weave textile composites. Utilizing the direct micromechanics method (DMM) via finite element modeling, three methods are presented: (a) direct simulation of a multi-layer plain weave textile composite; (b) micromechanical analysis of a single layer of interest from the force and moment resultants acting on that layer; and (c) application of the previously developed quadratic stress-gradient failure theory to the layer of interest. In comparison to direct modeling, the other two techniques show only 5% difference over a number of random test cases. Several practical design examples of strength prediction are included to illustrate the importance and accuracy of method implementation.

Keywords: A. Textile composites; B. Strength; C. Finite element analysis; C. Micro-mechanics

1. Introduction

Textile composites present several advantages towards the design of effective lightweight structures. The undulation of the woven or braided structure provides inherent out of plane reinforcement. Textile structures also provide inherent reinforcement in multiple directions. Both of these properties can be quite useful, especially in impact energy absorption applications. However, the increased microstructural complexity, as compared to traditional unidirectional composites, also presents the challenge of increased complexity of characterization and analysis.

An effective approach to failure prediction of textile composites must incorporate several key points: it should be based upon the unique characteristics of textile composites and reflect the consequent mechanical implications; it should be robust in its accommodation of various loading cases and in its ease of use; and it should be applicable to a layup of any number of layers.

The micro level scale of a textile composite is physically larger and geometrically more complex than a unidirectional composite. Thus many common assumptions and traditional analysis techniques break down [1]. The stress state and material properties are non-uniform, i.e., there is no single-layer stress analogous to the classical laminate theory. Though negligible across the typical several micron-sized dimension of a unidirectional RVE, even a mild stress gradient could represent a large stress difference at different points across the face of a typical millimeter-sized textile composite RVE. The importance and details of such considerations are presented in a parent study [1] by the authors of the current work. A follow-on study [2] then presented the development of quadratic failure criteria for effective prediction of failure of textile composites, with inclusion of the consideration of micro level stress gradients. As will be presented, the current study represents an extension of both of these previous works to realistic textile

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1359-8368/$ - see front matter
doi:10.1016/j.compositesb.2006.07.021

composite structural components by including analysis methods for multiple layers.

A more detailed survey of current research in the area of failure modeling of textile composites is offered in the parent study [1] to the current paper. A brief sampling of these works is presented.

Designs of current textile composite structures are often based upon well-known phenomenological failure criterion, predominantly the maximum stress criterion, maximum strain criterion, and quadratic interaction criterion, such as the Tsai–Hill and Tsai–Wu failure theories [3].

Initial and progressive failure of a plain weave composite using finite element analysis has provided insight into the failure modes under axial loading conditions [4,5]. Accurate stress distributions for plain weave composites in flexure have been investigated, and effective stiffness properties for multi-layer specimens have been predicted [6].

The binary model [7,8] utilizes 1-D line elements to represent fiber tow embedded within the bulk matrix. This allows for quick and efficient analysis of any textile weave. Some micro-level stress field detail is lost while still maintaining accurate macro-level representation.

The Mosaic model analytical method [9,10] represent a textile composite RVE as an assemblage of homogenized blocks for which classical laminated plate theory can be used to determine the global stiffness matrix. The method can readily be applied to in-plane stiffness characterization of multiple layer specimens, but effectiveness under bending conditions may be limited.

Effective prediction of compressive strength of braided textile composites using a detailed FEM micromechanical model and experimental verification has been performed [11,12]. Buckling analysis has been performed, and the effects of tow waviness and micro-architecture on the compressive strength are shown.

Previous work by the authors [1,2] extended a method, known as the direct micromechanics method [13] (DMM), to develop failure envelopes and a quadratic stress gradient failure theory for a plain-weave textile composite. This was developed for a single RVE and thus represents single-layer analyses.

When multiple layers are present, the layer material properties will remain the same, but the stiffness and strength of the overall composite laminate will change. To develop a fully general failure theory, results from single-layer analyses [1,2] must be adapted to accommodate a textile composite of an arbitrary number of layers. This allows material characterization simulations of a single RVE to be applicable to a lay-up of an arbitrary number of layers, eliminating the need for further material characterization.

In this regard, three analysis techniques are proposed: (a) multilayer direct micro-mechanics (MDMM), direct simulation of the multi-layer composite; (b) adapted direct micro-mechanics (ADMM), estimation of macro deformations in the layer of interest and application of single layer DMM [1]; and (c) quadratic stress gradient failure theory (QSFT), implementation of a phenomenological quadratic stress-gradient failure theory [2] to the layer of interest (without the requirement of determining a new set of failure coefficients as developed for a single layer).

Sections 2 and 3 of the current paper introduce an overview of the methods developed previously to determine stiffness and strength of a single-layer plain weave textile composite, as well as the development of a quadratic stress gradient failure theory based upon these results. The following Sections (4 through 5.4) then extend the procedures to accommodate multi-layer analysis, and various methods are employed as described in the preceding paragraph.

The entire body of work is then applied to several practical examples of strength prediction (Section 5.5) to illustrate their implementation. The results are compared to conventional methods utilizing common failure theories not specifically developed for textile composites. Design of a two-layer textile plate under uniform pressure is considered for several geometries and width-to-thickness ratios. Also shown is a test case of a pressure vessel in which stress-gradients are less prevalent.

2. Direct micromechanics method

Micromechanical analysis has been performed to determine the stiffness, strength, and failure envelopes of a plain weave textile composite. Details of this analysis are available in a previous study [1], but methods therein are presented here in an abbreviated fashion. This previous study of a single-layer textile composite prepares the groundwork and fundamentals that are built upon in the current work. In the previous study, stress gradient effects are investigated, and it is assumed that the stress state is not uniform across the RVE. This represents an extension of the micromechanical models used to predict the strength of textile composites [13–15]. The stress state is defined in terms of the well-known laminate theory force and moment resultants, $[N]$ and $[M]$, in which the latter term captures the presence of a stress distribution or gradient that is typically neglected. Furthermore, structural stiffness coefficients analogous to the $[A]$, $[B]$, $[D]$ matrices are defined.

The textile architecture under investigation is a plain weave, and this RVE is shown in Fig. 1, with dimensions as listed in Table 1. Constituent properties are shown in Tables 2 and 3 (note that fiber tows have the properties of a carbon epoxy composite at the micro level). This architecture was chosen from a literature source [16] that provided a complete and detailed description of the needed geometrical parameters. Given parameters are representative of micro-architectures as experimentally observed via SEM or standard microscope.

In the direct micromechanics method (DMM), the RVE is subjected to macroscopic force and moment resultants, from which the complete micro level stress field can be determined. By applying known independent unit strain and curvature cases, one can completely determine the

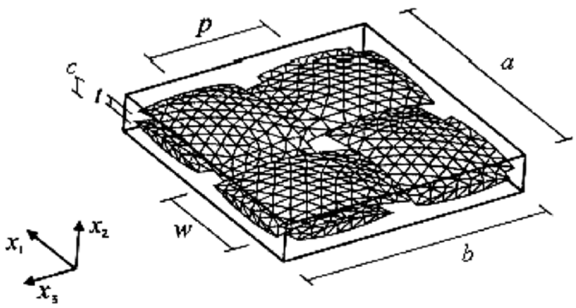

Fig. 1. RVE geometry of a plain weave textile composite [16].

constitutive [A], [B], [D] matrices. In this approach, these structural stiffness coefficients are computed directly from the micromechanical models, rather than from the lamination theories.

Strength can then be determined by evaluating the microstress field found via the DMM for a prescribed macro force and moment resultant (an input as might be determined from structural analysis, either analytical or finite element method). Strength is predicted by comparing the computed microstresses in each element against failure criteria for the constituent material. Failure is checked on an element-by-element basis, and the failure criterion of each element can be selected appropriately based upon whether it is a yarn or matrix element. For the isotropic matrix elements, the maximum principal stress criterion has been chosen to evaluate element failure. For fiber tow elements, the Tsai–Wu failure criterion is used. Element failure represents a failure initiation point beyond which property loss and ultimate failure will occur. This is analogous to a first-ply failure point of classical laminate theory.

3. A quadratic stress-gradient failure criterion in terms of force and moment resultants

To provide a method that may be utilized to determine effective strength prediction without the need for case-by-case DMM analysis, a quadratic stress-gradient failure criterion has been defined [2] for single-layer textile composites. Secondarily, this also serves to provide a concise numerical description of the varied failure spaces that a textile composite may exhibit. Given the quadratic interactive nature of the stress state in determination of failure, an expression of the below form has been developed to predict failure:

$$C_{ij}F_iF_j + D_iF_i = 1 \tag{1}$$

where F_i represent general normalized load terms ($N_xN_y N_{xy}M_xM_yM_{xy}$), and C_{ij} or D_i represent 27 failure coefficients such that Eq. (1) defines failure when its left hand side exceeds 1. Note again that inclusion of moment resultant terms represents the critical inclusion of the consideration of stress gradients that may exist across the characteristically large geometry of a textile RVE. Due to the large disparity in the magnitude of a typical force resultant as compared to a typical moment resultant, each load term F_{i} must be normalized, or Eq. (1) will be numerically ill conditioned. All force resultants are normalized with respect to the critical tensile force resultant N_x (6.40×10^3 N/m) while moment resultants are normalized with respect to the critical moment resultant M_x (1.82×10^{-4} N). Note that this normalization is reflected in the magnitude of the failure coefficients shown later in Tables 4 and 5.

Failure coefficients C_{ij} and D_i can be solved given a sufficient amount of known failure points. These failure points can be determined by the DMM or by physical tests.

Table 1
Dimensions of the representative volume element (RVE) of the plain woven composite (see Fig. 1)

Dimension	a, b	c	p	t	w
Length (mm)	1.68	0.254	0.84	0.066	0.70

Table 2
Fiber tow and matrix material properties [17]

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
AS/3501 graphite/epoxy (65% fiber volume)	138	9.0	6.9	0.30
3501 Matrix	3.5	3.5	1.3	0.35

Table 3
Fiber tow and matrix strength properties (MPa) [17]

Material	$S_L^{(+)}$	$S_L^{(-)}$	$S_T^{(+)}$	$S_T^{(-)}$	S_{LT}
3501/Graphite tow	1448	1172	48.3	60	62.1
3501 Matrix	70	70	70	70	–

Table 4
Normalized failure coefficients C_{ij} in the quadratic stress gradient failure theory

	$n = 1$	2	3	4	5	6
$m = 1$	1.02	−0.81	2.45	0.15	0.15	−0.09
2		1.02	2.45	0.15	0.15	−0.09
3			9.29	0.15	0.15	−1.28
4				1.00	−0.65	0.29
5					1.00	0.29
6						3.05

Coefficient C_{mn} is in mth row and nth column.

Table 5
Normalized failure coefficients D_i in the quadratic stress gradient failure theory

D_1	−0.011
D_2	−0.011
D_3	0.000
D_4	0.000
D_5	0.000
D_6	0.000

Coefficients C_{ii} and D_i can be solved by evaluating Eq. (1) with a single load of F_i and setting all other loads to zero. For example, in order to obtain C_{11} and D_1 a tensile and compressive load of $F_1 = N_x$ are applied, all others are set to zero, which may be substituted into Eq. (1) to generate two equations:

$$C_{11}N_{xT}^2 + D_1 N_{xT} = 1 \tag{2}$$

$$C_{11}N_{xC}^2 + D_1 N_{xC} = 1 \tag{3}$$

where N_{xT} and N_{xC}, respectively, are the tensile and compressive strengths in the x direction, as have been determined from DMM analysis. Thus, these two independent equations can be simultaneously solved to yield C_{11} and D_1.

Remaining coefficients C_{ij} can be solved by evaluating Eq. (1) under the loading condition given by $F_i = F_j$ as determined by the DMM results. All other loads are set to zero. Along with knowledge of previously determined coefficients C_{ii} and D_i, this allows for the solution of all remaining coefficients. For example, C_{14} can be determined by applying the maximum possible $F_1 = F_4$. The failure coefficient is then solved directly from Eq. (1).

The above procedure is similar to that used in deriving Tsai–Wu failure coefficients for unidirectional composites [3], with the additional inclusion of moment resultant terms and their associated failure coefficients, which accommodate the stress gradients in the analysis. The results of the above procedures are shown in Tables 4 and 5.

Coefficients D_3 through D_6 are equal to zero since positive and negative strengths are the same for any shear, moment, or twist loads for the particular woven composite considered in the example.

The same methods shown in this section can easily be applied to any textile architecture of interest. The 27-term quadratic failure equation has been found to be robust to adapt to the various forms of failure spaces that may be exhibited by a particular architecture [2]. It has been shown [2] that this method can be used to accurately predict failure for the plain weave textile composite under consideration. In load cases in which one, two, or three force or moment resultants are present, accuracy is shown to be good within a few percent. As load cases become more complicated, the overall agreement in these diverse failure spaces is seen to be quite suitable, but there will be "corners" or portions of the 6D failure space that will be missed with the essentially 6D ellipse space of the quadratic failure theory. Such cases exhibit an average error of 9.3% [2]. Predictions tend to be conservative in areas of disagreement.

4. Stiffness prediction of multi-layer textile composites

In general, a textile composite laminate may consist of any number of stacked layers. The results of the previous sections describe the fundamental groundwork for stiffness and strength prediction of a single RVE, which represent single-layer analyses. Such procedures must be modified to incorporate accommodation of multiple layers.

To provide a verifiable basis for multi-layer characterization, a direct simulation of the behavior of a two-layer composite has been performed. The single RVE is replaced with two stacked RVE's, which simulate the two-layer textile composite. Using the DMM procedures, unit strain and curvature cases have again been carried out to directly determine the constitutive matrices. Then macro level force and moment resultants can be computed, and the constitutive matrices are determined. A single layer of the plain weave textile under consideration will exhibit the following constitutive matrices

$$[A] = \begin{bmatrix} 4.14 & 0.52 & 0 \\ 0.52 & 4.14 & 0 \\ 0 & 0 & 0.18 \end{bmatrix} \times 10^6 \text{ (Pa m)} \quad [B] \approx 0$$
$$[D] = \begin{bmatrix} 7.70 & 2.53 & 0 \\ 2.53 & 7.70 & 0 \\ 0 & 0 & 1.35 \end{bmatrix} \times 10^{-3} \text{ (Pa m}^3\text{)} \tag{4}$$

For a two-layer textile, the constitutive matrices, as determined from a two-layer model utilizing the DMM (procedurally referred to in this paper as MDMM or multilayer direct micromechanics), are seen to be

$$[A] = \begin{bmatrix} 8.28 & 1.04 & 0 \\ 1.04 & 8.28 & 0 \\ 0 & 0 & 0.36 \end{bmatrix} \times 10^6 \text{ (Pa m)} \quad [B] \approx 0$$
$$[D] = \begin{bmatrix} 148.8 & 22.4 & 0 \\ 22.4 & 148.8 & 0 \\ 0 & 0 & 8.51 \end{bmatrix} \times 10^{-3} \text{ (Pa m}^3\text{)} \tag{5}$$

Bending stiffness follows a relationship analogous to the parallel axis theorem, which governs the increased moment of inertia of an area of material that is moved away from the bending axis. These expressions for the overall stiffness of a two-layer textile can be represented as

$$A_{ij}^{DL} = 2A_{ij}^{SL} \tag{6}$$

$$D_{ij}^{DL} = 2(D_{ij}^{SL} + A_{ij}^{SL} d^2) \tag{7}$$

where the superscripts DL and SL represent "double layer" or "single layer" properties, and d represents the distance from the center of a layer to the bending axis. Similar results have been shown in [6] for textile composites under flexure. From this expression, the double layer stiffness matrices (Eq. (5)) can effectively be calculated with knowledge only of the single layer stiffness properties (Eq. (4)). This can be extrapolated to a material of an arbitrary number of layers (N), once the DMM has been used to characterize a layer (n) or one RVE. Eqs. (8) and (9) may be used to evaluate the stiffness matrices and no further analysis is needed

$$A_{ij} = \sum_{n=1}^{N} A_{ij}^{n} \quad (8)$$

$$D_{ij} = \sum_{n=1}^{N} (D_{ij}^{n} + A_{ij}^{n} d_n^2) \quad (9)$$

Note that if a coupling matrix $[B]$ were present, this would need to be accounted for in the below expression of Eq. (10). Due to symmetry, this expression will evaluate to zero for the plain-weave architecture

$$B_{ij} = \sum_{n=1}^{N} (B_{ij}^{n} + A_{ij}^{n} d_n) \quad (10)$$

Table 7
Strength values for single-layer and two-layer textile composite as determined by MDMM

	Strength (single layer)	Strength (two layer)
N_x	6.40×10^3 Pa m (+)	1.29×10^4 Pa m (+)
	5.86×10^3 Pa m (−)	1.18×10^4 Pa m (−)
N_y	6.40×10^3 Pa m (+)	1.29×10^4 Pa m (+)
	5.86×10^3 Pa m (−)	1.18×10^4 Pa m (−)
N_{xy}	2.11×10^3 Pa m	4.18×10^4 Pa m
M_x	1.85×10^{-4} Pa m^2	1.91×10^{-3} Pa m^2
M_y	1.85×10^{-4} Pa m^2	1.91×10^{-3} Pa m^2
M_{xy}	1.06×10^{-4} Pa m^2	1.08×10^{-3} Pa m^2

The (+) and (−) signs, respectively, denote the tensile and compressive strengths.

5. Strength prediction of multi-layer textile composites

Once stiffness has been determined by the methods of the previous section, further analysis is needed to determine strength of a multi-layer textile composite. The following three sub-sections describe the various modeling approaches used to analyze laminated plain weave composite structures and comparison of various methodologies in modeling the strength of textile composites. For clarity, the various approaches are summarized in Table 6.

5.1. Direct FE simulation of multi-layer textile composites (MDMM)

The same methods described previously for single layer strength prediction can be used for direct simulation of a two layer RVE. This direct simulation paints an accurate picture of the load capacity of a multilayer textile, at the expense of model preparation and computational time. As the methods for the MDMM approach are essentially the same as the DMM described previously and in [1], with a two-layer RVE in place of the single RVE, the details are not repeated. The two sections to follow this describe two methods based upon these results (ADMM and QFT), which can be used to predict strength of a textile composite laminates of an arbitrary number of layers without having to employ direct FEM simulation.

Table 7 indicates the maximum allowable level of any force or moment resultant when it is the only load present. Single-layer strength and double-layer strength are compared. When applicable, a (+) or (−) indicates tensile or compressive strength, respectively. In essence, the in-plane strength properties do not change. The critical force resultant doubles as a result of the doubling of material present, but otherwise the load capacity is unchanged. The strength under bending will change significantly for the two-layer textile composite. As a direct consequence of increased bending stiffness, the critical applied moment is seen to increase approximately tenfold. Note that the relationship will depend on the thickness of each layer, and thus is an observation specific to this RVE micro geometry. The MDMM serves as a check upon which a more generalized approach may be developed, which can predict for an arbitrary number of layers under arbitrary load conditions with mixed load types.

5.2. Extension of the single layer DMM results to predict strength for multi-layer textile composites or adapted direct micromechanics method (ADMM)

In this method, the mid-plane strains and curvatures of the single layer of interest are calculated from the laminate strains and curvature. Then the direct micromechanics is applied to that layer in order to predict its strength. Thus the strength prediction involves extending the results of single layer FEM analysis directly from the DMM. Thus only one material characterization is needed (of a single layer) to predict strength for any number of layers.

The single-layer DMM was discussed in a Section 5.1. In the case of a single layer under bending, the sole "stress source" stems purely from the resulting curvature. Now in the case of two layers under bending, this stress source

Table 6
Summary of the various methods employed in multi-layer strength analysis

Method	Acronym	Summary
Direct micromechanics method (single layer)	DMM	Basic method used to characterize strength and stiffness of an RVE
Multi layer direct micromechanics method	MDMM	Direct simulation of two or more stacked RVEs used to characterize multiple layers
Adapted direct micromechanics method for multi-layer analysis	ADMM	Calculate the force and moment resultants carried by the layer of interest in a multilayer textile composite laminate and apply DMM
Quadratic stress-gradient failure theory	QSFT	Calculate the force and moment resultants carried by the layer of interest, and then use QSFT [2]
Classical lamination theory		Calculate the macro stresses in each layer and use classical failure theories such as Tsai–Wu or maximum stress theory

of curvature is still present, while normal strains that result from the layer offset from the axis of bending represent an additional stress source that must be accounted for. Thus, superposition of these effects must be applied to find the total stress field for a multi-layer textile under bending.

As detailed in classical laminate theory [17], the magnitude of the normal strain in offset layers will be directly proportional to the curvature (κ) that is present and the distance from the layer midplane to the bending axis (d)

$$\varepsilon_{ij} = \varepsilon_{ij}^{0} \pm d\kappa_{ij} \tag{11}$$

The sign of the distance d (positive or negative) depends on the position of the layer with respect to the mid-plane.

With this adjusted assessment of the strain and curvature in each layer, the DMM can be used to evaluate the failure envelope for any force and moment resultants. As detailed earlier, failure from the total stress field in each layer is then checked on an element-by-element basis to determine overall failure of the composite. Results and comparisons via this method will be shown after Section 5.3.

5.3. Implementation of the quadratic stress gradient failure theory (QSFT) to predict strength for multi-layer textile composites

The previously developed 27-term quadratic failure theory for textile composites [2], as determined from the single-layer DMM, can be implemented to predict failure for a multi-layer specimen. Once the original failure coefficients have been determined, no further FEM or experimental analysis will be needed. Implementation of this procedure is accomplished by calculating the force and moment resultants carried by the layer of interest and then applying the single layer quadratic failure theory (QSFT). First, the mid-plane multi-layer strains and curvatures are calculated from the applied macro-level force and moment resultant, along with the constitutive matrix representing the multi-layer (double-layer, DL, in the present example) material properties

$$\begin{Bmatrix} [\varepsilon_0] \\ [\kappa] \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix}_{\mathrm{DL}}^{-1} \begin{Bmatrix} [N] \\ [M] \end{Bmatrix} \tag{12}$$

The multi-layer midplane strain must now be modified to represent the actual strain state in each layer

$$\varepsilon_{ij} = \varepsilon_{ij}^{0} \pm d\kappa_{ij} \tag{13}$$

Note that in plate analysis, mid-plane curvature and layer-level curvature will always be the same.

The layer-level force and moment resultant are then calculated using this modified deformation from Eq. (12), along with the single layer (SL) constitutive matrix

$$\begin{Bmatrix} [N] \\ [M] \end{Bmatrix}_{\mathrm{SL}} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix}_{\mathrm{SL}} \begin{Bmatrix} [\varepsilon] \\ [\kappa] \end{Bmatrix} \tag{14}$$

The adjusted force and moment resultants capture what is seen in each layer offset from the bending axis. These are then directly input to the quadratic failure theory of Eq. (1) with coefficients as per Tables 4 and 5 (as developed from one layer or RVE). Failure analyses are performed independently in each layer. This is to say that, the single layer force and moment resultants for each layer must be independently calculated input to the quadratic stress-gradient failure theory (computations which can still be automated). Note that in the above-described procedure, a pure moment resultant applied to a multi-layer composite will correspond to both force and moment resultants in each layer.

5.4. Comparison of the results of multi-layer failure analysis methods

Several cases are now presented which illustrate the relative effectiveness of the multi-layer analysis methods shown in the preceding sections of this chapter. Direct FEM simulation provides the most accurate prediction of the stress field and failure envelope of the multi-layer textile composite. Thus it is taken as the reference strength against which other methods are compared. The two techniques for predicting failure of a multi-layer composite without additional material characterization tests can be compared to this in order to estimate their accuracy.

Comparison of predicted failure points (the maximum allowable force and moment resultants under combined loading) proves to be the best and most germane method of comparing the multiple prediction methods. To this end, failure has been predicted for a variety of load cases based on the data from the direct simulation of the MDMM. The results of the ADMM and the QSFT are then compared to this. Both methods are shown to compare well to the MDMM results, though use of the QSFT is computationally faster and more practical once failure coefficients have been determined. Tables 8 and 9 show a comparison of the various methods to calculate failure for a multi-layer textile. Load cases are shown in terms of a load ratio (α) defined as

$$\alpha_i = \frac{F_i}{F_1} \quad \text{or} \quad \alpha_i = \frac{F_i}{F_4} \quad (\text{if } F_1 \equiv 0) \tag{15}$$

Note that moment resultant load ratios are normalized by F_4 rather than F_1. By maintaining the same load ratios, all predicted failure loads would maintain a single ratio with respect to the benchmark MDMM failure points. Thus one ratio can characterize the congruence of these solutions. Error in a particular theory is defined as the deviation from the MDMM result. A negative difference indicates a conservative failure prediction, and a positive difference implies a non-conservative prediction.

For combined force and moment resultants, failure occurs in the tensile layer, in which the applied moment generates additional tensile forces that accelerate failure. The critical force resultant in this case is approximately

Table 8
Example load cases to determine the accuracy of multi-layer analysis methods

Load ratios (α)	MDMM	ADMM		QSFT	
	Calculated allowables	Calculated allowables	% Difference	Calculated allowables	% Difference
Case 1					
1	1.35E + 04	1.31E + 04	−3.0	1.30E + 04	−3.7
0	0	0		0	
0	0	0		0	
Case 2					
1	1.91E + 04	2.01E + 04	+5.2	2.02E + 04	+5.8
1	1.91E + 04	2.01E + 04		2.02E + 04	
0	0	0		0	
Case 3					
0	0	0		0	
0	0	0		0	
1	4.23E + 03	4.14E + 03	−2.1	4.18E + 03	−1.2
Case 4					
1	3.73E + 03	3.52E + 03	−5.6	3.42E + 03	−8.3
1	3.73E + 03	3.52E + 03		3.42E + 03	
1	3.73E + 03	3.52E + 03		3.42E + 03	

Accuracy is indicated by to the deviation from MDMM results. In these examples only in-plane loads are applied and there are no moment resultants, thus rows in each case show the load ratio (Eq. (15)) corresponding to $N_xN_yN_{xy}$.

Table 9
Further example load cases (including moment resultants) to determine the accuracy of multi-layer analysis methods

Load ratios (α)	MDMM	ADMM		QSFT	
	Calculated allowables	Calculated allowables	% Difference	Calculated allowables	% Difference
Case 1					
0					
0					
0					
1	1.68E − 03	1.60E − 03	−4.8	1.54E − 03	−8.3
0					
0					
Case 2					
0					
0					
0					
1	2.59E − 03	2.67E − 03	+3.1	2.72E − 03	+5.0
1	2.59E − 03	2.67E − 03		2.72E − 03	
0					
Case 3					
1	6.14E + 03	5.64e3	−8.1	5.50E + 03	−10.4
0					
0					
1	8.06E − 04	7.42E − 04		7.22E − 04	
0					
0					

Accuracy is indicated by the deviation from MDMM results. Rows correspond to load ratios (Eq. (15)) for $N_xN_yN_{xy}M_xM_yM_{xy}$.

halved. In the compressive layer, the applied moment offsets applied tensile force resultants. The critical force resultant is roughly doubled here, however, the tensile layer is the limiting case for ultimate failure.

For the load cases shown in Tables 8 and 9, the ADMM and the QSFT show an average of 5.2% and 5.5% deviation, respectively, with respect to multilayer direct micro mechanics. Most often, this error is conservative in comparison to the MDMM simulation.

5.5. Practical examples to illustrate strength prediction of a two-layer textile composite plate

In order to show the application of the preceding failure prediction approaches to practical examples, design of a two-layer plain weave-textile plate is considered. Also shown is an example of a closed-end thin-walled pressure vessel. Classical analysis procedures are employed to determine the loads within the plate (force and moment resultants) that are then input to the various failure analysis techniques.

As shown in Fig. 2, a uniform pressure is applied to a simply supported plate. Three different plate sizes, as shown in Table 10, are considered to explore the different mechanical regimes of varying width-to-thickness ratios and to consider a square versus rectangular geometry. Plate theory [18] is employed to determine the force and moment resultants at each point in the plate, which are then checked for failure.

Once moments and curvatures (per unit pressure) have been determined from plate theory, failure in the plate is analyzed via the multi-layer analysis techniques of the previous sections. From this the maximum allowable pressure can be determined, and results for each method are compared. The most reliable method is direct simulation through the MDMM. This again provides a basis of comparison for the remaining methods.

The second method represents a conventional approach, which employs failure analysis methods not developed for

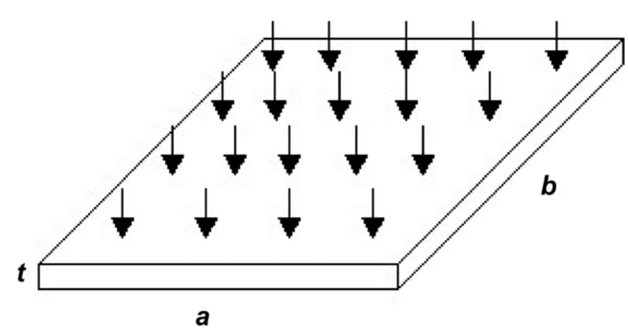

Fig. 2. Schematic of the simply supported two-layer textile plate under uniform pressure.

Table 10
Geometry of the simply supported textile plate under uniform pressure

	a	*b*	*t*
Case 1	0.102 m	0.102 m	0.508 mm
Case 2	0.0102 m	0.0102 m	0.508 mm
Case 3	0.102 m	0.051 m	0.508 mm

textile composites, and for which stress gradients are not considered. Classical analysis techniques are used to find strains and stresses, which are compared to a conventional failure theory. The third method is application of the quadratic stress-gradient failure theory. The fourth method is the aforementioned ADMM. For each case, the plate is discretized into a 21 by 21 point grid of points that are each checked for failure (441 total points).

The conventional method is accomplished by determining the curvature per unit pressure at each point via the shear deformable plate theory. Eq. (13) then determines strains as per classical lamination theory. Stress is calculated by approximating a stiffness matrix $[Q]$ from the $[A]$ matrix (determined via the DMM) as indicated by Eq. (16), and multiplying by the corresponding strain. It should be noted that this in itself can represent an improvement over conventional methods, as a stiffness matrix would generally be calculated from homogenized material properties or estimations rather than from direct simulation or experiment. However, unlike bending properties, these methods can often be acceptable for in-plane stiffness properties.

$$\sigma = Q\varepsilon \approx At\varepsilon \tag{16}$$

In the above, t is the thickness of a layer and A represents the in-plane stiffness matrix of a layer. This stress can then be compared to a maximum allowable stress via the Tsai–Wu failure theory (for which failure coefficients can be found via the DMM or experimental methods).

The procedures for the direct simulation (MDMM) as well as the QSFT and ADMM methods have been detailed above, which is not repeated. In these cases, the maximum allowable moment per unit pressure is found, and the maximum allowable pressure can then be compared.

Table 11 tabulates the maximum allowable pressure for each of the three geometries under consideration, for each of the four prediction methods. The relative accuracy of prediction is indicated as deviation (error) from that of MDMM direct simulation. Again, a positive error indicates a non-conservative prediction.

For all three cases, the QSFT and ADMM represent a significant improvement over the conventional approach. This is due to the presence of significant stress gradients across the thickness dimension of the RVE (as accounted for with the moment resultants), contrary to common isostrain assumptions used in textile micromechanics or failure theory development. The relative accuracy of conventional methods increases somewhat for Case 3. The disparity between the conventional and DMM-based approaches, which include consideration of stress gradients, will diminish as the relative presence of stress gradients diminish with respect to other loads present.

In general, the ADMM will be more accurate than the QSFT, as the QSFT is an approximation method which is slightly further removed from the developmental data. Though both methods involve multi-layer approximations, the QSFT must also approximate the DMM stress field data. For Case 1 versus Case 2, the agreement of the two mutli-layer analysis methods (ADMM or QSFT) with the direct FEM simulation is similar, for both mechanical regimes. Although the low aspect ratio plate is naturally able to withstand a much higher pressure, prediction accuracies are similar. These predictions are both non-conservative, though Tables 8 and 9 have shown that this is not generally true. For Case 3, these methods are conservative compared to the MDMM. In this case, at the failure point, there is a different load ratio ($M_x = 2M_y$, rather than the $M_x = M_y$ of Case 1 and 2), thus a different portion of the failure space is being predicted.

As most often seen in previous [1,2] results, the initial failure mode for these design cases is transverse failure of the fiber tow, beginning in the tensile layer. This represents a fiber pull-apart initiation, or an intra-tow matrix cracking. For the unbiased ($M_x = M_y$) biaxial bending of Cases 1 and 2, failure initiates in both fiber tows, as the transverse stresses will be equal for both tows.

In contrast to the design of a textile plate under uniform pressure, there are other common design cases for which there would be no improvement in accuracy in employing the QSFT or ADMM rather than conventional methods. We consider the problem of a thin-walled pressure vessel in which a biaxial stress state with negligible stress gradients will exist. In this case, conventional methods will predict similar maximum allowable pressure when compared to the QSFT or ADMM. The wall thickness is assumed as 0.508 mm and the radius to thickness ratio as 20. Force resultants are found from basic thin-walled pressure vessel theory [19]. Since there are no moment resultants in thin-walled cylinder, this example serves as a comparison of the MDMM simulation and the QSFT, contrasted to conventional methods, for a test case in which stress gradients are small. Results for this design case are shown in Table 12.

Table 11
Maximum allowable pressure for the textile plate of Fig. 2 for the 3 cases described in Table 10 as predicted from various multi-layer analysis methods

	Case 1 ($a = b$, $a/t = 200$)		Case 2 ($a = b$, $a/t = 20$)		Case 3 ($a/b = 2$, $a/t = 200$)	
	p_{max} (kPa)	% Difference	p_{max} (kPa)	% Difference	p_{max} (kPa)	% Difference
DDMM	19.5	N.A.	135.3	N.A.	34.7	N.A.
Conventional	24.0	23.1	169.1	25	40.6	17
QSFT	20.5	5.1	142.1	5.0	32.6	−6.1
ADMM	20.1	3.1	139.4	3.0	33.7	−2.9

The efficacy of each method is measured from its deviation from MDMM.

Table 12
Maximum allowable pressure for a textile composite pressure vessel as predicted by various multi-layer analysis methods

	p_{max} (MPa)	% Difference
MDMM	1.89	N.A.
Conventional	1.93	+2.1
QSFT	1.86	−1.6

As expected, it is seen that predicted maximum allowable pressures are similar for all methods. Although there are stress gradients along the radial direction, the variation is relatively small, thus the moment resultant that is present will be nearly negligible. These results will hold true for any thin-walled pressure vessel.

6. Summary

In this paper micromechanical methods are used to analyze the failure of multilayer textile composite. A two-layer plain-woven graphite/epoxy composite is used as an example. Three methods are proposed and they are compared with conventional laminated plate theory type analysis. The first method is the multi-layer direct micromechanics (MDMM) which is computationally intensive, but can be considered as more accurate than any of the other methods. The second method is adapted direct micromechanics (ADMM) in which a single layer of interest is analyzed using micro-mechanics. The force and moment resultants in that layer are obtained from structural analysis. The third method is the simplest, in which a quadratic stress gradient failure theory (QSFT) is applied to the layer of interest. In the fourth method, conventional analysis, the stresses are obtained using laminated plate theory and stress based failure theories are used. It is found that ADMM and QSFT compare well with the detailed and expensive MDMM. The two theories predict failure loads within 6% compared to MDMM. The ADMM is still expensive because of the use of micromechanics. However, QSFT is a phenomenological model and is highly amenable to failure analysis of textile composite plates. The conventional theory fails to capture the stress gradient effects, and hence is not suitable for textile composites. Several simple but significant design cases have been presented as a practical application of the methods presented in this dissertation. Multi-layer failure prediction methods have been shown to be sufficiently accurate, and the importance of the consideration of stress gradients in a common design situation is shown.

Acknowledgements

This work was performed under Army Research Office contract DAAD19-02-1-0330 with Dr. Bruce LaMattina as the Grant Monitor. This support is gratefully acknowledged.

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