# Evaluation of Failure Criteria for Fiber Composites Using Finite Element Micromechanics

H. ZHU AND B. V. SANKAR*
*Department of Aerospace Engineering*
*Mechanics & Engineering Science*
*231 Aerospace Building*
*University of Florida*
*Gainesville, FL 32611-6250*

R. V. MARREY
*Advance USA*
*Old Lyme, Connecticut*

(Received January 2, 1997)
(Revised June 27, 1997)

**ABSTRACT:** A micromechanical analysis of the unit cell of a unidirectional composite is performed using the finite element method. The circular fibers are assumed to be packed in a periodic square array. Assuming that the failure criteria for the fiber and matrix materials and also for the fiber-matrix interface are known, the failure envelope of the composite is developed using the microstresses computed in the unit cell analysis. This method is referred to as the Direct Micromechanics Method (DMM). The micromechanical methods were also used to simulate different tests to determine the strength coefficients in phenomenological failure criteria such as maximum stress, maximum strain and Tsai-Wu theories. The failure envelopes from the phenomenological failure criteria are compared with those of the DMM for the cases of biaxial and off-axis loading of a model unidirectional composite material. It is found that none of the phenomenological criteria compare well with the DMM in the entire range. A conservative failure envelope obtained using a combination of maximum stress and Tsai-Wu criteria seems to be the best choice for predicting the failure of unidirectional fiber composites.

**KEY WORDS:** direct micromechanics method, failure criteria, failure envelopes, micromechanics, off-axis test, periodic boundary conditions, unit-cell analysis, unidirectional fiber composites.

*Author to whom correspondence should be addressed.



0021-9983/98/08 0766–17 $10.00/0

## INTRODUCTION

MICROMECHANICAL METHODS HAVE been used in analyzing fiber composite for more than 30 years. With the ever increasing computing capabilities, more and more detailed micromechanical analyses are being performed. However, most often micromechanics is used for predicting thermo-elastic constants and other transport properties such as thermal conductivity [1–7]. Dvorak et al. [8] used micromechanics for predicting the yield surface of several metal matrix composites. Lin et al. [9] performed finite element micromechanical analysis of boron/epoxy and boron/aluminum composites to determine the elastic-plastic behavior under uniaxial loading. Ishikawa [10] used analytical micromechanics to study the effects of thermal residual stresses on the strength. Adams [11] used micromechanical methods to predict the nonlinear temperature dependent stress-strain behavior. Recently Whitcomb and Srirengan [12] used three-dimensional finite element micromechanical analysis to predict the progressive failure of a plain weave textile composite under in-plane extension.

Although micromechanical models have been successfully employed in predicting thermo-elastic constants of fiber reinforced composite material, their use for strength prediction under multiaxial loading conditions is not practical. Hence phenomenological failure criteria are still the popular choice in the industry. The various strength coefficients in the phenomenological criteria are measured by testing unidirectional composites under different combinations of loads until failure. There are three major types of engineering failure criteria for unidirectional fiber composites: (a) maximum stress criterion; (b) maximum strain criterion; and (c) quadratic interaction criteria. Tsai-Hill and Tsai-Wu are the two major quadratic failure criteria. Although these failure criteria are widely used, there is no mechanistic explanation why these criteria should work or what their limitations are. For example the form of the Tsai-Hill failure criterion is based on Hill's yield criterion for anisotropic materials undergoing plastic deformation [13].

In this paper we perform a finite element based micromechanical failure analysis on the unit cell of a unidirectional fiber composite. The micromechanical analysis computes the microstress field within the unit cell for a given homogeneous macrostress state acting on the composite. We assume that the failure criteria for the fiber and matrix materials, and also for the fiber-matrix interface are known. Then from the microstress field we can determine if the composite will fail under a given set of macrostresses. In this paper this procedure is referred to as the Direct Micromechanics Method (DMM). Thus DMM can be used to draw failure envelopes in various stress spaces such as biaxial state of stress ($\sigma_L$ and $\sigma_T$)** with or without inplane shear stress $\tau_{LT}$. The DMM can also be used to determine the off-axis strength of a unidirectional composite at various loading directions. Since the DMM can be thought of as numerical simulation of various strength tests on a composite, they can also be used to determine the strength coefficients in the

**$\sigma_L$ and $\sigma_T$ are the longitudinal and transverse normal stresses, respectively. $\tau_{LT}$ is the shear stress in the $LT$ plane.

aforementioned phenomenological failure criteria. The strength coefficients can then be used to draw the failure envelopes for respective phenomenological criteria. In this paper the DMM failure envelopes are compared with the phenomenological models, and based on the results the applicability and limitations of each criterion are discussed.

## DIRECT MICROMECHANICS METHOD

### Unit Cell Analysis

The micromechanical analysis of a unidirectional fiber composite is performed by analyzing the unit cell of the composite using the finite element method. In the present study, we assume that uniform macrostress exists through the composite. It is assumed that the fibers are circular in cross section packed in a square array. Thus the unit cell or the representative volume element is a square. The unit cell shown is in Figure 1. The unit cell analysis assumes that the composite is under a uniform state of strain at the macroscopic scale which are called the macroscale strains or macrostrains, and the corresponding stresses are called macrostresses. However the actual stresses in the fiber and the matrix within the unit cell will have spatial variation. These stresses are called microstresses. The macrostresses are average stresses required to create a given state of macro-deformations, and they can be computed from the microstresses obtained from the finite element analysis. The macrostresses and macrostrains are related by the elastic constants of the homogenized composite $[C]$:

$$\{\sigma^M\} = [C]\{\varepsilon^M\} \tag{1}$$

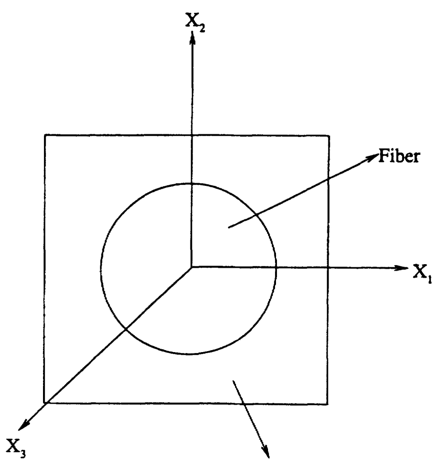


*Figure 1. Unit cell coordinate system.*

If the composite is subjected to a macroscopically homogeneous deformation then all unit cells will have identical microstress and microstrain fields. Continuity of stresses across a unit cell then requires that traction be equal and opposite at the corresponding points on opposite face of the unit cell (periodic boundary condition). Since the displacement gradients are constant for a homogeneous deformation, the displacements at corresponding points on opposite faces of the unit cell differ only by a constant. A macroscopically homogeneous deformation can be expressed by the boundary displacements:

$$u_i = H_{ij} x_j \qquad (i = 1,3; j = 1,2) \tag{2}$$

where $H_{ij}$ are the displacement gradients. Then the periodic displacement conditions on the faces $x_i = 0$ and $x_i = L$ are

$$\begin{aligned} u_i(L, x_2) - u_i(0, x_2) &= H_{i1} L \\ u_i(x_1, L) - u_i(x_1, 0) &= H_{i2} L \end{aligned} \tag{3}$$

The traction boundary condition on the faces $x_i = 0$ and $x_i = L$ are:

$$\begin{aligned} T_i(L, x_2) &= -T_i(0, x_2) \\ T_i(x_1, L) &= -T_i(x_1, 0) \end{aligned} \tag{4}$$

The above boundary conditions can be ensured by using multi-point constraint elements in the finite element model.

In the DMM the unit cell is subjected to six linearly independent macroscopic deformations. In each deformation case one of the six macrostrains are assumed to be nonzero and the rest of the macrostrains are set equal to zero. The six cases are: Case 1: $\varepsilon_{11}^M = 1$; Case 2: $\varepsilon_{22}^M = 1$; Case 3: $\varepsilon_{33}^M = 1$; Case 4: $\gamma_{12}^M = 1$; Case 5: $\gamma_{23}^M = 1$; Case 6: $\gamma_{31}^M = 1$.

For cases 1 through 4, eight-node isoparametric generalized plane strain elements and multi-point constraint elements were used [14]; for cases 5 and 6, eight-node plane elements capable of out of plane deformation were used. The derivation of stiffness matrix of the out of plane shear element is given in the Appendix. FE analysis was used to compute the microstresses in each element and also at the fiber matrix interface for each case of deformation given in Table 1. The macrostresses corresponding to each case can be computed by averaging the microstresses over the entire area of the unit cell:

$$\sigma_{ij}^M = \frac{1}{A}\int_A \sigma_{ij}\, dA = \frac{1}{A}\sum_{i=1}^{n}\int_{A_i} \sigma_{ij}\, dA_i \tag{5}$$

where $A$ is area of the unit cell, $A_i$ is the area of the $i$th element, and $n$ is the number of elements.

***Table 1. Periodic boundary conditions for the FE model of the unit cell.***

| Case | Constraints between Left and Right Faces | Constraints between Top and Bottom Faces | Out of Plane Strains |
|---|---|---|---|
| $\varepsilon_{11} = 1$ | $u_1(L,x_2)-u_1(0,x_2) = L$<br>$u_2(L,x_2)-u_2(0,x_2) = 0$ | $u_i(x_1,L)-u_i(x_1,0) = 0$<br>$i = 1,2$ | $\varepsilon_{33} = 0, \gamma_{31} = 0, \gamma_{23} = 0$ |
| $\varepsilon_{22} = 1$ | $u_i(L,x_2)-u_i(0,x_2) = 0$<br>$i = 1,2$ | $u_1(x_1,L)-u_1(x_1,0) = 0$<br>$u_2(x_1,L)-u_2(x_1,0) = L$ | $\varepsilon_{33} = 0, \gamma_{31} = 0, \gamma_{23} = 0$ |
| $\varepsilon_{33} = 1$ | $u_i(L,x_2)-u_i(0,x_2) = 0$<br>$i = 1,2$ | $u_i(x_1,L)-u_i(x_1,0) = 0$<br>$i = 1,2$ | $\varepsilon_{33} = 1, \gamma_{31} = 0, \gamma_{23} = 0$ |
| $\gamma_{12} = 1$ | $u_1(L,x_2)-u_1(0,x_2) = 0$<br>$u_2(L,x_2)-u_2(0,x_2) = L$ | $u_i(X_1,L)-u_i(x_1,0) = 0$<br>$i = 1,2$ | $\varepsilon_{33} = 0, \gamma_{31} = 0, \gamma_{23} = 0$ |
| $\gamma_{23} = 1$ | $u_3(L,x_2)-u_3(0,x_2) = 0$ | $u_3(x_1,L)-u_3(x_1,0) = L$ | $\varepsilon_{33} = 0, \gamma_{31} = 0$ |
| $\gamma_{31} = 1$ | $u_3(L,x_2)-u_3(0,x_2) = L$ | $u_3(x_1,L)-u_3(x_1,0) = 0$ | $\varepsilon_{33} = 0, \gamma_{23} = 0$ |

## Evaluation of Elastic Constants

The DMM requires the evaluation of elastic constants to determine the macrostrains for a given state of macrostress. Further the computed elastic constants can be compared to the available analytical results as a verification of the FE mode. In implementing the procedure for obtaining the elastic constants of the composite, first, we choose that only one macroscopic strain (say, $\varepsilon_{11}^M = 1$) exists and the others are zero. The corresponding macrostresses can be computed by the method mentioned above. Substituting the value of the macrostrain and macrostresses in Equation (1), the stiffness coefficients in a column corresponding to the non-zero strain can be evaluated. The same procedure can be repeated for the other strain components to obtain all the stiffness coefficients. Once the $[C]$ matrix is determined, the elastic constants can be obtained from the $[S]$ matrix which is the inverse of $[C]$. It can be shown that the $[C]$ obtained from the micromechanics will always be symmetric [15]. In the unit cell analysis care should be taken in discretizing the unit cell such that opposite faces of the unit cell have identical nodes so that periodic boundary conditions can be implemented using multi-point constraints.

## Micromechanics of Failure

In the micromechanical failure analysis we are concerned with the failure of the fiber, matrix or the interface for a given macroscopic stress state. The failure can be predicted by checking each finite element in the unit cell model. Thus for a given state of macrostresses we need information on microstresses in each element. First the macrostrains for a given macrostress state can be found from constitutive relation (Equation 1) of the composite as:

$$\{\varepsilon^M\} = [C^{-1}]\{\sigma^M\} \tag{6}$$

From unit cell analysis we have already found the microstresses in each finite element for each of the six linearly independent macrostrain component. Thus the microstresses for a given macrostress state can be obtained by superposition. This can be expressed as

$$\{\sigma^{(e)}\} = [F^{(e)}]\{\varepsilon^{M}\} \tag{7}$$

where $\{\sigma^{(e)}\}$ is the microstress in Element *e*, and the matrix $[F^{(e)}]$ contains the microstresses in Element *e* for various states of unit macrostrains. For example, the first column $F_{i1}$ contains the six microstresses in Element *e* for a unit macrostrain $\varepsilon_1^M$. In using Equations (6) and (7) we have tacitly assumed that there are no thermal residual stresses in the material. In the present study the microstresses were computed at the central integration point of each element and also at the midpoints of the edges that are common to fiber and matrix elements. We assume that failure criteria for the matrix, fiber or interface materials are known. It is also assumed that the composite has failed even if only one of the fiber or matrix element fails or if fiber-matrix interface fails in one of the elements. Although this assumption is very restrictive, it can be considered to represent the initial failure of the composite, and it is consistently applied to both the DMM and in the development of phenomenological criteria. Such point stress criteria for failure have been used before by many researchers, e.g., Adams and Doner [16]. In the present study we have used two types of failure criteria for the fiber and matrix: (1) maximum principal stress criterion; and (2) von Mises yield criterion which is referred to as quadratic criterion in this paper. For the interface we used the maximum tensile interface stress and maximum interfacial shear stress criteria. Thus we can generate failure envelopes for the composite in various stress spaces. A flow chart that describes the DMM is shown in Figure 2.

## Determination of Strength Coefficients

Although the DMM seems to be straight forward, it is not convenient in the routine design of composite structures. Hence it will be desirable to have failure criteria in terms of macroscopic stresses. For example Tsai-Wu, Maximum Stress, and Maximum Strain criteria are phenomenological failure criteria that are expressed in terms of the macrostresses. Typically these criteria have some constants in the equation which can be thought of strength parameters of the composite. These parameters are usually determined by conducting various tests on composite specimens. In the present study we propose a numerical simulation of the same tests using the Direct Micromechanics Method to determine the strength parameters. The unit cell analysis is used to simulate various tests that are conducted to measure the strength parameters. In principle we are trying to curve-fit the DMM failure envelopes using the strength parameters or strength coefficients. Let us take an example to show how to determine the strength coefficients in Tsai-Wu failure criteria [17]. The form of Tsai-Wu criterion in two dimensions is as follows:

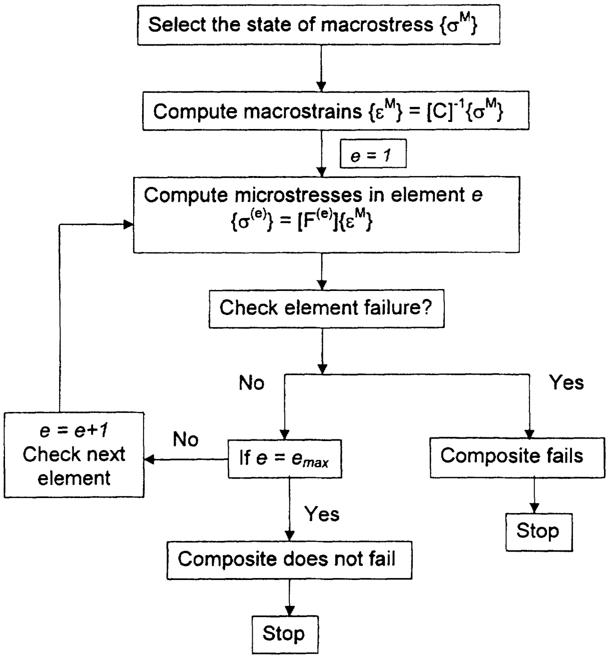


***Figure 2.*** *Flow chart for direct micromechanics method.*

$$F_{11}\sigma_{11}^2 + F_{33}\sigma_{33}^2 + F_{55}\tau_{31}^2 + 2F_{13}\sigma_{11}\sigma_{33} + F_1\sigma_{11} + F_3\sigma_{33} = 1 \tag{8}$$

Here $\sigma_{11}$ is the transverse normal macrostress, $\sigma_{33}$ is the normal macrostress in the direction of fiber and $\tau_{31}$ is the in plane macro-shear stress. $F_{11}$, $F_{33}$, $F_{55}$, $F_{13}$, $F_1$ and $F_3$ are strength coefficients which need to be determined. In order to obtain the strength coefficients $F_{11}$ and $F_1$, we assume that only the macroscopic stress $\sigma_{11}$ exists and other macroscopic stresses are zero in the composite. Then Equation (8) takes the following form:

$$F_{11}\sigma_{11}^2 + F_1\sigma_{11} = 1 \tag{9}$$

First we assume that only macro-stress $\sigma_{11} = 1$, and apply micromechanics analysis to obtain the microscopic stresses in each element of fiber and matrix. Based on these information we can determine which element fails first, and the maximum stress $\sigma_{11TU}$ for corresponding failure. Second we assume only macro-stress $\sigma_{11} = -1$ exists and repeat the above procedure to determine the compressive strength $\sigma_{11CU}$. Then we can calculate $F_{11}$ and $F_1$ as follows:

$$F_{11} = -\frac{1}{\sigma_{11TU}\sigma_{11CU}} \tag{10}$$

$$F_1 = \frac{\sigma_{11TU} + \sigma_{11CU}}{\sigma_{11TU}\sigma_{11CU}} \tag{11}$$

Using similar procedures, we can evaluate the strength coefficients $F_{33}$, $F_3$ and $F_{55}$. The expressions take the form:

$$F_{33} = -\frac{1}{\sigma_{33TU}\sigma_{33CU}} \tag{12}$$

$$F_3 = \frac{\sigma_{33TU} + \sigma_{33CU}}{\sigma_{33TU}\sigma_{33CU}} \tag{13}$$

$$F_{55} = \frac{1}{\tau_{31U}^2} \tag{14}$$

The procedure for finding $F_{13}$ is as follows. We assume that the macroscopic stress $\tau_{31}$ is zero, and $\sigma_{11}$ and $\sigma_{33}$ are tensile and equal to unity. By applying the similar procedure described above, we obtain the maximum macroscopic stress $\sigma_{11max}$ or $\sigma_{33max}$ ($\sigma_{11max} = \sigma_{33max}$) which can cause failure. Thus the coefficient $F_{13}$ is determined from Equation 8 as:

$$F_{13} = \frac{1}{2\sigma_{11max}^2}[1 - (F_{11} + F_{33})\sigma_{11max}^2 - (F_1 + F_3)\sigma_{11max}] \tag{15}$$

## RESULTS AND DISCUSSION

The unidirectional fiber composite was assumed to have circular fibers packed in a square array. The dimensions of the unit cell were such that the fiber volume fraction was equal to 63%. The fiber and matrix materials were assumed isotropic. The Young's modulus ($E$), Poisson's ratio ($\nu$) and the tensile strength ($\sigma_u$) of the fiber (subscript $f$) and matrix materials (subscript $m$) were: $E_f = 130$ GPa, $\nu_f = 0.3$, $\sigma_{uf} = 2.8$ GPa, $E_m = 3.5$ GPa, $\nu_m = 0.35$, and $\sigma_{um} = 70$ MPa. The properties we have assumed are close to that of Kevlar/epoxy [18]. The fiber-matrix interface is assumed to be perfect until failure occurred due to either normal tensile stresses or shear stresses. The interface strengths were 30 MPa in tension and 28 MPa in shear. The interface strengths are rough estimates based on the transverse tensile and shear strength of Kevlar/epoxy composites. The composite was assumed to be cured at room temperature and hence the thermal stresses due to fabrication were ignored.

As explanation on the choice of the material properties is in order. The objective of this research was to compare various phenomenological criteria with the DMM. As such we could have assumed arbitrary properties for the fiber and matrix materials. However, we were curious to compare the results with available experimental data, and hence chose Kevlar/epoxy as a candidate material.

The results for a unidirectional composite can be divided into four parts: (1) results for elastic constants; (2) results for strength properties; (3) failure envelope for biaxial state of stress with or without inplane shear stress; and (4) off-axis tensile strength. The results for elastic constants given in Table 2 provide a check for our micromechanical analysis. In Table 2 the elastic constants are compared with those obtained using Halpin-Tsai equations [18]. The agreement in elastic constants is excellent.

We have used a three-letter notation to refer to various combinations of failure criteria that have been used. The first letter refers to the failure criterion used for the fiber and the second letter for the matrix material. In these notations $Q$ and $M$ refer to quadratic (von Mises criterion) and maximum principal stress failure criteria, respectively. The third letter, either $Y$ or $N$, denotes if an interface failure criterion was used or not. If the third letter is $Y$, then it means that the interface failure was considered in the micromechanical analysis. For example, $QMY$ means that the quadratic failure criterion was used for fiber, maximum principal stress criterion for matrix and the interface failure was considered. When the maximum principal stress criterion was used, the constituent material (either the fiber or matrix) was assumed to fail only in tension like some brittle materials.

The results for strength properties of the composite for various combinations of constituent failure criteria are presented in Table 3. The longitudinal compressive strength was not computed because it is believed that failure under longitudinal compression is more an instability phenomenon rather than failure of constituent materials, and the present analysis is not suitable for it. As the constituent properties we have chosen were close to those of Kevlar and epoxy, published strength properties of Kevlar/epoxy composite [18,19] are also listed in Table 3.

From the results on Table 3 several observations can be made. The longitudinal strength in tension does not depend on the particular failure criterion for fiber or matrix materials or the interface conditions assumed here. The longitudinal strength of 1.31 GPa seems to be much less than that will be predicted by some simple models [18]. In the present case the ultimate tensile strain of fiber and matrix materials are 0.0215 and 0.02, respectively. Thus the matrix controls the failure and the ultimate tensile strength of the composite is given by 0.02 $E_L$ = 1.68 GPa. However the simple rule of mixture type models do not account for the stress concentration that will occur in the matrix locally due to constraining effects of the fiber. Since the DMM is based on maximum stress at a single point (point stress criterion), it is expected to give a very conservative result for strength values. In

***Table 2. Elastic constants of the unidirectional composite.***

| Method | $E_L$(GPa) | $E_T$ (GPa) | $G_{LT}$ (GPa) | $\nu_{LT}$ |
|---|---|---|---|---|
| Numerical simulation | 84.03 | 19.88 | 5.12 | 0.31 |
| Halpin-Tsai Equation | 83.08 | 18.09 | 5.17 | 0.32 |

**Table 3. DMM results for strength values and comparison with published data.**

| Method | $\sigma_{LU}$ (GPa) | $\sigma_{TU}$ (MPa) | $\sigma'_{TU}$ (MPa) | $\tau_{LTU}$ (MPa) |
|---|---|---|---|---|
| QQN | 1.31 | 69.25 | 69.25 | 22.63 |
| QQY | 1.31 | 16.30 | 60.47 | 15.47 |
| QMN | 1.31 | 37.44 | 203.36 | 39.19 |
| QMY | 1.31 | 16.29 | 60.47 | 15.47 |
| MQN | 1.31 | 69.55 | 69.25 | 22.63 |
| MQY | 1.31 | 16.30 | 60.47 | 15.47 |
| MMN | 1.31 | 37.44 | 203.06 | 39.19 |
| MMY | 1.31 | 16.29 | 60.47 | 15.47 |
| Reference [18] | 1.40 | 12 | 53 | 34 |
| Reference [19] | 1.40 | 10 | 50 | 30 |

fact the predicted longitudinal tensile strength (1.31 GPa) is somewhat closer to the published strength of Kevlar/epoxy (1.40 GPa).

The transverse tensile strength $\sigma_{TU}$ is very much affected by the interfacial strength, and also by the type of failure criterion used for the matrix material. When the interfacial failure was not considered (the cases ending with $N$, e.g., QQN, QMN etc.) a quadratic criterion for matrix gives higher strength and the maximum stress criterion yields a lower strength. In the case of transverse compressive strength $\sigma'_{TU}$ a slightly different trend is observed. The quadratic criterion for matrix yields lower compressive strength where as maximum stress criterion results in a higher value. The interface has the same effect as in the case of transverse tensile strength in these cases. The effects of various failure criteria of the constituents on shear strength ($\tau_{LTU}$) is very similar to the transverse compressive strength. It might be noted that for QMN and MMN cases, a very high transverse compressive strength is predicted. This is because of the combination of maximum principal stress criterion (Q<u>M</u>N) for the matrix—which does not consider failure due to compression—and also the fact that the interface failure is not considered (QM<u>N</u>).

The micromechanics was applied to the case wherein the composite is subjected to a state of biaxial stress with $\sigma_L$ and $\sigma_T$ being the two nonzero stresses. The results were plotted as failure envelopes in the ($\sigma_L$, $\sigma_T$) space. Sample envelopes for the case QQN are shown in Figure 3. The results for all the cases are summarized in Table 4. In Figure 3 we have presented the envelope obtained by DMM and also the envelopes obtained by phenomenological criteria such as Tsai-Wu, maximum stress and maximum strain criteria. From Figure 3 it can be seen that the maximum strain criterion does not compare with the micromechanical results very well. In fact this was true for all cases studied in this research. In all the eight cases neither the Tsai-Wu nor the maximum stress criterion fit the direct micromechanics results in the entire space. Between the two criteria maximum stress criteria was better in most cases. From Table 4 one can note that the failure criteria for the matrix and also consideration of interface failure play a dominant role in the type of crite-

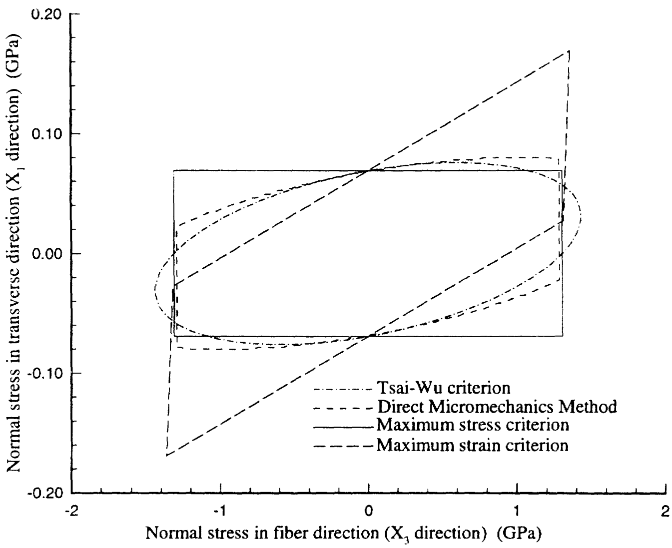


***Figure 3.*** *Failure envelopes for QQN case (no in-plane shear stress).*

ria for the composite. For example, if the matrix fails by maximum stress criterion and if the interface failure is also considered, then the maximum stress theory is good for the composite. However the combination of both Tsai-Wu and maximum stress criteria was much better in all cases considered. A similar observation has been made by Daniel and Ishai [20] based on some experimental results. They recommend the use of several failure criteria and determine the most conservative envelope in each quadrant.

The next case considered was again a biaxial state of stress with a constant in-plane shear given by $\tau_{LT} = 10$ MPa. The results for this case are summarized in Ta-

***Table 4. Comparison of various failure criteria for the case of biaxial loading of the composite with $\tau_{LT} = 0$.***

| Case | Combination of Tsai-Wu and Maximum Stress Criteria | Tsai-Wu Criterion | Maximum Stress Criterion |
|---|---|---|---|
| QQN | Best | Good | Fair |
| QQY | Best | Fair | Good |
| QMN | Best | Poor | Good |
| QMY | Best | Poor | Good |
| MQN | Best | Good | Fair |
| MQY | Best | Fair | Fair |
| MMN | Best | Poor | Good |
| MMY | Best | Poor | Good |

**Table 5. Comparison of various failure criteria for the case of biaxial loading of the composite with $\tau_{LT}$ = 10 MPa.**

| Case | Combination of Tsai-Wu and Maximum Stress Criteria | Tsai-Wu Criterion | Maximum Stress Criterion |
|---|---|---|---|
| QQN | Excellent | Excellent | Fair |
| QQY | Excellent | Good | Fair |
| QMN | Excellent | Poor | Good |
| QMY | Excellent | Poor | Good |
| MQN | Excellent | Excellent | Poor |
| MQY | Excellent | Good | Poor |
| MMN | Excellent | Poor | Good |
| MMY | Good | Poor | Fair |

ble 5, and sample failure envelopes for MMY case is given in Figure 4. Again the results are very similar to the case without any inplane shear just discussed. In general if the matrix failure is by quadratic criterion, then Tsai-Wu criterion is good for the composite also. Again, the combination of Tsai-Wu and maximum stress criteria is the safest for all cases.

The micromechanical analyses were used to simulate off-axis tensile tests for various loading directions with respect to the fiber orientation. The off-axis strength was computed by direct micromechanical analysis and also using the Tsai-Wu and maximum stress criteria. The strength values given in Table 3 were used in the Tsai-Wu and maximum stress criteria. Sample figures for QMN case showing the variation of strength with fiber orientatin are given in Figures 5 and 6. The results are summarized in Table 6. No one criterion compared well with the direct micromechanics in the entire range of fiber orientation ($0 < \theta < 90$). From Table 6, one can note that the Tsai-Wu criterion fits well for $0 < \theta < 30$, the maximum stress criterion is good for $60 < \theta < 90$, and for $30 < \theta < 60$ both criteria must be used

**Table 6. Comparison of various failure criteria for simulated off-axis tension tests.**

| Case | $0 < \theta < 30$ | $30 < \theta < 60$ | $60 < \theta < 90$ |
|---|---|---|---|
| QQN | T-W | T-W | T-W |
| QQY | T-W | T-W ∩ M.S | M.S |
| QMN | T-W | T-W ∩ M.S | M.S |
| QMY | T-W | T-W ∩ M.S | M.S |
| MQN | T-W | T-W | T-W |
| MQY | T-W | T-W ∩ M.S | M.S |
| MMN | T-W | T-W ∩ M.S | M.S |
| MMY | T-W | T-W ∩ M.S | M.S |

T-W: Tsai-Wu criterion; M.S: Maximum stress criterion; T-W ∩ M.S: Combination of Tsai-Wu criterion and Maximum stress criterion.

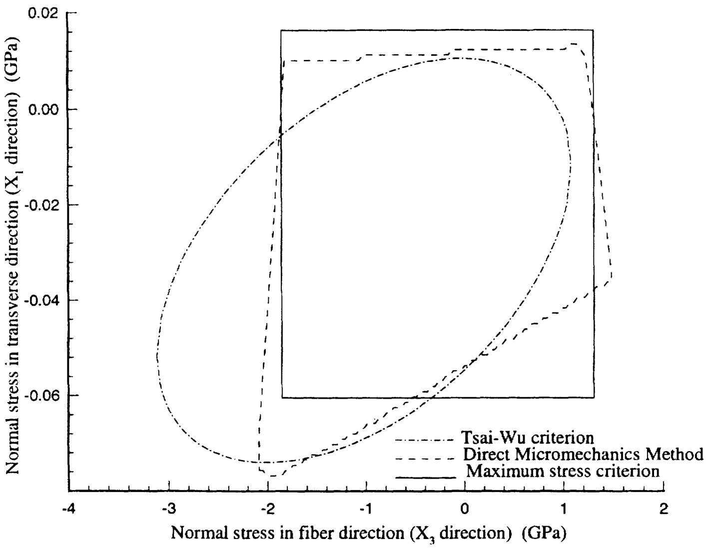


*Figure 4. Failure envelopes for MMY case (in-plane shear stress = 10 MPa).*

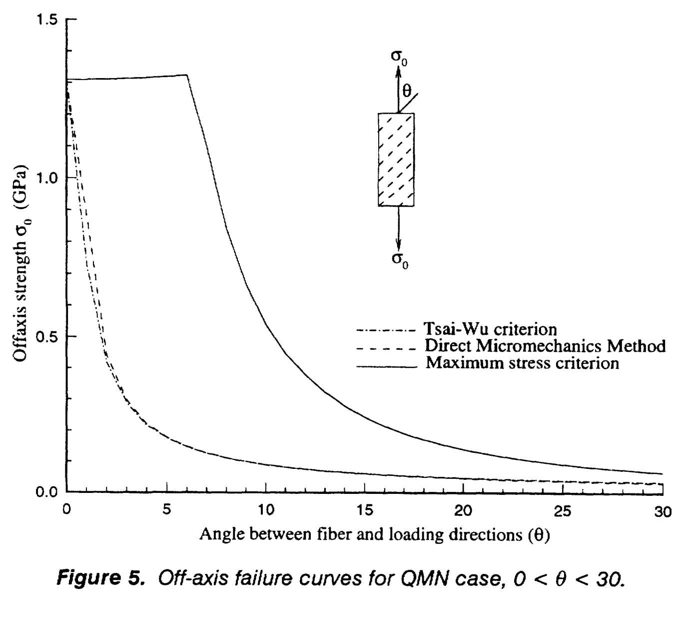


*Figure 5. Off-axis failure curves for QMN case, 0 < θ < 30.*

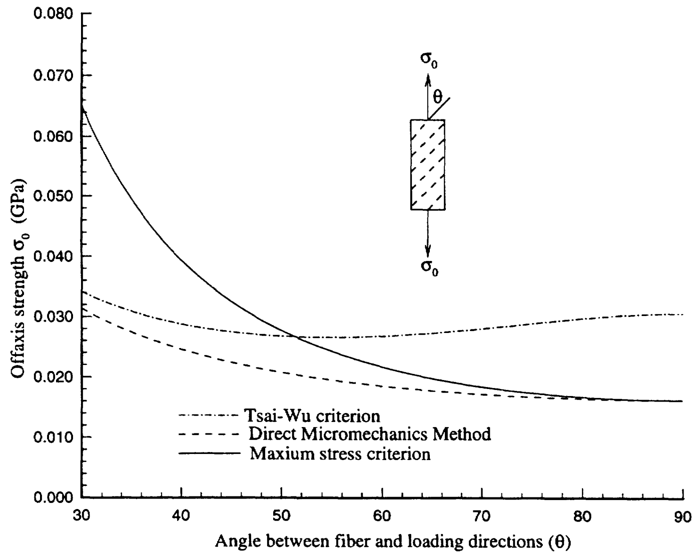


***Figure 6.*** *Off-axis failure curves for QMN case, 30 < θ < 90.*

in most cases except for QQN and MQN. The quadratic criterion can be chosen for both QQN and MQN cases.

## CONCLUSIONS

From the above discussion we can reach the following conclusions:

1. The elastic constants obtained by direct micromechanics method match well with those from the Halpin-Tsai equations.
2. According to the Direct Micromechanics Method the combination of Tsai-Wu and maximum stress criteria offers a conservative failure envelope in the entire space of biaxial state of stress.
3. The failure criteria for the matrix and also the interfacial condition play a dominant role in the phenomenological failure criteria of the composite.
4. In off-axis tensile test simulations, no one criterion compared well with the direct micromechanics in the entire range of fiber orientation ($0 < \theta < 90$). The Tsai-Wu criterion fits well for $0 < \theta < 30$, and the maximum stress criterion is good for $60 < \theta < 90$. For $30 < \theta < 60$ both criteria must be used and the lower of the two should be considered as the composite off-axis strength.

## ACKNOWLEDGEMENT

This research was partially supported by a NASA Langley Research Center grant (NAG 1222-1) to the University of Florida. Mr. Wade C. Jackson was the grant monitor.

## APPENDIX

In this section the stiffness matrix for the eight node out of plane shear element is derived. The out of plane displacement $u_3$ can be interpolated as:

$$u_3 = \sum_{i=1}^{8} N_i(\xi_1, \xi_2) q_{3i} \tag{A1}$$

Where the shape functions are given by:

$$\begin{aligned}
N_1 &= -\frac{1}{4}(1-\xi_1)(1-\xi_2)(\xi_1+\xi_2+1) & N_5 &= \frac{1}{2}(1-\xi_1^2)(1-\xi_2) \\
N_2 &= \frac{1}{4}(1+\xi_1)(1-\xi_2)(\xi_1-\xi_2-1) & N_6 &= \frac{1}{2}(1+\xi_1)(1-\xi_2^2) \\
N_3 &= \frac{1}{4}(1+\xi_1)(1+\xi_2)(\xi_1+\xi_2-1) & N_7 &= \frac{1}{2}(1-\xi_1^2)(1+\xi_2) \\
N_4 &= \frac{1}{4}(1-\xi_1)(1+\xi_2)(\xi_2-\xi_1-1) & N_8 &= \frac{1}{2}(1-\xi_1)(1-\xi_2^2)
\end{aligned} \tag{A2}$$

and $q_{3i}$ is the displacement of $i$th node in the $x_3$ direction.

The out of plane shear strains $\gamma_{23,}$, and $\gamma_{31}$ can be obtained in terms of the nodal displacements as follows:

$$\begin{Bmatrix} \gamma_{23} \\ \gamma_{13} \end{Bmatrix} = [B]\{q\} \tag{A3}$$

Where:

$$[B]_{2\times 8} = \begin{bmatrix} \frac{\partial N_1}{\partial x_1} & \frac{\partial N_2}{\partial x_1} & \cdots & \frac{\partial N_8}{\partial x_1} \\ \frac{\partial N_1}{\partial x_2} & \frac{\partial N_2}{\partial x_2} & \cdots & \frac{\partial N_8}{\partial x_2} \end{bmatrix} \tag{A4}$$

$$\{q\}^T = \lfloor q_{31} q_{32} \cdots q_{37} q_{38} \rfloor \tag{A5}$$

It is also assumed that the fiber and matrix are isotropic and no coupling between normal and shear deformation exists.

The strain energy density related to shear strains g23 and g31 can be expressed by

$$U_0 = \frac{1}{2}\begin{Bmatrix}\gamma_{23}\\ \gamma_{31}\end{Bmatrix}^T [G]\begin{Bmatrix}\gamma_{23}\\ \gamma_{31}\end{Bmatrix} \tag{A6}$$

Where $[G]$ is the matrix of shear moduli:

$$[G] = \begin{bmatrix} G_{23} & 0 \\ 0 & G_{31}\end{bmatrix} \tag{A7}$$

Then the strain energy in an element can be written as:

$$U_e = \int\int U_0 \, dx_1 dx_2 \tag{A8}$$

The element stiffness matrix related to shear strains $\gamma_{23}$ and $\gamma_{31}$ can be obtained by using the principle of minimum potential energy, i.e., differentiating the strain energy with respect to $\{q\}$. So the element stiffness matrix $[K]_e$ can be obtained as:

$$[K_e] = \int\int [B]^T[G][B] dx_1 dx_2 \tag{A9}$$

$[K_e]$ can also be obtained by performing the integration in the natural coordinates $\xi_1$–$\xi_2$:

$$[K]_e = \int_{-1}^{+1}\int_{-1}^{+1} [B]^T[G][B]|J| d\xi_1 d\xi_2 \tag{A10}$$

Where $|J|$ is the determinant of Jacobian matrix [21]. Now it is convenient to use the Gauss quadrature and evaluate the above integral using numerical integration [21]:

$$[K_e] = \sum_{m=1}^{3}\sum_{n=1}^{3} [B]^T[G][B] W_m W_n |J| \tag{A11}$$

where $W_m$, $W_n$ are weighing factors for the Gaussian point ($\xi_{1m}$,$\xi_{2m}$).

## REFERENCES

1. Dasgupta, A., S. Bhandarkar, M. Pecht and D. Barkar. 1990. "Thermoelastic Properties of Woven-Fabric Composites using Homogenization Techniques," *Proceedings of the American Society for Composites,* Fifth Technical Conference, Lansing, MI, pp. 1001–1010.
2. Agarwal, R. K. and A. Dasgupta. 1993. "Prediction of Electrical Properties of Plain-Weave Fabric Composites for Printed Wiring Board Design," *J. Electronic Packaging,* 115:219–224.

3. Foye, R. L. 1993. "Approximating the Stress Field within the Unit Cell of a Fabric Reinforced Composite Using Replacement Elements," NASA CR-191422.
4. Hart, R. 1994. "A Micromechanical Model for Predicting Thermal Conductivities in Textile Composites," Master Thesis, University of Florida, Gainesville, FL.
5. Marrey, R. V. and B. V. Sankar. 1995. "Micromechanical Models for Textile Structural Composites," NASA CR 198229.
6. Naik, R. A. 1994. "Analysis of Woven and Braided Fabric Reinforced Composites," NASA CR-194930.
7. Whitcomb, J. D. 1991. "Three-Dimensional Stress Analysis of Plain Weave Composites," *Composite Materials Fatigue and Fracture (Third Volume),* ASTM STP 1110, pp. 417–438.
8. Dvorak, G. J., M. S. M. Rao and J. Q. Tarn. 1973. "Yielding in Unidirectional Composites Under External Loads and Temperature Changes," *J. Composites Materials,* 7:194–216.
9. Lin, T. H., D. Salinas and Y. M. Ito. 1972. "Elastic-Plastic Analysis of Unidirectional Composites," *J. Composite Mateirals,* pp. 48–60.
10. Ishikawa, T. 1982. "Strength and Thermal Residual Stress of Unidirectional Composites," *J. Composite Materials,* pp. 40–52.
11. Adams, D. F. "Micromechanical Analysis of Composite Materials: Applications to Practical Problems," Seminar Notes, Technomic Publishing Co., Lancaster, PA.
12. Whitcomb, J. and K. Srirengan. 1995. "Effect of Various Approximations on Predicted Progressive Failure in Plain Weave Composites," *Mechanics of Textile Composites Conference, Part 2,* C. C. Poe and C. E. Harris, eds., NASA CP 3311, NASA Langley Research Center, Hampton, VA.
13. Tsai, S. W. 1968. "Strength Theories of Filamentary Structures," in *Fundamental Aspects of Fiber Reinforced Plastic Composites,* R. T. Schwartz and H. S. Schwartz, eds., Interscience, NY, Chapter 1.
14. Zhu, H. 1994. "A Micromechanics Approach for Predicting the Strength of Fiber Reinforced Composite Materials," Master Thesis, University of Florida, Gainesville, FL.
15. Marrey, R. V. and B. V. Sankar. 1997. "A Micromechanical Model for Textile Composite Plates," *J. Composite Materials,* 31(12):1187–1213.
16. Adams, D. F. and D. Doner. 1967. "Transverse Normal Loading of a Unidirectional Composite," *J. Composite Materials,* 1:152–164.
17. Tsai, S. W. and H. T. Hahn. 1980. *Introduction to Composite Materials,* Technomic Publishing Co., Lancaster, PA.
18. Agarwal, B. D. and L. J. Broutman. 1990. *Analysis and Performance of Fiber Composites,* New York, NY: John Wiley & Sons, Inc.
19. Hoskin, B. C. and A. A. Baker. 1986. *Composite Materials for Aircraft Structures,* New York, NY: American Institute of Aeronautics and Astronautics, Inc.
20. Daniel, I. M. and O. Ishai. 1994. *Engineering Mechanics of Composite Materials,* New York, NY: Oxford University Press.
21. William, W. and P. R. Johnston. 1984. *Finite Elements for Structural Analysis,* Englewood Cliffs, NJ: Prentice-Hall.