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A Plate Finite Element for Modeling Delaminations

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ABSTRACT: Laminate constitutive relations are derived for a composite plate, the midplane of which is offset from the reference x-y plane. The equations are similar to the conventional laminates except that the definition of the $[A]$, $[B]$, and $[D]$ matrices are different. This offsetting procedure is convenient for finite element modeling of delaminations. Starting with the three-dimensional J-integral, an expression for strain energy release rate for delaminations is derived. Unlike the crack closure method, the G can be derived with one single computation; further, the distribution of G along the delamination front can also be obtained. The method is illustrated for an anisotropic DCB specimen.

INTRODUCTION

WITH THE INCREASING use of laminated composites in a variety of structures, there is a need for efficient analytical/numerical methods for the purpose of analysis and design. Composite structures may delaminate during processing or service. Although nondestructive inspection techniques are available to locate delamination damage, they are expensive and cannot be frequently used. Hence, the designer has to allow for some delaminations in designing a composite structure. Then it is necessary to have analysis tools that can be used to study the effects of delaminations on the performance of the structure.

The effects of delaminations are: (a) reduction in static load carrying capacity of the structure; (b) an additional mode of instability is introduced, for the sublaminates may buckle at a reduced magnitude of inplane loads; (c) delaminations may become unstable under dynamic loads, such as impact, and lead to catastrophic failure; (d) the natural frequencies of vibration and mode shapes may be altered; and (e) structural damping may increase due to the friction between delamination surfaces. The strain energy release rate (G) is a convenient parameter to predict the onset of delamination propagation. In analyzing delaminations under static loads, G has to be evaluated along the delamination front. The delamination will propagate at sites where G is greater than a critical value G_c for

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the particular material and layup. A similar fracture mechanics approach can be used to study dynamic delamination propagation and post-buckling of delaminated plates. The objective of our research is to develop a shear deformable, laminated plate finite element with nodes offset to either surface of the plate, and to demonstrate its efficient use in analyzing all of the above problems. However, the present article will be concerned with the development of the plate element, its efficiency and accuracy compared to other formulations, and its usefulness in computing the strain energy release rate along the delamination front.

Our research group has developed a beam finite element with offset nodes for analyzing static analysis of delaminated beams (Sankar, 1991), hygro-thermal and free edge stress analysis (Sankar and Pinheiro, 1990), dynamic delamination propagation due to impact loading (Sankar, Hu and Sun, 1989; Hu, 1990; Sankar and Hu, 1991), and damping due to constrained viscoelastic layers (Sun, Sankar and Rao, 1990; Rao, Sankar and Sun, 1990). Buckling of delaminated plates has been studied by Shivakumar and Whitcomb (1985), and post-buckling of elliptical delaminations was studied by Whitcomb and Shivakumar (1990) and Flanagan (1987). Grady and Sun (1986) performed impact tests on delaminated beams to study crack propagation under dynamic loading. Crews, Shivakumar and Raju (1989) performed a three-dimensional analysis of DCB specimens to find the effects of anticlastic curvature on G distribution.

In the following sections, we describe the derivation of a laminated plate finite element with offset nodes, and its application in computing the strain energy release rate along the delamination front. An expression for strain energy release rate along the delamination front for plate-like structures is derived from the three-dimensional J-integral. Numerical examples are given for the case of a double cantilever beam specimen.

FINITE ELEMENT FORMULATION

A plate finite element model is developed for the analysis of delaminations in laminated composite plates. The relations between the force and moment resultants, and the strains and curvatures, are derived for a reference plane located at arbitrary distance z_0 from the midplane of the laminate. With such a formulation, it is possible to easily derive the element matrices for nodes located at a non-zero distance away from the midplane. Another feature of the element is that the coupling between bending, stretching and twisting is completely accounted for. The derivation is based on the Reissner-Mindlin theory, which accommodates transverse shear strains and requires C^0 continuity only. Most current plate elements are derived based on this theory since a variety of interpolatory schemes can be used, and the problems associated with the classical theory are avoided. The assumed displacement field is

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) + z\theta_1(x,y) \\
v(x,y,z) &= v_0(x,y) + z\theta_2(x,y) \\
w(x,y,z) &= w(x,y)
\end{aligned}
\tag{1}
$$

where

u = displacement in the positive x direction
v = displacement in the positive y direction
w = displacement in the positive z direction
θ_1,θ_2 = rotations
u_0,v_0 = displacements in the plane of definition of the nodes

The stress-strain relation for each ply with respect to the laminate axes is

$$\begin{Bmatrix} \sigma_I \\ \sigma_S \end{Bmatrix} = \begin{bmatrix} [\bar{Q}] & [0] \\ [0] & [\bar{Q}_S] \end{bmatrix} \begin{Bmatrix} \epsilon_I \\ \epsilon_S \end{Bmatrix} \tag{2}$$

$[\bar{Q}]$ and $[\bar{Q}_S]$ are the transformed ply stiffnesses, and σ_I and σ_S stand for inplane stress and transverse shear stresses, respectively. The strain-displacement relations are

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \\ \epsilon_{xz} \\ \epsilon_{yz} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \\ w_{0,x} + \theta_1 \\ w_{0,y} + \theta_2 \end{Bmatrix} + z \begin{Bmatrix} \theta_{1,x} \\ \theta_{2,y} \\ \theta_{1,y} + \theta_{2,x} \\ 0 \\ 0 \end{Bmatrix} \tag{3}$$

By using the above stress-strain and strain displacements, the strain energy per unit area of the laminate is calculated as

$$U = \frac{1}{2}\{E\}^T[S]\{E\} \tag{4}$$

where the vector $\{E\}$ is

$$\{E\} = [u_{0,x}v_{0,y}(u_{0,y} + v_{0,x})(w_{0,x} + \theta_1)(w_{0,y} + \theta_2)\theta_{1,x}\theta_{2,y}(\theta_{1,y} + \theta_{2,x})]^T \tag{5}$$

and, the matrix $[S]$ is

$$[S] = \begin{bmatrix} [A] & [B] & [0] \\ [B] & [D] & [0] \\ [0] & [0] & [G] \end{bmatrix} \tag{6}$$

The submatrices $[A]$, $[B]$, $[D]$ and $[G]$ are defined below.

$$A_{ij} = \int_{z_0-h/2}^{z_0+h/2} [\bar{Q}]dz \qquad i,j = 1,2,3 \tag{7}$$

$$B_{ij} = \int_{z_0 - h/2}^{z_0 + h/2} z[\bar{Q}]dz \qquad i,j = 1,2,3 \tag{8}$$

$$D_{ij} = \int_{z_0 - h/2}^{z_0 + h/2} z^2[\bar{Q}]dz \qquad i,j = 1,2,3 \tag{9}$$

$$G_{ij} = \int_{z_0 - h/2}^{z_0 + h/2} [\bar{Q}_s]dz \qquad i,j = 1,2 \tag{10}$$

z_0 is the distance of the midplane of the laminate from the x-y reference plane, and h is the thickness of the laminate.

By using this formulation, the stiffness matrix of the isoparametric, variable-eight-node, laminated plate element with node-offset capability is calculated. The calculations follow standard finite element discretization procedure. When thin plates are modeled using this element, shear locking problems, which are inherently present in such formulations, can cause the solution to be very stiff. Reduced integration of the overall energy is often used to avoid this problem, but this inexact evaluation of the stiffness matrix may introduce spurious modes which can corrupt the solution.

AN EXPRESSION FOR STRAIN ENERGY RELEASE RATE

We assume that there is a single dominant delamination in the plate. Further, we assume that the delamination will continue to grow in the same plane, and the possibility of matrix cracking and delamination initiation in adjacent interfaces is not considered. The entire laminate is divided into two sublaminates, one on either side of the plane of delamination. The two sublaminates are modeled by offset node finite elements such that in the uncracked portion of the plate, they share common nodes. In the delaminated portion, the sublaminates may be connected by gap elements to monitor contact between crack surfaces.

Consider a plane normal to the delamination front and perpendicular to the plate (Figure 1). Let us denote the plane as the 1–3 plane. Let the 1-axis be normal to the delamination front. The 2-axis is on the plane of delamination and is tangential to the delamination front at the point where G has to be found. Consider a zero-area path Γ on the 1–3 plane surrounding the crack tip. The strain energy release rate G can be derived as (Shih, Moran and Nakamura, 1988)

$$G = J = \int_{\Gamma} [W\delta_{1j} - \sigma_{ij}u_{i,1}]n_j d\Gamma \tag{11}$$

where W is the strain energy density, n_j are the direction cosines of the normal

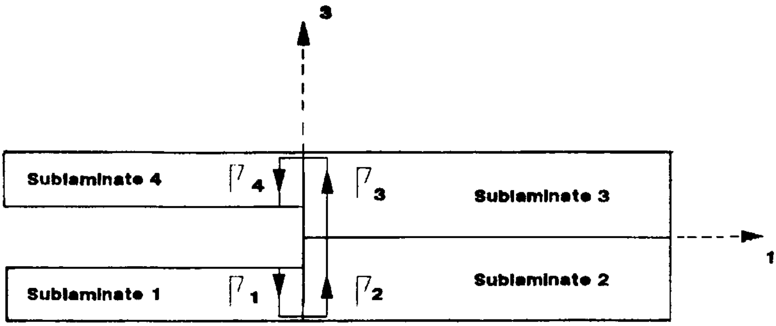

***Figure 1.** Zero-area path for the J-integral.*

to Γ, σ and u are the stress and displacement components, δ is the Kronecker delta, and sum over repeated indices is assumed.

Consider the above integral for the path Γ_1 in the bottom sublaminate (Figure 1). For this path, $n_1 = -1$, and $n_2 = n_3 = 0$. Hence, the integral in Equation (11) becomes

$$G^{(1)} = J^{(1)} = \int_{\Gamma_1} (-W + \sigma_{i1} u_{i,1}) d\Gamma \tag{12}$$

The strain energy density for an elastic solid can be written as

$$W = \frac{1}{2} \sigma_{ij} u_{i,j} \tag{13}$$

Then

$$\sigma_{i1} u_{i,1} = 2W - \Psi \tag{14}$$

where Ψ has the units of W and is given by

$$\Psi = \sigma_{i2} u_{i,2} + \sigma_{i3} u_{i,3} \tag{15}$$

Substituting from Equations (14) and (15) into Equation (12), the J-integral for path Γ_1 becomes

$$J^{(1)} = U^{(1)} - \phi^{(1)} \tag{16}$$

where $U^{(1)}$ is the strain energy per unit are of the sublaminate 1, and $\phi^{(1)}$ is also an energy quantity per unit area given by

$$\phi^{(1)} = \int_{\Gamma_1} \Psi^{(1)} d\Gamma \tag{17}$$

Similarly, the integral for other parts of the path Γ can be derived as

$$J^{(2)} = -U^{(2)} + \phi^{(2)} \tag{18}$$

$$J^{(3)} = -U^{(3)} + \phi^{(3)} \tag{19}$$

$$J^{(4)} = U^{(4)} - \phi^{(4)} \tag{20}$$

Thus, the strain energy release rate G is simply the sum of the integrals for the four sublaminates:

$$G = J = (U^{(1)} + U^{(4)} - U^{(2)} - U^{(3)}) + (-\phi^{(1)} + \phi^{(2)} + \phi^{(3)} - \phi^{(4)}) \tag{21}$$

The stress components σ_{i2} and σ_{i3} and the displacement gradients $u_{i,2}$ and $u_{i,3}$ are continuous between sublaminates 1 and 2, and hence, ϕ is continuous across the zero-area path we have chosen for the evaluation of the J-integral, i.e., $\phi^{(1)} = \phi^{(2)}$. Similarly, $\phi^{(3)} = \phi^{(4)}$. Hence, the expression for G in Equation (21) becomes

$$G = J = U^{(1)} + U^{(4)} - U^{(2)} - U^{(3)} \tag{22}$$

A physical interpretation of the results in Equation (22) is as follows. Let the delamination extend by a small amount $\lambda(s)$, where s is the curvilinear coordinate along the delamination front. Along a small length ds, the area of the new crack surface, dA, created is equal to $\lambda(s)ds$. Now the plate gains dA of sublaminates 1 and 4, and loses equal area of sublaminates 2 and 3. Thus, the change in the strain energy of the plate is

$$\Delta U = \oint (U^{(1)} + U^{(4)} - U^{(2)} - U^{(3)})\lambda(s)ds \tag{23}$$

On the other hand, for constant loading of the plate, ΔU should also be equal to

$$\Delta U = \oint G(s)\lambda(s)ds \tag{24}$$

where $G(s)$ is the strain energy release rate. Thus, from Equations (23) and (24)

$$\oint (U^{(1)} + U^{(4)} - U^{(2)} - U^{(3)})\lambda(s)ds = \oint G(s)\lambda(s)ds \tag{25}$$

If Equation (25) should be true for any arbitrary $\lambda(s)$, then $G(s)$ should be identically equal to $(U^{(1)} + U^{(4)} - U^{(2)} - U^{(3)})$.

This method is valid as long as the delamination is big enough to treat the sublaminates as plates. Unlike the crack closure method, the present method can compute $G(s)$ from a single finite element run of the problem. The force and moment resultants are computed from the finite element results and the strain energy density at the nodes along the delamination front can be computed using Equation (4).

NUMERICAL EXAMPLES

Double cantilever beams made up of aluminum, 0 and 90 unidirectional graphite/epoxy composites are considered. The beam was modeled by using eight-node isoparametric elements. The dimensions of the beam and load applied are shown in Figure 2. The material properties for aluminum are $E = 71$ GPa and $\nu_{12} = 0.3$. The material properties for graphite epoxy are $E_1 = 134$ GPa, $E_2 = 13$ GPa, $G_{12} = 6.8$ GPa, $\nu_{12} = 0.35$ and $\nu_{23} = 0.34$. The results for G along the delaminaion front, and the w-displacement of nodes at a distance of 0.78125 mm from the delamination front, are shown in Figures 3–5. The G variation is more or less uniform, except at the ends. A similar trend was observed by Crews, Shivakumar and Raju (1989) using three-dimensional FE analysis. It should be noted that gap elements were not used in the present study, and hence there is interpenetration of nodes near the free edges in some cases. Hence, the G values in the vicinity of the free edges cannot be considered as realistic. Finer discretization near the free edges and use of gap elements to monitor crack closure are being implemented to obtain more exact estimates of $G(s)$. The present method can be applied to a variety of delamination problems, e.g., elliptical delaminations under compressive loads.

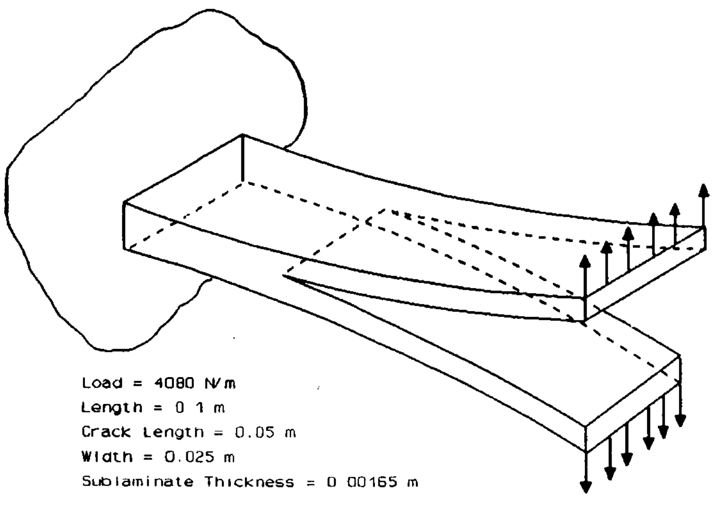

Figure 2. *Double cantilever beam specimen.*

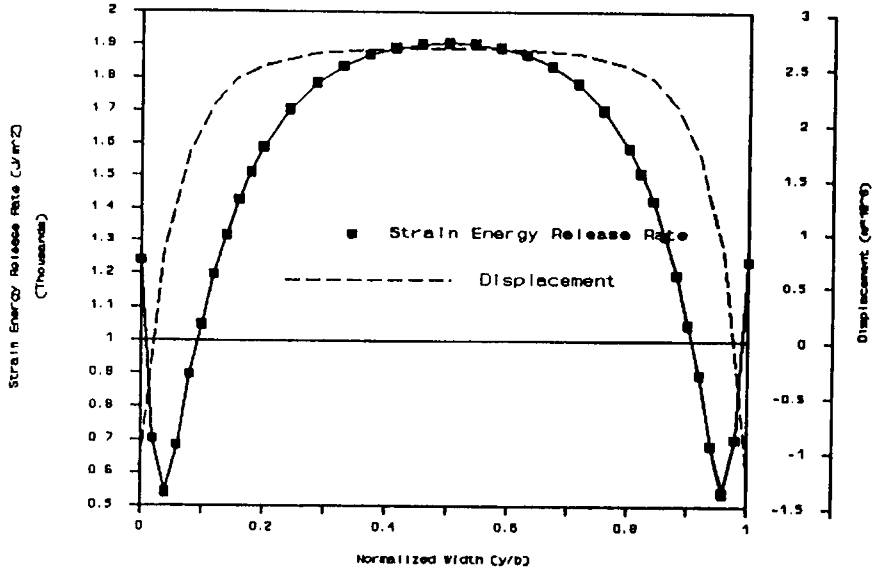

Figure 3. Variation of strain energy release rate and displacement at the crack tip (aluminum).

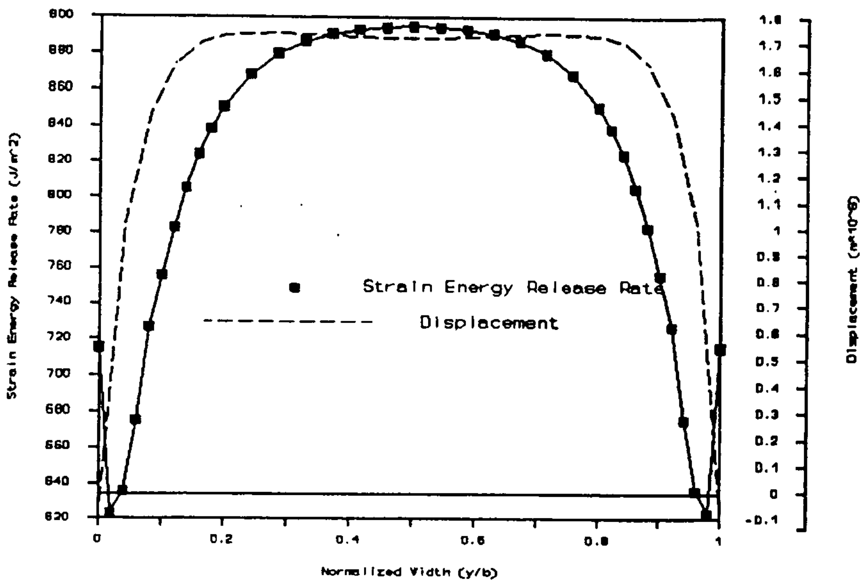

Figure 4. Variation of strain energy release rate and displacement at the crack tip (0° graphite/epoxy).

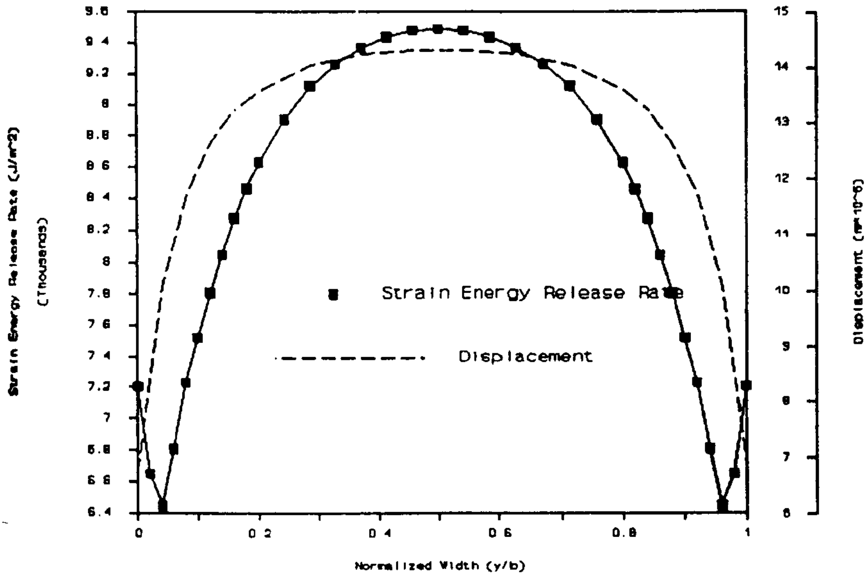

Figure 5. *Variation of strain energy release rate and displacement at the crack tip (90° graphite/epoxy).*

SUMMARY

A shear deformable, laminated plate element with nodes offset to either one of the plate surfaces has been developed. This element is convenient in modeling delaminations. A simple expression for strain energy release rate has been derived starting from the three-dimensional J-integral. The expression has the advantage of computing the strain energy release rate distribution along the delamination front in one single computation.

ACKNOWLEDGEMENTS

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