Pergamon

0013-7944(94)00302-5

EVALUATION OF BIMATERIAL STRESS INTENSITY FACTORS USING A FINITE ELEMENT–BOUNDARY ELEMENT ALTERNATING METHOD

S. LAHIRI, B. V. SANKAR and P. A. MATAGA†

Center for Studies of Advanced Structural Composites, Department of Aerospace Engineering, Mechanics & Engineering Science, P.O. Box 116250, University of Florida, Gainesville, FL 32611-6250, U.S.A.

Abstract—The finite element–boundary element alternating method is extended to the analysis of a crack in a bimaterial test specimen. The crack is perpendicular to, and terminates at, the interface between two dissimilar materials. The finite element method is used to model the finite-size specimen without the crack. The displacement discontinuity method (boundary element method) is used to find the stresses in the infinite bimaterial medium due to crack surface tractions. The stress intensity factors are extracted by using a closed form solution for a semi-infinite crack in an infinite bimaterial medium opened by a pair of wedge forces. The alternating method is verified by solving several bimaterial crack problems for which solutions are known. A parametric study is performed for a specimen proposed for evaluating the fracture toughness of bimaterial interfaces.

1. INTRODUCTION

THE ANALYSIS of the problem of a crack perpendicular to, and terminating at, a bimaterial interface was first analyzed by Zak and Williams [1]. However, it has only been in recent years, since composites technology started gaining popularity, first in the aerospace industry and then in spin-off industries like the automobile and consumer industries, that engineers and scientists felt the need to formulate better failure prediction models for cracks in multilayered dissimilar materials. For example, in a crossply laminate subjected to tension in the 0° direction, the matrix cracks appear in the 90° layers and terminate at the 0°/90° interface [2]. Whether or not these cracks will propagate under the given loading depends on the fracture toughness of the interface. Further, the cracks have two options: to propagate along the interface or into the 0° layer, breaking the fibers. Similar situations arise in ceramic composites also [3].

Currently, there are analysis methods available for determining the near-tip stress field for cracks in infinite specimens, terminating at a bimaterial interface [4, 5, 13]. However, for cracks in finite-size specimens, a method for computing stress fields in the vicinity of the crack tip is yet to be developed. Also, the prediction of crack-tip propagation is an exercise that is yet to be accomplished for cracks terminating at bimaterial interfaces in finite-size specimens. In the present work, we are attempting to calculate a parameter associated with the stress field in the vicinity of a crack tip terminating at the interface of two dissimilar materials in finite-size specimens. This parameter is believed to be responsible for crack propagation, and by means of this quantity, it may be possible to establish crack propagation criteria for problems such as we are interested in. While it is possible to use "brute-force" finite element methods, these kinds of problems need extremely fine meshes at the crack tips in order to capture the correct stress singularities. Hence, accurate stress intensity factors may be obtained only at the cost of running time-consuming and cost-intensive finite element programs.

While testing is a necessity for the understanding of failure mechanisms and also for developing failure criteria, efficient analysis methods must be available for extracting the crack tip parameters from the load and specimen geometry. Consequently, the present study is concerned with the development of a practical analysis procedure for evaluating stress intensity factors in cracked bimaterial test specimens (see Fig. 1). Although a crack may have two crack tips, the term "crack tip" in the present paper refers to the tip perpendicular to and touching the bimaterial interface.

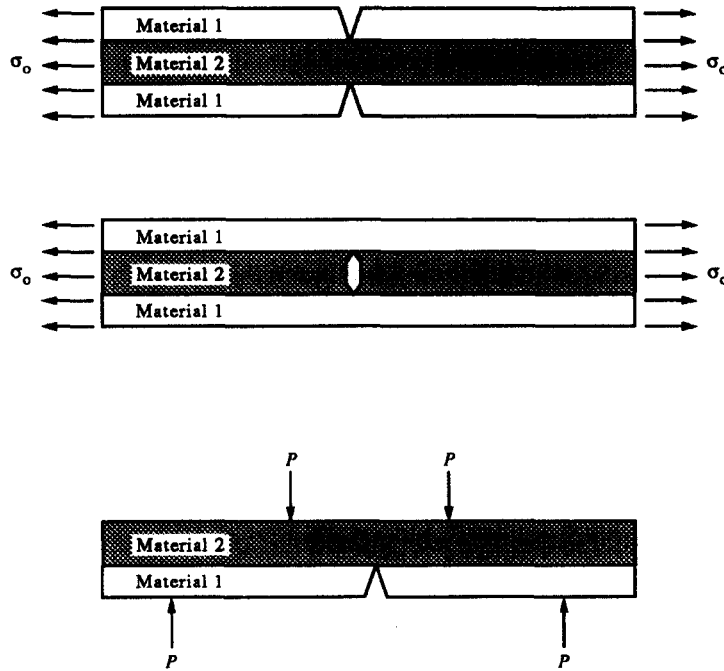†Present address: AT & T Bell Laboratories, Chicago, IL, U.S.A.

Fig. 1. Various bimaterial test specimens.

2. CRACK TIP STRESS FIELDS IN BIMATERIAL MEDIA

The problem with cracks in a bimaterial medium is the nature of the stress singularity. If both materials happen to be the same, then the stresses possess a $1/\sqrt{r}$ singularity at the crack tip. In that case, the strain energy release rate exists and can be computed using conventional methods. From Zak and Williams [1] we know that in the case of bimaterial cracks, the near-tip fields vary as

$$\sigma \sim r^{\lambda-1}$$

$$\epsilon \sim r^{\lambda-1} \tag{1}$$

$$u \sim r^{\lambda}$$

where σ, ϵ and u are the near-tip stresses, strains and displacements, respectively, and r is the distance from the crack tip. In the case of bimaterial systems, λ, which is a parameter based on the stiffness properties of the two materials, can be lesser or greater than $\frac{1}{2}$. The strength of the stress singularity, given by $(\lambda - 1)$, can hence be lesser or greater than $-\frac{1}{2}$ for bimaterial systems; λ is exactly equal to $\frac{1}{2}$ when both materials are the same, and the singularity for such cases is equal to $-\frac{1}{2}$.

The strain energy release rate, designated by G, can always be computed for cracks in homogeneous media. However, as the crack approaches a bimaterial interface, G goes to zero or infinity as shown in Fig. 2. So, in reality, this means that for cracks in bimaterial media, the parameter G cannot be computed and hence cannot be used as a crack propagation criterion. This has been borne out by the fact that

$$G \sim b^{2\lambda-1} \tag{2}$$

where b is the distance between the crack tip and the interface [6]. Hence, it may be observed that for $\lambda > \frac{1}{2}$, $G \to 0$ as $b \to 0$ and for $\lambda < \frac{1}{2}$, $G \to \infty$ as $b \to 0$. Hence, in both instances a finite value of G does not exist.

Some suggestions for computing G in dissimilar media have been the use of virtual crack closure or crack extension methods. The problem with these methods is that as the crack extension Δa becomes very small (see Fig. 3) the value of G goes to zero or infinity, depending on which
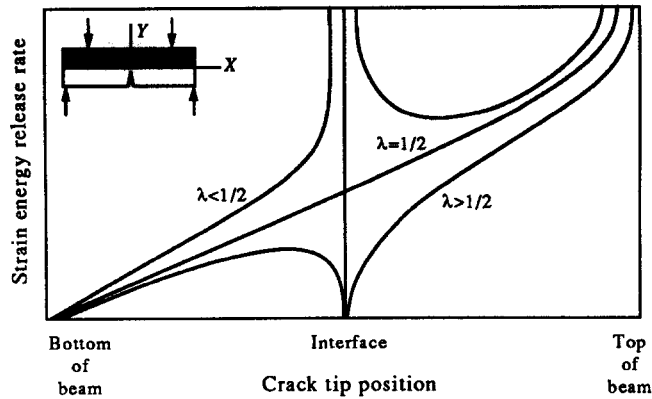
Fig. 2. Schematic of strain energy release rate variation for a cracked bimaterial beam under bending loads.

medium the crack resides in. Hence, G does not approach a unique value as Δa goes to zero and so it cannot be used as a criterion. Ahmad [7] addressed the elastic problem of a crack normal to a bimaterial interface, using finite element analysis, in the context of unidirectional fiber composites. The structure of asymptotic crack-tip stress fields was obtained numerically. The numerical results thus obtained were then used to formulate criteria for assessing cracking normal to the fiber, interface splitting and fiber pull-out.

In the present study, a finite element–boundary element alternating method is proposed for the problem of a crack terminating at a bimaterial interface in a finite sized specimen. In this paper, the symbol K_1 is used to designate the Mode I stress intensity factor for the case when $\lambda = \frac{1}{2}$ whereas the symbol Q_1 is used to designate the corresponding factor for the cases when λ is other than $\frac{1}{2}$.

3. METHODS OF ANALYSIS

In this section, we describe the various analysis tools that have been used in determining the stress intensity factor Q_1 in finite sized bimaterial specimens. A description of superposition and alternating methods can be found in Atluri and Nishioka [8, 12]. The basic principle of the alternating method in the present context is depicted in Fig. 4. Figure 4(a) shows a typical finite-size bimaterial specimen containing a crack and subjected to some external loading. We would like to consider this as a superposition of three different problems, as shown in Fig. 4(b)–(d). The loading case shown in Fig. 4(b) represents the external forces acting on the uncracked specimen. The specimen in Fig. 4(c) possesses a crack, and hence captures the singularities in the problem. It
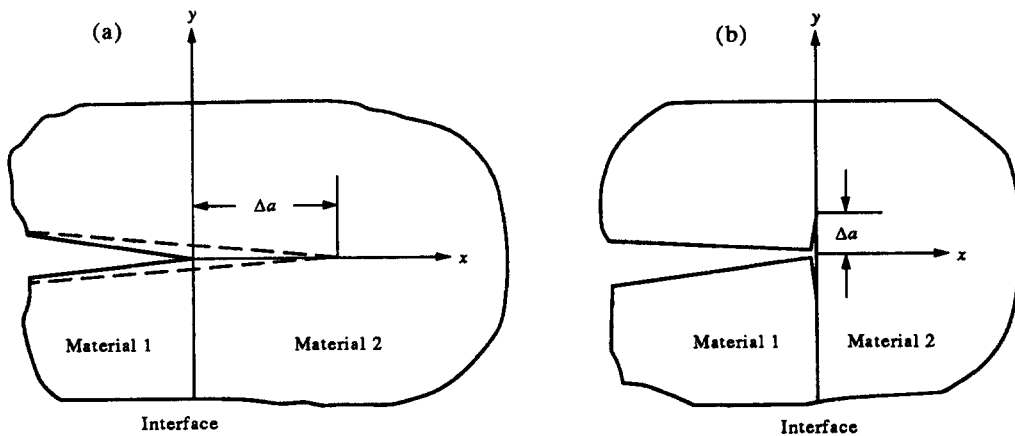

Fig. 3. Application of crack closure method to compute G: (a) crack perpendicular to the interface; (b) crack parallel to the interface.
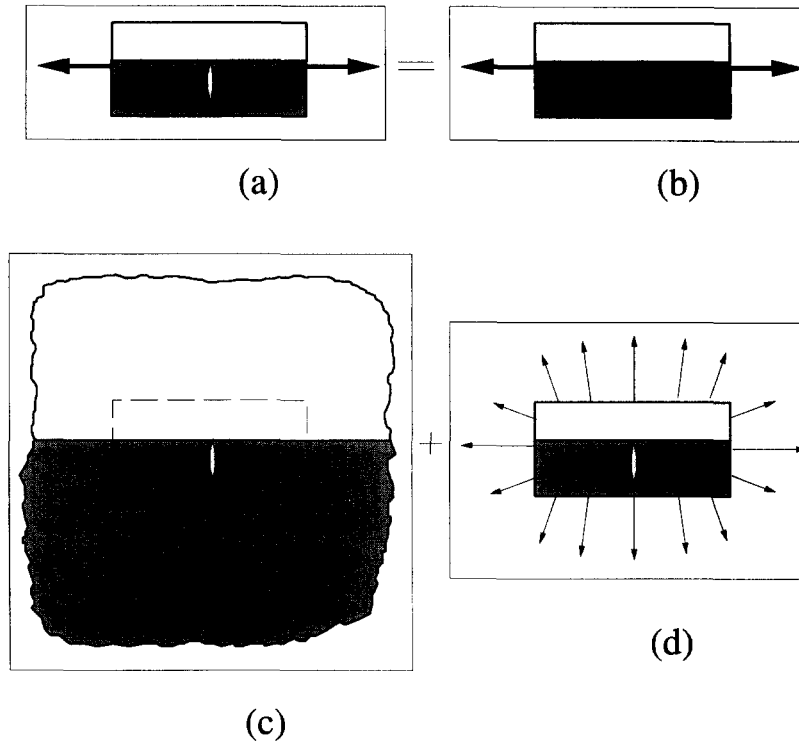
Fig. 4. Superposition scheme for alternating method.

should be mentioned that this specimen is embedded in an infinite bimaterial medium, so that analytical solutions can be used to describe the stresses near the crack tip. In Fig. 4(d), tractions are applied on the surface of the specimen so that the tractions generated in the problem depicted in Fig. 4(c) may be cancelled to make the outer surfaces traction-free. It is interesting to note that the problem represented by Fig. 4(d) is very similar to the problem we started with [Fig. 4(a)], but for the loads acting on the surface of the specimen. In general, the loads in Fig. 4(d) will be much smaller than those in Fig. 4(a) and these small loads will be referred to as "residual loads". The superposition procedure can now be applied to the loading in Fig. 4(d) to obtain the next set of residual loads. The iteration procedure is continued until the residual loads become negligibly small in some sense. The sum of crack surface pressures from the infinite medium problem [Fig. 4(c)] are then used to compute the stress intensity factor using the formulation derived by Cook and Erdogan [9].

The finite element method (FEM) is used to solve the problem depicted in Fig. 4(b). Due to the simple geometry of the problem, four-node plane rectangular elements can be used. Further, since there is no crack in the model, there is no need for fine meshing at the crack tip. A uniformly coarse mesh is used throughout the model. In the following sections, the boundary element method (BEM), a method for computing the stress intensity factor for a crack in an infinite bimaterial medium, and the steps involved in the alternating method, are briefly described.

3.1. Boundary element method

One of the tools used in the development of the present numerical method is a BEM due to Crouch [5]. This method consists of placing N displacement discontinuities of unknown magnitudes along the boundaries of the region to be analyzed (crack surfaces in the present case), then setting up and solving a system of algebraic equations to obtain the discontinuity values that produce prescribed boundary tractions. One may define a displacement discontinuity as a line crack whose opposing faces have been displaced relative to one another by a constant amount. Displacement discontinuities may also be thought of as a continuous distribution of dislocations. A constant displacement discontinuity may be formed by placing a pair of equal and opposite edge dislocations at the two ends of a line segment.
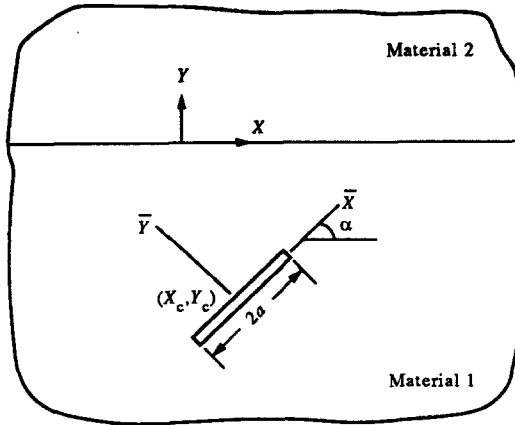
Fig. 5. Displacement discontinuity in an infinite bimaterial medium.

With reference to Fig. 5, we may define the displacement discontinuity d_i as the difference in displacements between the two sides of the line segment as follows:

$$d_{\bar{x}} = u_{\bar{x}}(\bar{x}, 0^+) - u_{\bar{x}}(\bar{x}, 0^-), \quad -a < \bar{x} < a \tag{3}$$

$$d_{\bar{y}} = u_{\bar{y}}(\bar{x}, 0^+) - u_{\bar{y}}(\bar{x}, 0^-), \quad -a < \bar{x} < a \tag{4}$$

where $u_{\bar{x}}$ and $u_{\bar{y}}$ are displacements in the $\bar{x}$ and $\bar{y}$ directions respectively.

Analytical expressions for stresses and displacements at any arbitrary point (x, y) in the infinite bimaterial solid can be derived in terms of $d_{\bar{x}}$ and $d_{\bar{y}}$, the length $2a$ over which the discontinuity exists, and the location and orientation of the discontinuity specified by (x_c, y_c) and α [5]. For example

$$\sigma_{xx}(x, y) = \sigma_{xx}(x, y, x_c, y_c, a, \alpha, d_{\bar{x}}, d_{\bar{y}}) \tag{5}$$

$$u_x(x, y) = u_x(x, y, x_c, y_c, a, \alpha, d_{\bar{x}}, d_{\bar{y}}). \tag{6}$$

Similar expressions may be derived for σ_{yy}, τ_{xy} and u_y also.

A knowledge of the stresses and displacements at any point in the bimaterial medium can be used indirectly to solve the problem of a crack subjected to a known pressure or crack surface traction distributions.

Now let us consider the actual problem of a pressurized crack in an infinite bimaterial medium. It is required to find the crack surface displacements, and also the displacement and stress fields in the medium. The crack is divided into N segments. The crack surface displacement distribution
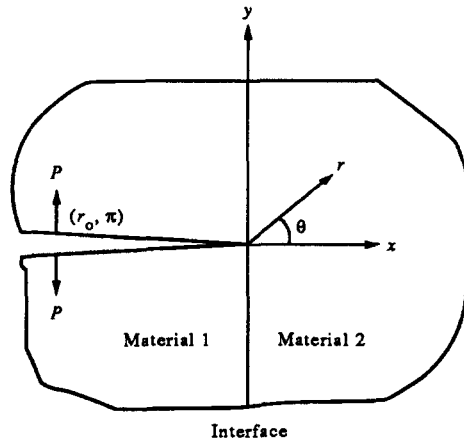

Fig. 6. A semi-infinite crack in an infinite bimaterial medium.

is assumed to be such that the displacement discontinuity is constant over each segment. The stresses and hence the tractions at the midpoint of each segment are expressed as a linear function of the unknown displacement discontinuities using eq. (5) and two other similar equations for σ_{yy} and τ_{xy}. The unknown discontinuities are solved from the condition that the tractions on the crack surfaces should be equal to the applied pressures. Once the displacement discontinuities are known, the stresses and displacements at any arbitrary point can be calculated using eqs (5) and (6). An example of such a problem for the case of a crack not touching the interface is given in Crouch and Starfield [10]. For the case of a crack touching the interface, the procedures are similar except that the distance between the crack tip and the interface is taken as a very small fraction of the actual crack length, say, $10^{-3}a$, in order to avoid numerical difficulties.

3.2. Evaluation of the stress intensity factor

Cook and Erdogan [9] have provided the analytical solution to the problem of a semi-infinite perpendicular crack terminating at a bimaterial interface, using a conjunction of Mellin transforms and dislocation theory to derive the integral equations. Referring to Fig. 6, Material 1 contains a semi-infinite crack along $\theta = \pi$ which is subjected to a pair of concentrated wedge forces P at $r = r_0$. The strength of the stress singularity at the crack tip for a given stress component may be expressed as:

$$Q_{ij}(\theta) = \lim_{r \to 0} \sqrt{2\pi} r^{1-\lambda} \sigma_{ij}(r, \theta). \tag{7}$$

The parameter λ may be solved from the characteristic equation:

$$2\alpha \cos \pi\lambda - (\beta\lambda^2 + \gamma) = 0 \tag{8}$$

where

$$m = (\mu_2/\mu_1)$$

$$\alpha = (m + \kappa_2)(1 + \kappa_1)$$

$$\beta = -4(m + \kappa_2)(1 - m) \tag{9}$$

$$\gamma = (1 - m)(m + \kappa_2) + (1 + m\kappa_1)(m + \kappa_2) - m(1 + \kappa_1)(1 + m\kappa_2).$$

In the above equation, μ's are the shear moduli, κ's are the Muskhelishvili constants, and the subscripts refer to Materials 1 or 2.

For the present problem we define the stress intensity factor Q_0 due to a pair of wedge forces P at $r = r_0$ as

$$Q_0 = \lim_{r \to 0} \sqrt{2\pi} r^{1-\lambda} \sigma_{2\theta\theta}(r, 0). \tag{10}$$

The explicit expression for Q_0 is given in Cook and Erdogan [10] as

$$Q_0 = C\sqrt{2\pi} P r_0^{-\lambda} \tag{11}$$

where the factor C is given by

$$C = \frac{m(1 + \kappa_1)[(1 - 2\lambda)(m + \kappa_2) + (1 + 2\lambda)(1 + m\kappa_1)]}{[2\pi\alpha \sin \pi\lambda + 2\beta\lambda]}. \tag{12}$$

The relevance of the results in eqs (10) and (11) for our problem is explained below. As mentioned earlier the singularities are contained in the infinite domain problem [Fig. 4(c)], and we use the BEM to solve this problem. Theoretically it should be possible to evaluate the stress intensity factor either from the crack surface opening displacements behind the crack tip or the stresses σ_{yy} ahead of the crack tip. However, our experience shows that the singularity in the present case is limited to a very small distance ahead of the crack tip, and a very large number of crack discretizations are needed to capture the singularities. Similar observations were made by Ahmad [7] also. Further,
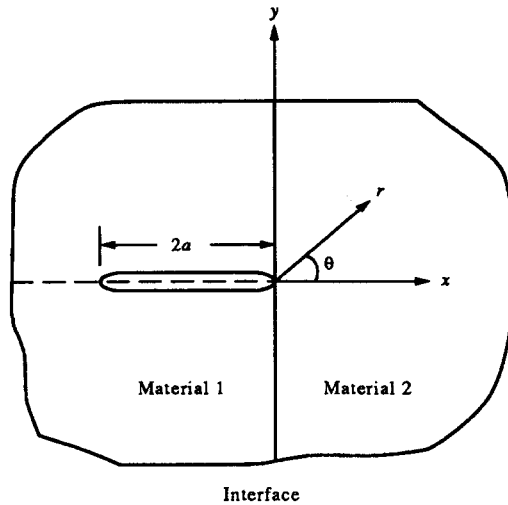
Fig. 7. Geometry for a finite crack touching a bimaterial interface.

this is compounded by the fact that we are assuming a constant displacement discontinuity over each crack segment. Nevertheless, the solutions for stresses and displacements away from the crack tip were found to be good. This is the nature of the solution we would like to exploit in evaluating the stress intensity factor using eqs (10) and (11).

The procedure for computing the stress intensity factor is as follows. The crack surface pressures are obtained from the converged solution of the alternating method (see Section 3.3). Using the BEM one can then solve for the corresponding displacement discontinuities. The stress component σ_{yy} is found along the line segment ($\theta = \pi, 2a < r < \infty$) (Fig. 7). The stress intensity factor for the crack tip at the interface can be computed using Q_0 in eq. (11) as the Green's function:

$$Q_1 = C\sqrt{2\pi}\left[\int_0^{2a} p(r)r^{-\lambda}\,dr - \int_{2a}^{\infty} \sigma_{yy}(r)r^{-\lambda}\,dr \right]. \qquad (13)$$

Here, $p(r)$ are the crack surface pressures, $\sigma_{yy}(r)$ are the tractions ahead of the remote crack tip and the factor C is defined in eq. (12). Since $p(r)$ is constant over each crack segment, the first term on the right-hand side of eq. (13) can be evaluated in closed form. The second term may be integrated using either a Riemann summation or an appropriate quadrature scheme. Although the second integral in eq. (13) extends to infinity, in numerical integration a finite value of the upper limit was used.

3.3. The alternating method

The alternating method uses a combination of FEM and BEM. The FEM is used to solve for the stresses in an uncracked finite body, and the BEM to provide the solution to a pressurized crack in an infinite bimaterial body. The alternating method manages to overcome some of the technical barriers discussed earlier. For example, the extremely fine order of meshing required at the crack tip has been eliminated. This has been made possible because the problem has been split up into two major components. The first is the modeling of the uncracked body using FEM and the second is the modeling of the cracked regime using BEM. Due to the superposition scheme discussed below, we can use relatively coarse meshing for the FE model and slightly finer meshing for the BE model. The steps involved in the execution of the alternating methods are as follows:

(1) Solve the problem of the uncracked body under the given external loads by using FEM. The pressures that would act on the crack surfaces are computed in this step.

(2) Since the crack surfaces are stress-free, the crack surface pressures computed in Step 1 have to be reversed, and applied to the same finite body. However, since we do not have analytical expressions for a finite-crack body, the reversed crack surface pressures are applied on the crack in an infinite bimaterial medium. The BEM explained earlier is used to find the tractions and displacements on surfaces which correspond to the actual surfaces of the specimen.

(3) Since the external surfaces of the body are stress-free (we have taken into account the applied loads in Step 1), the tractions computed in Step 2 have to be reversed and applied to the surfaces of the finite body. We can use the FE model to solve this problem. Now, one may observe that we have reverted back to a problem which is similar to the initial problem except for the external loads acting on the body. As mentioned in Section 3, we will refer to these loads as "residual loads".

(4) Step 1 is repeated with the residual loads acting as external loads.

The residual loads will reduce after each cycle of the iteration. The convergence is checked by comparing similar quantities for two successive iterations. This, for example, can be the norm of the vector of crack surface pressures computed at the end of Step 1. The stresses and displacements for the given problem are obtained by superposing the corresponding results from all the steps used. However, for evaluating the stress singularity we need only the crack surface pressures acting in the infinite medium derived from the results of Step 1.

In the above description we assumed that only tractions are prescribed on the surface of the body. If displacements are prescribed, then the corresponding residual displacements have to be computed in Step 3.

The actual implementation of the alternating method is as follows. The finite element equations can be cast in the form:

$$[K]\{q\} = \{R\} - [K_p]\{q_p\} \tag{14}$$

where

$[K]$ = global stiffness matrix
$\{q\}$ = unknown displacements
$\{R\}$ = applied forces
$[K_p]$ = elements of stiffness matrix associated with prescribed displacements
$\{q_p\}$ = prescribed displacements.

The crack surface pressures $\{p\}$ can be computed from the displacements $\{q\}$ as

$$-\{p\} = [S]\{q\}. \tag{15}$$

Since a negative sign is used with $\{p\}$ in eq. (15), the crack surface pressures in the BEM will then be $\{p\}$ with a positive sign. The BEM solves for the displacement discontinuities $\{d\}$ from $\{p\}$:

$$[A]\{d\} = \{p\} \tag{16}$$

in which $[A]$ is computed using eq. (5). Once $\{d\}$ is found, the tractions and displacements needed in Step 3 can be computed using eqs (5) and (6) as

$$\{t\} = [B]\{d\} \tag{17}$$

$$\{u\} = [C]\{d\} \tag{18}$$

where $\{\ \}$ = stresses due to $\{p\}$ at finite number of points on the imaginary surface corresponding to the boundary of the problem;

$\{t\}$ = tractions due to $\{p\}$ at finite number of points on the imaginary surface corresponding to the boundary of the specimen = $\{\ \}[n]$;
$\{u\}$ = displacements due to $\{p\}$ at finite number of points on the imaginary surface referred to above $[n]$ = direction cosines normal to the boundary segment under consideration.

The sizes of the $[A]$, $[B]$ and $[C]$ matrices are as follows:

$[A]$ = (NMUL) × (NMUL)
$[B]$ = (3 × NLOCS) × (NMUL)
$[C]$ = (2 × NLOCD) × (NMUL)

Table 1. Comparison of normalized stress intensity factors Q_1 for a crack in an infinite bimaterial medium

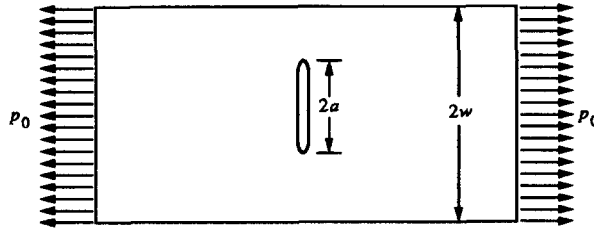| μ_2/μ_1	λ	$p = -p_0$ (Q_{exact})	$p = -p_0$ (Q_{numer})	$p = -p_0\rho$ (Q_{exact})	$p = -p_0\rho$ (Q_{numer})	$p = -p_0\rho^2$ (Q_{exact})	$p = -p_0\rho^2$ (Q_{numer})
23.08	0.6619	2.7845	2.8829	-1.6867	-1.6268	1.6660	1.7078
0.043	0.1752	0.0700	0.0673	-0.0303	-0.0317	0.0314	0.0285

Fig. 8. Center crack in a finite width plate.

where

  NSEG = number of crack discretizations in the BE model;
  NMUL = 2 × NSEG;
  NLOCS = number of prescribed traction location points;
  NLOCD = number of prescribed displacement location points;

The matrices $[K]$, $[K_p]$, $[S]$, $[A]$, $[B]$ and $[C]$ were first computed for the given model and stored in memory. To start the cycle, the applied loads $\{R\}$ and prescribed displacements $\{q_p\}$ are used in eq. (14) to solve for $\{q\}$, then $\{t\}$ and $\{u\}$ are computed using eqs (15)–(18). The second cycle starts with $-\{t\}$ as the applied load ($\{R\}$) and $-\{u\}$ as the prescribed displacements, $\{q_p\}$. The convergence is checked by comparing $\|\{p\}\|$ between two consecutive cycles.

4. RESULTS AND DISCUSSION

Examples have been chosen such that the various numerical tools used in this paper may be verified by comparison with known results. The combination of the BEM (Section 3.1) and the procedure for evaluating the stress intensity factor, Q (Section 3.2) was verified by comparing with the closed form solutions for a finite crack in an infinite bimaterial medium [9]. The problem
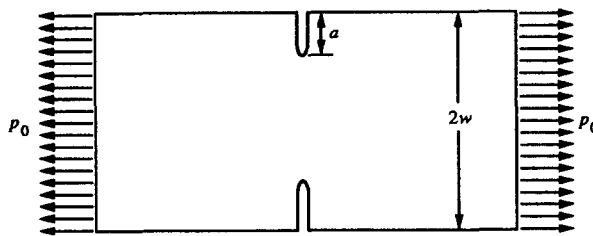

Fig. 9. Symmetric edge cracks in a finite width plate.

Table 2. Comparison of normalized stress intensity factors for finite sized isotropic specimens

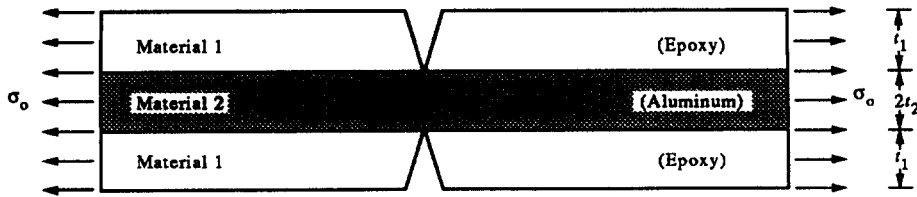| Plate problem	K_{exact}	K_{numer} (alt. meth.)
(1) One centrally located crack	2.7576	2.8100
(2) Two symmetric edge cracks	2.8617	2.8812

Fig. 10. A cracked bimaterial beam under uniaxial tension.

considered was that of a crack of length $2a$ residing in Material 1 (Fig. 7). One of the crack tips terminated at the bimaterial interface. Three types of crack surface pressures were considered: (a) constant pressure $p = -p_0$; (b) linearly varying pressure given by $p = -p_0\rho$; and (c) quadratic variation of pressure given by $p = -p_0\rho^2$, where $\rho = (r - a)/a$. The numerical values of p_0 and a were 1 psi and 1 in., respectively. The results for the stress intensity factor, Q_1, obtained from the present numerical method are compared with the exact solution in Table 1. It may be noted that the present numerical method is in good agreement with the closed form solutions of Cook and Erdogan [9]. In Table 1, μ's are the shear moduli of the respective materials, λ is the bimaterial parameter given by eq. (8), Q_{exact} refers to the results of Cook and Erdogan [9] and Q_{numer} are the results obtained using the present method.

Next, we evaluated the present scheme of the alternating method by considering a homogeneous, isotropic, finite-size cracked specimen. In the first example, we considered a plate with a center crack subjected to uniaxial stresses perpendicular to the crack (Fig. 8). In the second example, the center crack was replaced by two edge cracks (Fig. 9). The stress intensity factors were normalized with respect to $\sigma_0\sqrt{(\pi a)}$. The normalized stress intensity factors for both cases are compared with the exact solution in Table 2. It may be seen that the agreement is quite good. The exact solutions to these two problems can be found in Hellan [11].

Having verified that the proposed methodologies work for infinite bimaterial media as well as for finite isotropic specimens, we proceeded to demonstrate the usefulness of the method by
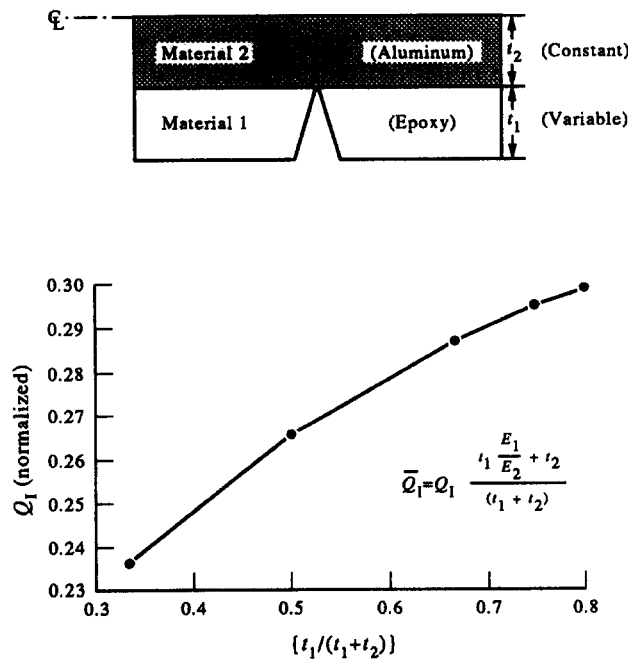

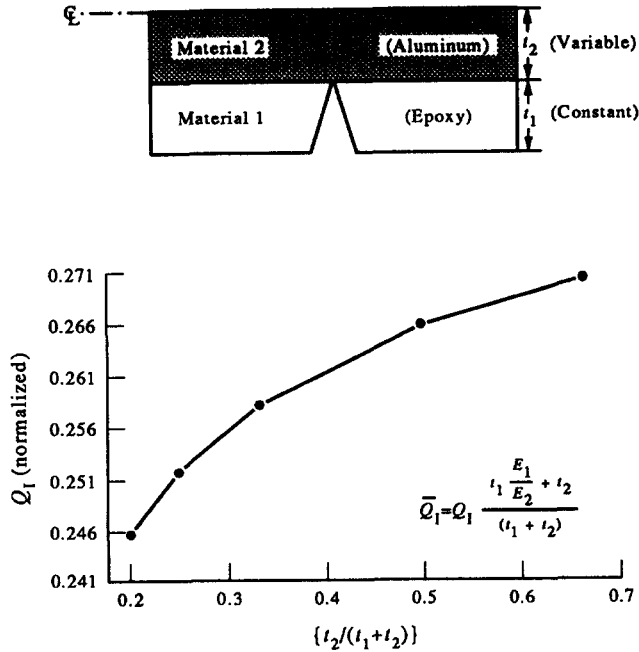Fig. 11. Plot of normalized Q_1 vs normalized thickness t_1.

Fig. 12. Plot of normalized Q_I vs normalized thickness t_2.

computing the stress intensity factor, Q_I, in finite-size cracked bimaterial specimens. The specimen considered is depicted in Fig. 10. The materials and dimensions are typical of laboratory specimens that can be used to measure the critical stress intensity factor, Q_{cr}.

The specimens shown in Figs 11–14 are all of the same length, subjected to the same uniaxial tension, $\sigma_0 = 6894.7573$ Pa (1 psi), whereas the volume fractions of the two materials have been varied in order to compute the different values of Q_I. In the first case, the crack length has been
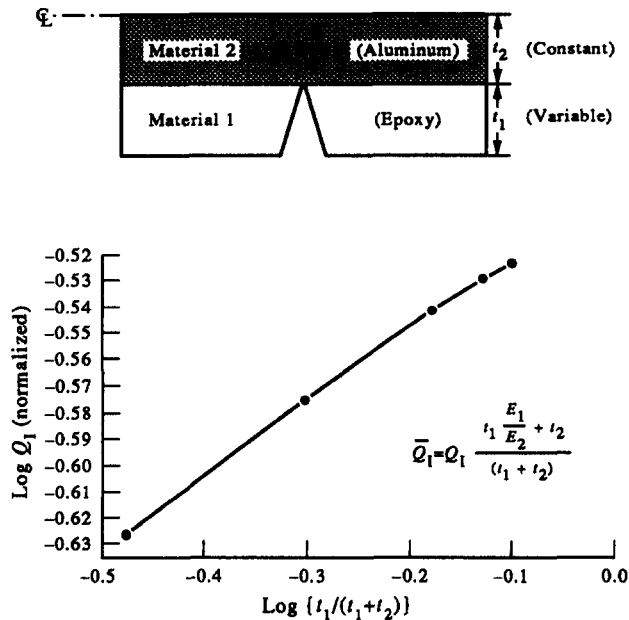

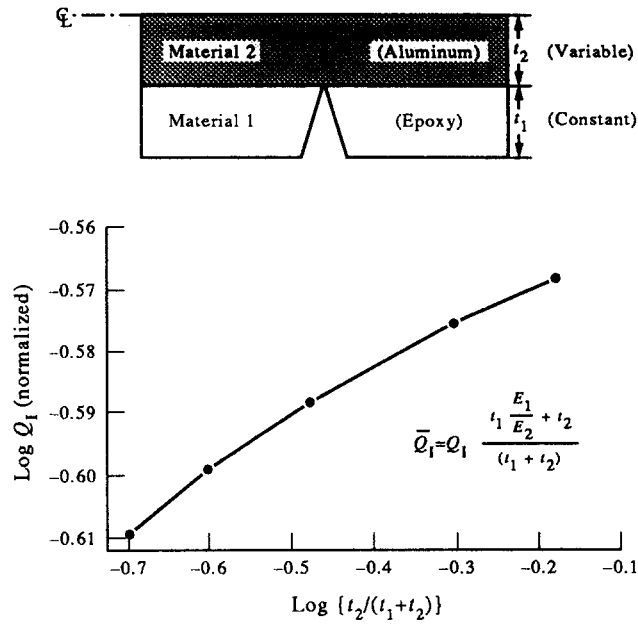Fig. 13. Log–log plot of normalized Q_I vs normalized thickness t_1.

Fig. 14. Log–log plot of normalized Q_1 vs normalized thickness t_2.

varied, keeping the thickness of the aluminum layer constant, and in the second case, the thickness of the aluminum layer has been varied keeping the crack length constant.

In the first case, the half-thickness of the aluminum layer has been set at 5.08 mm (0.2 in.) while the crack length, and in turn, the thickness of each epoxy layer has been varied from 2.54 mm (0.1 in.) to 20.32 mm (0.8 in.) for the five different examples analyzed. The total thicknesses have been varied from 15.24 mm (0.6 in.) to 50.8 mm (2.0 in.) while the thickness ratios have been varied from 0.25 to 2.0, respectively. In the second case, the thickness of each epoxy layer, and in turn, the crack length has been set to 5.08 mm (0.2 in.) while the half-thickness of the aluminum layer has been varied from 1.27 mm (0.05 in.) to 10.16 mm (0.4 in.) for the five different examples analyzed. The total thicknesses have been varied from 12.7 mm (0.5 in.) to 30.48 mm (1.2 in.) while the thickness ratios have been varied from 0.25 to 2.0, respectively. The ratio of the shear moduli is $\mu_2/\mu_1 = 23.08$. Table 3 tabulates Q_1 values for Case 1, and Table 4 tabulates the same for Case 2.

It should be noted that in all examples the crack length is equal to the thickness of the epoxy layer. The normalization was carried out with respect to the material properties of the stiffer layer,

Table 3. Stress intensity factors for a cracked bimaterial tension specimen in which the crack length has been varied while the stiffer layer thickness has been kept constant

Crack length (m)	Overall thickness (m)	t_2/t_1	Q_1 (Pa $\cdot$ m$^{1-\lambda}$)
0.00254	0.01524	2.00	691.3245
0.00508	0.02032	1.00	1013.8378
0.01016	0.03048	0.50	1575.9730
0.01524	0.04064	0.33	2078.0696
0.02032	0.05080	0.25	2533.9837

Table 4. Stress intensity factor for a cracked bimaterial tension specimen in which the crack length has been kept constant while the stiffer layer thickness has been varied

Crack length (m)	Overall thickness (m)	t_2/t_1	Q_1 (Pa $\cdot$ m$^{1-\lambda}$)
0.00508	0.03048	2.00	789.4915
0.00508	0.02032	1.00	1013.8378
0.00508	0.01524	0.50	1418.3876
0.00508	0.01355	0.33	1774.4072
0.00508	0.01270	0.25	2085.9306

viz. the aluminum layer. The normalized Q_1 was defined as

$$\bar{Q}_1 = Q_1 \frac{t_1\left(\dfrac{E_1}{E_2}\right) + t_2}{t_1 + t_2}. \tag{19}$$

This was an arbitrary choice since, at this point of time, the exact functional relationships between the various parameters and Q_1 have not yet been experimentally established.

The plots of normalized Q_1 are displayed in Figs 11–14. The plots also display linearities when normalized Q_1 values are plotted against normalized thickness ratios. The correlation coefficients for the two regular plots are 0.9921 and 0.9699 for Cases 1 and 2, respectively. The correlation coefficients for the log–log plots for Cases 1 and 2 are 0.9992 and 0.9948, respectively. A correlation coefficient of 1 indicates an exactly linear relationship. Hence, these values indicate that the log–log plots are more linear in nature than the regular plots and should be the ones to dictate the linear propensities. The key factor to be determined in these plots are the slopes of the lines, which may have an important bearing upon the relationship between Q_1 and the relevant geometric parameters for finite-size cracked bimaterial bodies. This relationship, when concluded, will lead to important approximations for the critical loads at which the cracks start initiating.

One of the aims of this study, as mentioned above, is to find empirical relations between Q_1 and various crack parameters similar to the ones found in fracture handbooks where the stress intensity factors for various specimen geometries and loading conditions are listed in terms of normalized loads and crack lengths for isotropic homogeneous materials. This kind of study has not been carried out for finite-sized cracked bimaterial specimens prior to this. If such empirical formulas can be established for different crack geometries and loading conditions, it would be an invaluable help for engineers and scientists. It would enable us to select appropriate specimen dimensions for testing purposes and also provide an estimate of failure loads according to our proposed hypothesis.

5. CONCLUSIONS

A finite element–boundary element alternating method has been developed for analyzing a cracked bimaterial test specimen. The crack is perpendicular to, and terminates at, the bimaterial interface. The FEM is used to model the specimen without the crack. The BEM (displacement discontinuity method) is used to find the stresses in the infinite bimaterial medium due to crack surface tractions. The stress intensity factor for the crack tip touching the interface is extracted by using a closed form solution for a semi-infinite crack opened by a pair of wedge forces. The method has been verified by applying it to several infinite domain bimaterial crack problems and finite-sized homogeneous medium problems for which exact solutions are available. The method was then applied to a bimaterial cracked test specimen. The stress intensity factors were computed for various crack lengths and specimen widths. Future work will be concerned with testing bimaterial specimens for measuring the critical value of the load at which crack propagation occurs and evaluating the critical stress intensity factor with this value to see if it can be used as a failure criterion. A series of parametric numerical analyses have been carried out to see whether empirical relationships may be established between Q_1 and the various specimen geometries and loads, such that a listing of such empirical formulas may be made for finite-size cracked bimaterial specimens.

REFERENCES

[1] A. R. Zak and M. L. Williams, Crack point stress singularities at a bi-material interface. *J. appl. Mech.* **30**, 142 (1963).
[2] K. C. Jen and C. T. Sun, Matrix cracking and delamination prediction in graphite/epoxy laminates. *Proc. Am. Soc. Comp., 5th Technical Conf.* (pp. 350–360) (1990).
[3] O. Sbaizero, P. G. Charalambides and A. G. Evans, Delamination cracking in a laminated ceramic matrix composite, in *The Processing and Mechanical Properties of High Temperature/High Performance Composites.* Annual Report, DARPA and ONR (1988–1989).

[4] Z. Suo, Singularities interacting with interfaces and cracks—1. Isotropic materials. Harvard University Report (1988).

[5] S. L. Crouch, Solution of plane elasticity problems by the displacement discontinuity method. *Int. J. numer. Meth. Engng* **10**, 301–343 (1976).

[6] B. V. Sankar, P. A. Mataga, S. Lahiri and Q. Y. Yao, Fiber–matrix interface failure in brittle matrix composites. *Advanced Composite Materials*, pp. 601–609. *Proc. American Ceramics Society's Conf.* Orlando (1990).

[7] J. Ahmad, A micromechanics analysis of cracks in unidirectional fiber composites. *J. appl. Mech.* **58**, 964–972 (1991).

[8] S. N. Atluri (Ed.) and T. Nishioka, Computational methods for three-dimensional problems of fracture, in *Computational Methods in the Mechanics of Fracture*, Vol. 2, pp. 254–283. North Holland, Amsterdam (1986).

[9] T. S. Cook and F. Erdogan, Stresses in bonded materials with a crack perpendicular to the interface. *Int. J. Engng Sci.* **10**, 677–697 (1972).

[10] S. L. Crouch and A. M. Starfield, *Boundary Element Methods in Solid Mechanics*, George Allen and Unwin (1983).

[11] K. Hellan, *Introduction to Fracture Mechanics*, pp. 243–245. McGraw-Hill, New York (1984).

[12] T. Nishioka and S. N. Atluri, Analytical solution for embedded elliptical cracks and finite element alternating method for elliptical surface cracks subjected to arbitrary loadings. *Engng Fracture Mech.* **17**(3), 247–268 (1983).

[13] M. L. Williams, On the stress at the base of a stationary crack. *J. appl. Mech.* **24**, 109–114 (1957).