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# Passive damping of prestressed composite structures

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**Abstract**

This paper presents some analysis techniques to estimate the passive damping ability of viscoelastically damped, fiber–reinforced, polymer composite materials. The potential use of passive damping treatments to further enhance the damping ability of composite structural elements is discussed. Experimental comparisons are provided wherever possible.

**Key words** damping, loss factor, finite element

## 1. Introduction

The ability to tailor the properties of composite materials / structures to suit a specific requirement has been recognized as a primary advantage gained by using composite materials. The inherent damping ability of a composite material is one such property which can be adjusted by varying the different fiber and matrix related parameters. Several automotive and aerospace related applications of polymer reinforced composite materials require improved damping characteristics from the materials. The loss in damping ability due to the absence of such damping mechanisms is felt severely in, for example, space structures for communications and space telescopes, where vibration due to maneuvering is an important design concern. Active damping mechanisms may be employed to reduce this problem, but the advantages of higher passive damping are obvious. Also, passive damping is necessary for structural stability.

## 2. Formulation of finite element equations

Finite element equations were developed by using the principle of incremental virtual work [10]. In the current problem, a sequence of two motions is considered to include the effect of a static preload. The first increment is due to the application of known static preload, and the second increment is due to a dynamic load superposed on this known static configuration. The total Lagrangian definition of motion is used where all static and kinematic variables are referred back to the natural undeformed configuration. The formulation discussed here includes all kinematic nonlinear effects due to large strain and rotation, however, the final equations are linearized so that both increments of motion, the part due to the static load and the part due to the dynamic load are linear elastic. The formulation models a stress stiffening effect which causes a change in the stiffness within the element. Physically, it represents the coupling between inplane and transverse deflections within the structure, and is modelled as a higher order effect over the small deflection solution. The stress stiffening matrix is represented by an additional stiffness matrix, $[K_g]$. The mechanism is often used in flexible structures to increase the lateral load carrying capacity of member.

Consider a body whose initial configuration is denoted by $C_0$ and in which cartesian coordinates $X_i$ are assigned to a point in the structure. After subsequent deformation of the body, the position of the same particle is given by, $x_i$, in its current configuration $C_1$. This state is the intermediate state caused by a known preload. State $C_2$ is the final state to be determined after the final increment of load is applied. The configuration in the final state is actually evaluated by calculating the incremental solution between the states $C_1$ and $C_2$ and updating the $C_1$ state deformation.

The principle of virtual work [10] in state $\alpha$ is written using indicial notation as,

$$\int_{V0} ({}_\alpha T_{ij}\delta_\alpha E_{ij} + {}_0\rho_\alpha \ddot{u}_i \delta u_i)\mathrm{d}V = \int_{V0} {}_\alpha f_i \delta u_i \mathrm{d}V + \int_{\partial V0} {}_\alpha t_i \delta u_i \mathrm{d}A$$

where

$T_{ij}$ = Piola – Kirchhoff stress tensor
$E_{ij}$ = Green's strain tensor
$f_i$ = body force
$t_i$ = surface traction
$u_i$ = displacement
$\ddot{u}_i$ = acceleration

predict the linear, damped, steady–state response of structures about a linear preloaded equilibrium configuration. By following the finite element discretization procedure, the equations governing the time dependent response of a prestressed body is obtained as

$$[[K]+[K_g]]\{U\}+[M]\{\ddot{U}\}=\{F\}$$

where

$[K]$ = global stiffness matrix
$[K_g]$ = global stress stiffening matrix
$[M]$ = global mass matrix
$\{U\}$ = global displacement vector
$\{\ddot{U}\}$ = global acceleration vector

An excitation force, harmonic in time, is considered; that is,

$$F=fe^{i\omega t}$$

where $\omega$ is the forcing frequency, $t$ the time and $i=(-1)^{1/2}$. The response due to the applied force is assumed to be harmonic and vibrating at the excitation frequency. The equation of motion reduces to

$$[[K]+[K_g]]\{U\}-\omega^2[M]\{U\}=\{f\}$$

where $[K]$ and $\{U\}$ are complex valued.

$$[K]=[K]^*\{[I]+i[\eta]\}$$
$$\{U\}=\{U\}'+i\{U\}''$$

Substituting into the above equation and equating the real and imaginary parts, we have

$$([K]^*+[K_g])\{U\}'-[K]^*[\eta]\{U\}''-\omega^2[M]\{U\}'=\{f\}$$
$$([K]^*+[K_g])\{U\}''+[K]^*[\eta]\{U\}'-\omega^2[M][U]''=\{0\}$$

The matrix on the left–hand side is called the displacement impedance matrix. For each frequency of excitation, the displacement per unit applied force is calculated by solving the system of complex–valued simultaneous linear equations and the response function is thereby generated. The loss factor is calculated from the generated response spectrum by using the half–power–bandwidth technique [11], or from the real part of the spectrum as shown in Fig.1(b).

$$[[K]+[K_g]]\{U\}=\omega^2[M]\{U\}$$

The natural frequencies and mode shapes are calculated about the initially stressed state for the undamped system. The stiffness matrix and the corresponding nodal displacements are real since only the real eigenvalue problem is solved. Modal damping is calculated by this technique by using the modal strain energy method. This technique is valid only for systems with relatively small levels of damping where the mode shapes and frequencies of the damped and undamped structure are similar, therefore, causing the fraction of strain energy stored in each element to also be similar. The system loss factor is calculated as the weighted sum of the loss factors of each individual element, where the weighting factor is the fraction of the strain energy stored in the element. Due to the stiffening effect of the prestress a significant amount of energy will be stored in the beam due to an apparent stiffness increase. The weighting factors have to be modified to account for this increase in stored energy since the dissipated energy per cycle does not change very much. The loss factor for vibration about prestressed configuration is calculated by

$$\eta=\sum_{i=1}^{i=n}\eta_i E_i^m \Big/ \sum_{i=1}^{i=n}E_i$$

where

$n$ = total number of elements

$\eta_i$ = loss factor of the $i$th element

$E_i^m$ = elastic strain energy stored in $i$th element calculated as $1/2\{u\}^T[k]\{u\}$

$E_i$ = sum of elastic strain energy and strain energy due to preload in the $i$th element calculated as $1/2\{u\}^T[k+k_g]\{u\}$

The modal strain energy based method is the most popular because of the computational efficiency of the method. A variation of the modal strain energy method has been used [12] where the displacement impedance matrix, discussed under the direct frequency response method, is solved over a range of frequencies to locate the resonant frequency. The deflected shape at resonance of the damped structure is used to calculate the strain energy fractions stored in the element. The accuracy of this result depends on factors obviously and the computational benefits of the method discussed earlier are lost. Even though an assumption of small damping is made in the formulation, accurate results have been reported for core loss factors as high as 1.

## 4. Experimental procedure

The most common methods used to measure damping are the free vibration decay method,

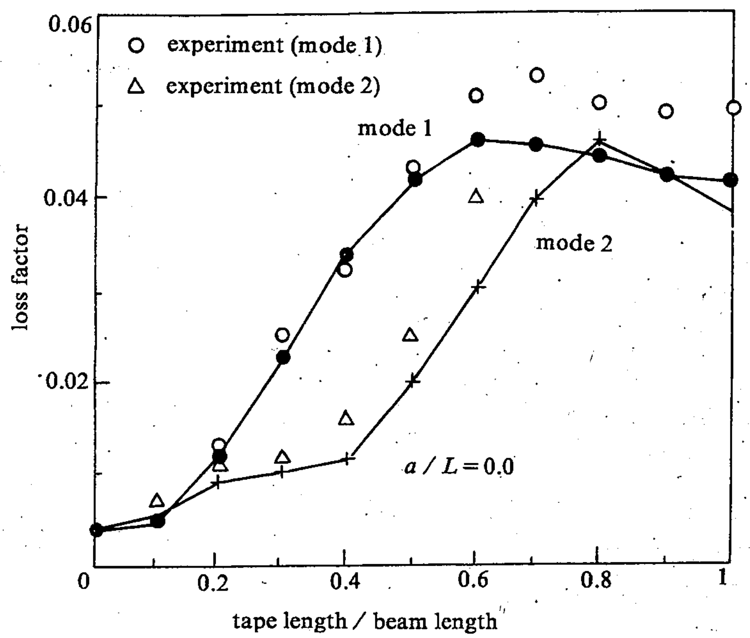


Fig.3 Analytical and experimental results (glass / epoxy base beam)

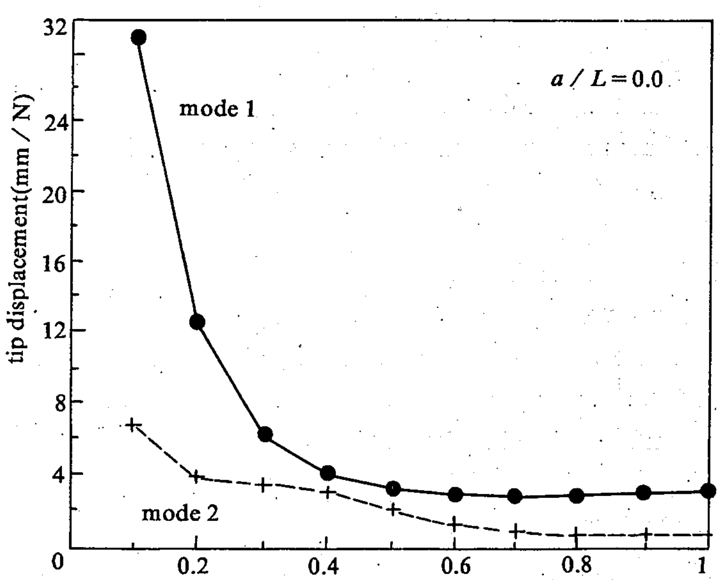


Fig.4 Variation of tip displacement (glass / epoxy base beam)

preload of 45N, the analytical and experimental first mode frequencies are significantly different. At higher preloads the results of frequency are in good agreement with the experimental results before any failure is initiated. For the loss factors, however, the analytical and experimental results are very close for the taped beams and differ considerably for the bare beams. The physical explanation is not clear at this moment. Failure is believed to be initiated at the point at which the loss factor shows an increase with increasing preload in the experimental result. At high preloads the difference between finite element results and experimentally measured ones is due to the problems being associated with measurement of small levels of damping.

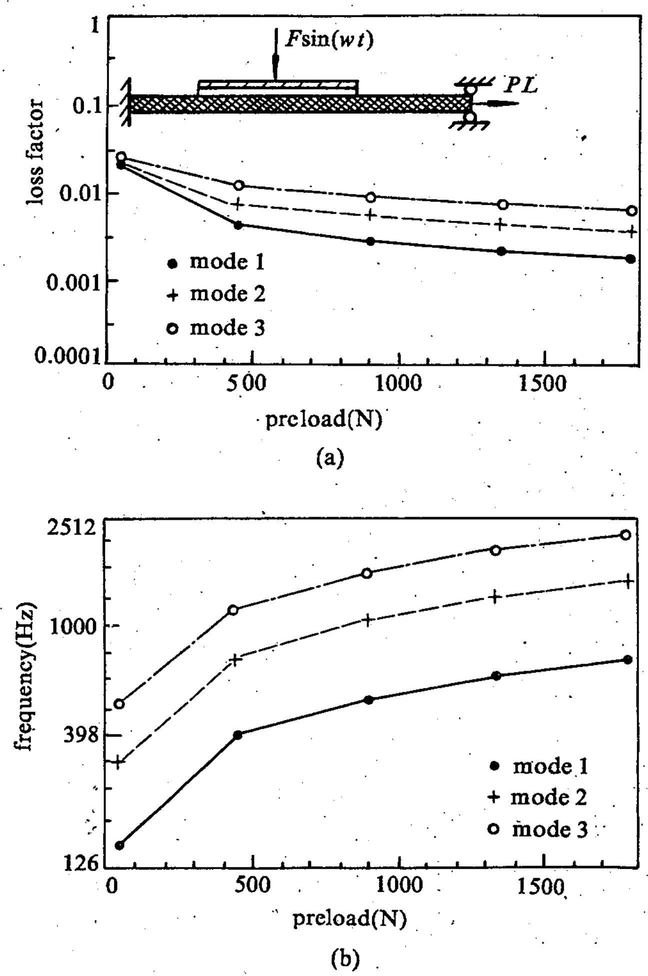


Fig.5 Variation of loss factor and frequency with preload (20% taped from the root).

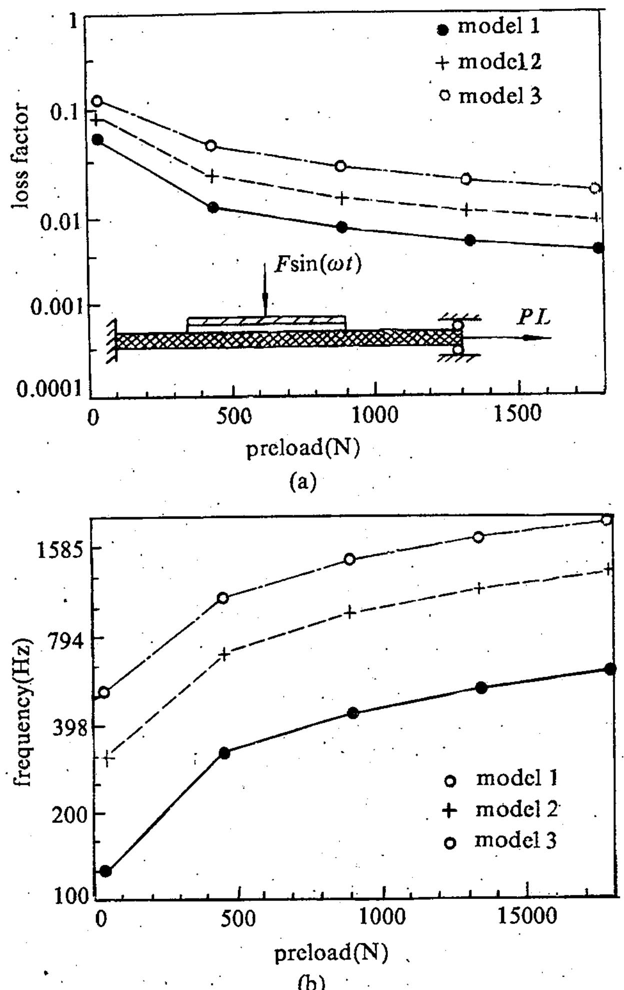


Fig.7 Variation of loss factor and frequency with preload (60% taped from the root).

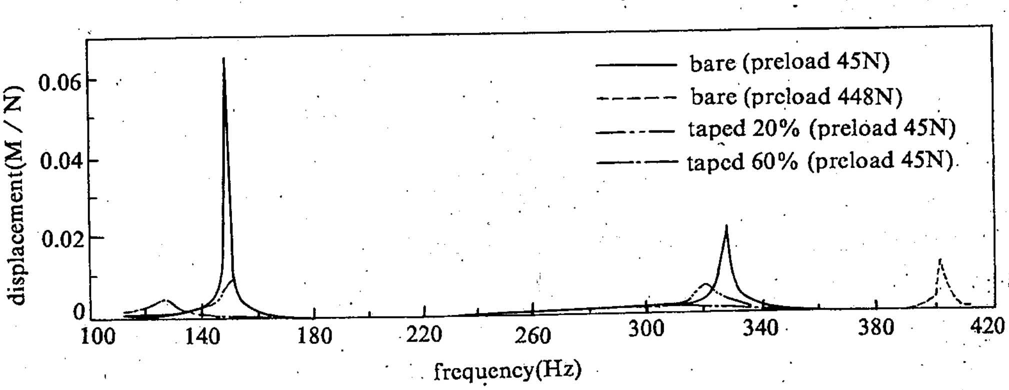


Fig.8 Response spectra for different preloads and tape lengths.

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