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Optimization of functionally graded metallic foam insulation under transient heat transfer conditions

H. Zhu, B.V. Sankar, R.T. Haftka, S. Venkataraman, M. Blosser

Abstract The problem of minimizing the maximum temperature of a structure insulated by a functionally graded metal foam insulation under transient heat conduction is studied. First, the performance of insulation designed for steady-state conditions is compared with uniform solidity insulation. It is found that the optimum steady-state insulation performs poorly under transient conditions. Then, the maximum structural temperature of a two-layer insulation with constant solidity for each layer is minimized by varying the solidity profile for a given total thickness and mass. It is found that the cooler inner layer of the optimal design has high solidity, while the hotter outer layer has low solidity. This is in contrast to the steady-state optimum, where the solidity profile is the reverse.

Key words functionally graded materials, metallic foam, optimization, thermal protection systems

1 Introduction

Thermal protection systems (TPS) on reusable launch vehicles have to be designed to keep the maximum temperature of the structure, e.g. cryogenic fuel tank structure, below a specified safe limit. A significant component of the total vehicle weight is the weight of the thermal protection system. A recent study of modeling and performance issues in TPS design conducted by Blosser (2002) has shown that currently existing Saffil-foam-filled TPS tile weighs 5.85 to 19.3 kg/m^2 (1.2 to 3.96 lb/ft^2), while the structural weight ranges from 4.64 to 8.54 kg/m^2 (0.95 to 1.75 lb/ft^2). The weight of the insulation depends greatly on the choice of the insulation and structural material. Figure 1 shows the components of a TPS tile. The volume enclosed is filled with the Saffil (fibrous type) insulation. Since the Saffil insulation is flexible and cannot be attached directly, it needs encapsulation in a foil and the secondary TPS support structure. The sandwich panel on the outside surface of the TPS tile provides a smooth and rigid outer surface and also acts as a mass heat sink. The analysis performed by Blosser (2002) for the material and design choices clearly indicates that the design of the TPS is highly influenced by the mass and heat capacities of the different components.

Functionally graded (FG) foams can significantly improve the performance of the insulation. For steady-state heat flow, Venkataraman *et al.* (2002) optimized the solidity profile or volume fraction of the foam in order to minimize the transmitted heat through foam for given mass or to minimize thickness for a specified maximum transmitted heat. It should be noted that solidity or volume fraction of foam is defined as the ratio between the volume of the solid material and the volume of foam. Zhu *et al.* (2003) minimized the mass of foam insulation for specified heat transfer. Unlike in steady-state conditions, mass plays an important role on the temperature history under transient heating conditions due to the heat capacity of insulation materials. The objective of this paper is to develop a methodology for optimizing functionally graded foam insulations taking into consideration the transient heat transfer conditions that arise during reentry of a space vehicle. We simplify the design task to a point design. We choose a point on the area of the surface of the vehicle to demonstrate the present methodologies and investigate the one-dimensional heat transfer problem at that point. The heat load is also simplified. First, we compare the performance of insulation designed for steady-state conditions with that of uniform solidity insulation. Then, the maximum structural temperature of a two-layer foam insulation with constant solidity for each layer is minimized by varying the solidity profile. We

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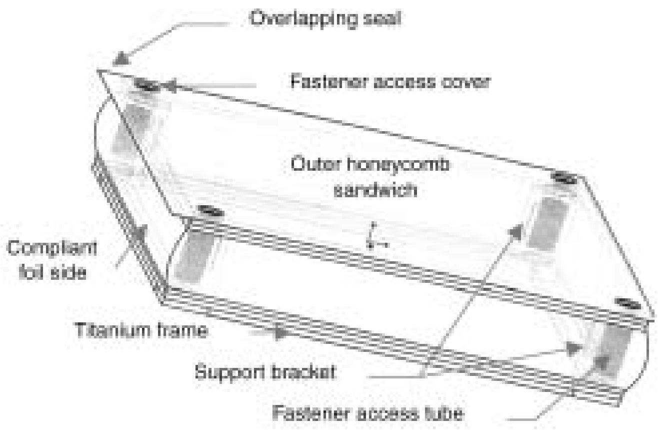

Fig. 1 Components of a metal thermal protection system (TPS) tile (Blosser 2002)

found that the cooler inner layer of the optimal design has high solidity, while the hotter outer layer has low solidity, which is contrary to our belief that the steady-state optimal design is a good approximation of transient optimal design.

2 Analysis

In this section we identify the necessary information and modeling details required to perform transient thermal analysis of the TPS insulation. Figure 2 shows a schematic of the simplified problem. The foam insulation is subjected to a transient heat flux $q_i(t)$ incident on the outside surface. The foam insulation is attached to the structural mass that makes the wall of the RLV tank. We assume that the amount of cooling heat load q_c that can be removed from the structure internally is negligibly small. The objective of the optimization is to ensure that the maximum temperature in the structural mass T_s is below a specified limit T_{slim} for all times during reentry and after landing. A similar constraint should also be imposed for the maximum temperature in the insulation T_{imax} to ensure that it does not exceed the maximum limiting temperature of the insulation material. However, we want the temperature at the outside wall to be as high as it can be so that most of the incident heat can be rejected by radiation at this surface. In this paper, we consider the first constraint on maximum structural temperature limit only.

The heat flux into the vehicle during reentry is given in Blosser *et al.* (2002). The heat flux varies significantly over the surface. For our study we choose a location on the lower surface referred to as station 413 (STA 413) in Blosser *et al.* (2002) as a representative point for the point design. The heat flux calculated by Blosser *et al.* (2002) for that location is reproduced in Fig. 3. The assumptions and the calculations used to obtain the heat flux are discussed in detail by Blosser *et al.* (2002).

For the preliminary phase of this study, we simplify the heat flux history functions as constant heat load:

$$q_i(t) = \begin{cases} q_{max} & 0 < t < t_0 \\ 0 & t_0 < t < t_f \end{cases}$$

where $q_{max} = 5.6745 \times 10^4\,\mathrm{W/m^2}$ ($5.0\,\mathrm{BTU/ft^2\text{-}s}$) is the maximum intensity of the heat flux, t_0 is the final time of the reentry (when the vehicle has landed). In our current study t_0 is 2000 seconds and t_f is 5000 seconds. The pressure differences are ignored, as they appear to be small in value.

The heat transfer in the TPS is assumed to be one dimensional. The finite width effects of the TPS insulation and the heat shorts resulting from the support structure around the perimeter of the TPS tile are ignored. The structural mass on the inside corresponds to the mass of the stiffened panel shell used for the RLV tank construction. The insulation itself is made of a titanium open-cell foam material, which is significantly less efficient than Saffil insulation, but it has the potential to carry structural loads. The foam is idealized as having rectangular cells of uniform size. The volume fraction (ρ) of metal in the foam, referred to as solidity, varies from 0.01 to 0.11. The variation in solidity is achieved by tailoring the cell size, while keeping the strut diameter fixed at 0.05 mm (0.002 inch). We assume titanium has a tem-

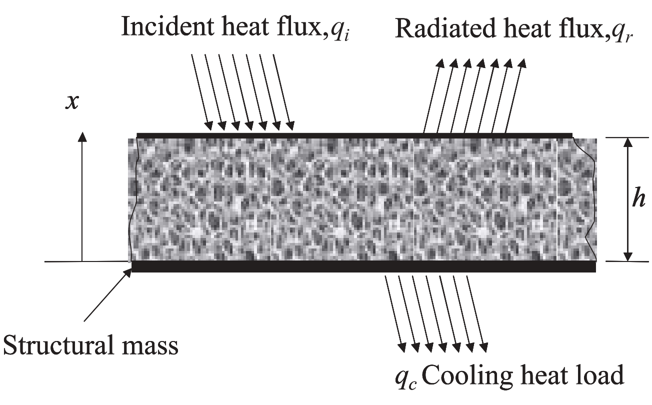

Fig. 2 Schematic of the heat transfer in the insulation on a reusable launch vehicle structure

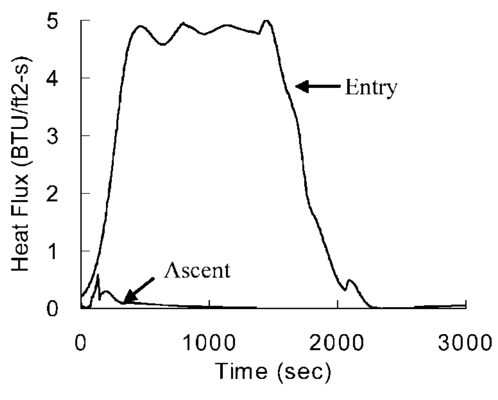

Fig. 3 Heat flux at a representative point on the windward side during ascent and reentry (Blosser 2002)

perature independent constant heat capacity C_p equal to 418.78 J/kg/K (0.1 Btu/lb$_{\mathrm{m}}$/°R).

Heat transfer in foams proceeds by three modes: conduction through the solid material, conduction in the gas filling the foam and radiation inside the foam. The model used to calculate the heat transfer coefficient in the foam is discussed by Venkataraman *et al.* (2002). At high temperatures radiation dominates the heat transfer, and at low temperatures conduction dominates the heat transfer. To minimize radiation, we require higher solidity foams (smaller foam pore sizes) while to minimize conduction we need low solidity foams (large foam pore sizes). Since there is a temperature gradient through the insulation, an optimum insulation requires different solidities in different regions. Optimum solidity profiles of graded foams that minimized heat transmitted to the structure under steady-state heat transfer conditions are presented in Venkataraman *et al*. (2002). The minimum mass design for functionally graded foam insulation under the same condition is presented in Zhu *et al.* (2003). In this paper we focus on the transient problem.

The governing equation for this one-dimensional heat conduction problem is given by:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \rho_d C_p \frac{\partial T}{\partial t} \tag{1}$$

where ρ_d is the insulation density and k is the thermal conductivity.

We use explicit finite difference (FD) method to discretize (1) as described in Appendix A, which also describes the discretization scheme and boundary condition implementation for the two-layer design described in Sect. 4. It should be noted that the thermal conductivities in the insulation are interpolated by the approach of summing the thermal resistance, which gives a harmonic mean of thermal conductivities at the interface between two adjacent grid points (Jaluria and Torrance 1986).

3
Performance of steady-state design

We start by assessing the performance of FG insulation designed for steady-state conditions by comparing them with uniform solidity insulation. The solidity profile of the FG insulation is designed to maximize the hot-side temperature of insulation, when it is subjected to a constant heat flux q_{max} under steady-state condition. Maximizing the outside temperature is equivalent to minimizing the heat flux into the insulation, which, as described in Venkataraman *et al*. (2002), requires minimization of the effective conductivity at every point for the given temperature at that point. The solidity of the uniform TPS is chosen such that it has the same thickness and mass as the graded insulation. In this paper the insulation thickness is chosen as 0.2 m, which is greater than the designs in Venkataraman *et al*. (2002) in order to keep the temperatures of the insulation and the structure in a reasonable range. The graded insulation areal density (m_p) is obtained as 20.9 kg/m^2 (4.29 lb/ft^2). The solidity of the corresponding uniform insulation is 0.0236, as shown in Fig. 4. The initial temperature of the structure and insulation panel is 300 K. Aluminum is selected as the structure at the cool side with a thickness of 2.2 mm (0.0866 inch) (Blosser *et al*., 2002). It has an areal density (m_s) of 6.1 kg/m^2 (1.25 lb/ft^2). The heat capacity C_{ps} of the structural mass is assumed to be temperature independent and equal to 494.16 J/kg/K (0.118 Btu/lb$_{\mathrm{m}}$/°R). The structural mass is insulated at the cool side ($x = 0$) so that there is no heat transfer out of the structure. The ambient temperature for $t \geq 2000$ seconds, when the aerodynamic heating stops, is assumed to be 300 K.

The maximum structural temperature of the uniform insulation is lower than that in graded insulation, as shown in Fig. 5. That is to say, the optimal solidity profile we obtained under steady-state conditions is far from optimal to protect the inner structure in the transient case.

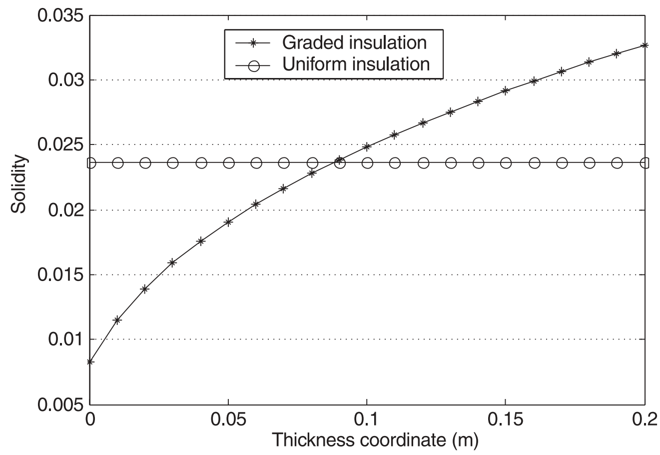

Fig. 4 Solidity profiles for graded insulation designed for the steady-state conditions and uniform insulation of same mass

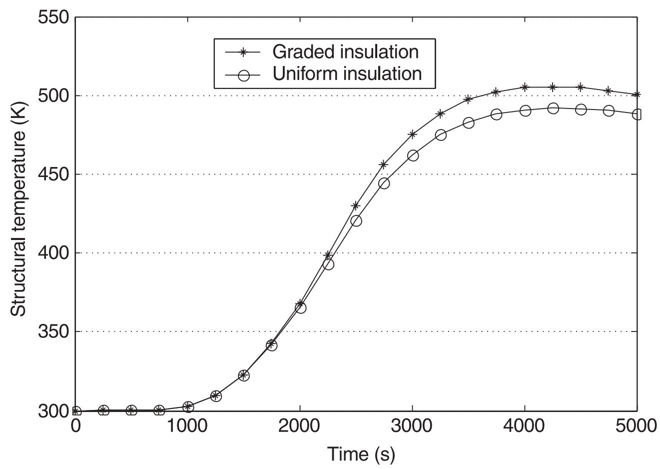

Fig. 5 Solidity profiles for graded insulation designed for the steady-state conditions and uniform insulation of the same mass

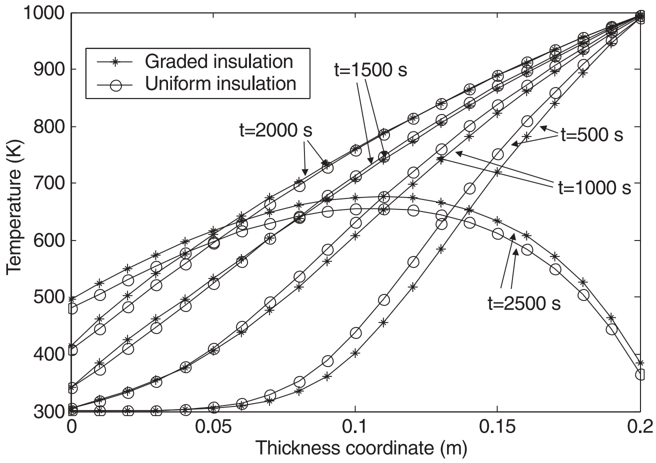

Fig. 6 Temperature profiles in a graded insulation designed for steady-state conditions and a uniform insulation of the same mass at different times

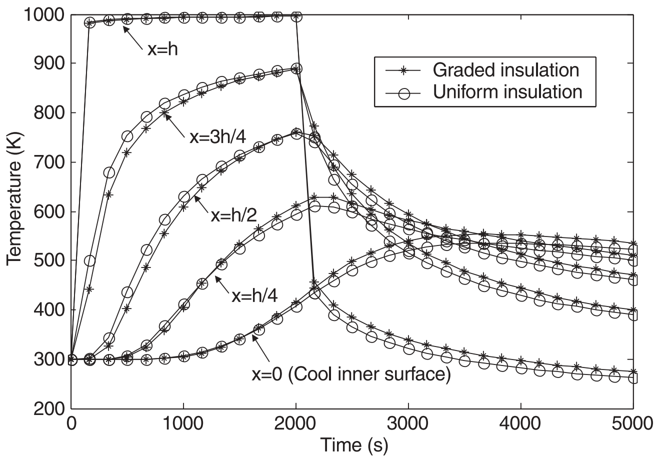

Fig. 7 Temperature histories of a graded insulation designed for the steady-state conditions and a uniform insulation of the same mass at different locations

Figure 6 gives the temperature profiles for both graded insulation and uniform insulation at different times. It explains why the steady-state optimum is not useful for transient conditions. It can be seen from Figs. 5 through 7 that for the early times the graded insulation provides better protection. However, later, and especially once the applied heat is removed, the uniform insulation becomes superior. The FG insulation has much more material on the hot side and that material continues to send heat to the structure after the external heating ends.

4 Two-layer design with transient analysis

Our ultimate goal is to design a functionally graded insulation for a given critical condition at the chosen point. However, we first solve the simpler problem, insulation with two uniform layers, to gain understanding of the effects of using functionally graded insulations. We consider an insulation panel with fixed thickness $h = 0.2$ m and fixed areal mass density $m_p = 22.15\,\mathrm{kg/m^2}$ ($4.54\,\mathrm{lb/ft^2}$), which is slightly higher than the mass in the previous section. The mass is chosen such that the average solidity of insulation is 0.025. As before, the mass of the structure is $m_s = 6.1\,\mathrm{kg/m^2}$ ($1.25\,\mathrm{lb/ft^2}$). The foam insulation is made of two uniform layers, layer 1 (cool side) and layer 2 (hot side), with thickness and solidity h_1, ρ_1, h_2, ρ_2, respectively.

The division of mass and thickness between the two layers is optimized numerically for minimizing the maximum temperature of the structure. We add the constraints that the solidities ρ_1 and ρ_2 should be between 0.01 and 0.11. The problem is formulated as:

$$\underset{h_1,\rho_1}{Minimize}\left\{\max_t\left[T_{str}\left(h_1,\rho_1,t\right)\right]\right\}$$

such that

$$h_1\rho_1 + h_2\rho_2 = \frac{m_p}{\rho_{Ti}}$$

$$0.01 \le h_1 \le 0.19$$

$$0.01 \le h_2 \le 0.19$$

$$0.01 \le \rho_1 \le 0.11$$

$$0.01 \le \rho_2 \le 0.11 \tag{2}$$

where T_{str} is the temperature of structure and ρ_{Ti} is the density of titanium $4431.8\,\mathrm{kg/m^3}$. We use the Matlab$^{\mathrm{TM}}$ function (*fmincon*) for constrained nonlinear optimization to solve the optimization problem. A sequential quadratic programming (SQP) method is used in this function. The optimization search starts from a uniform design, which has $\rho_1 = \rho_2 = 0.025$ and $h_1 = h_2 = h/2$. Surprisingly, for the present transient problem the optimum cooler inner layer had high solidity, while the hotter outer layer had low solidity, which is the opposite of what we would expect on the basis of conductivity. In order to clarify the results we also analyzed an extreme design, which had the minimum allowable solidity in the outer layer (0.01), the maximum allowable solidity (0.11) in the inner layer, and a uniform design, which had uniform solidity (0.025) through the whole insulation. These three designs are compared in Table 1.

Figure 8 shows the temperature history of the structure for these three designs. It can be seen that with uniform insulation the structure heats up more slowly, but it continues to heat much longer. For the extreme design the structure heats up fast, but peaks much earlier than the other two designs.

Close inspection of the history of heat absorbed by the various insulations explains the behavior. Figure 9 shows that during the heating period the uniform design absorbs more heat, but it also loses more heat during the cooling period. Figure 10 shows that, in terms of total heat captured from the outside, the uniform design ex-

Table 1 Two-layer designs for minimizing the maximum temperature of structure under transient conditions

	Optimal two-layer design	Extreme two-layer design	Uniform design
Cool-side h_1 (m)	0.0162	0.0300	N.A.
Cool-side solidity ρ_1	0.1100	0.1100	0.0250
Hot-side h_2 (m)	0.1838	0.1700	N.A.
Hot-side solidity ρ_2	0.0175	0.0100	0.0250
T_{str}^{max} (K)*	471.1	498.8	488.6
Time at which T_{str}^{max} is reached (s)	4203.7	3115.0	4572.0

$^*T_{str}^{max}$ is the maximum temperature of structure.

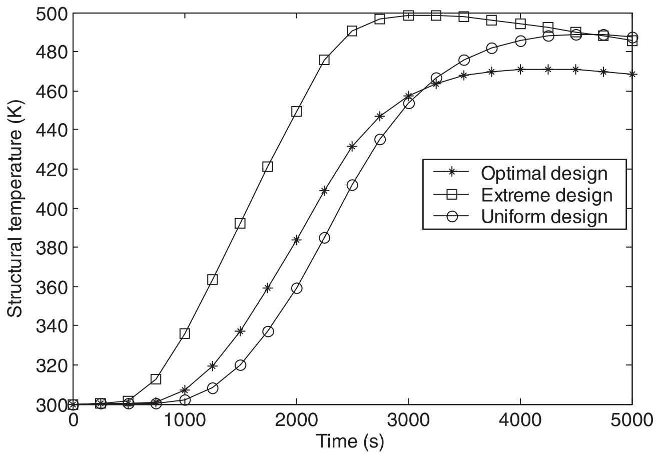

Fig. 8 Structural temperature history for various two-layer designs

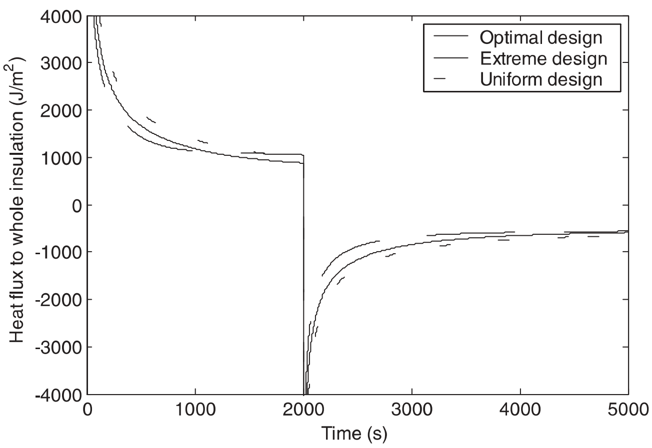

Fig. 9 History of heat flux to insulation in various two-layer designs

ceeds the optimum design throughout the entire history, even though the difference decreases during the cooling period.

As can be seen from these figures, it is useful to deal separately with the heating period and the cooling period of the outer surface. During the heating period, the optimum and extreme designs with low thermal mass on the outside can heat up more rapidly, as shown in Fig. 11, and radiate out more heat than the uniform design. The extreme design rejects heat more efficiently than the other two designs, and it lets in only 2.73% of the applied heat. The optimum design lets in only 2.91% of the applied heat, while the uniform design allows in 3.37%. Figure 11 shows that the extreme design outer surface heats up

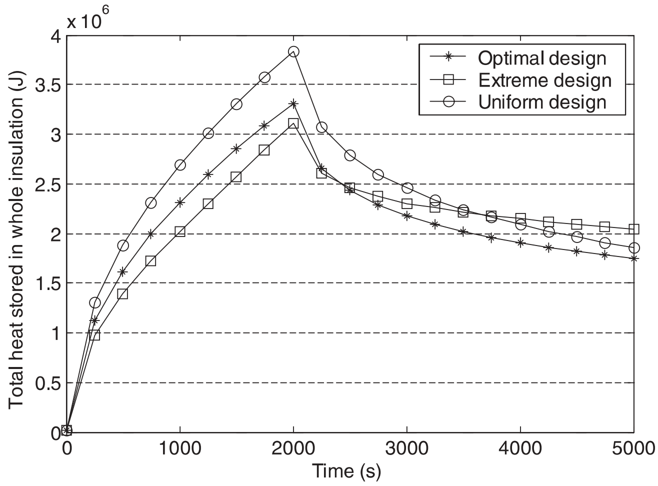

Fig. 10 History of heat stored in the whole insulation in the two-layer designs

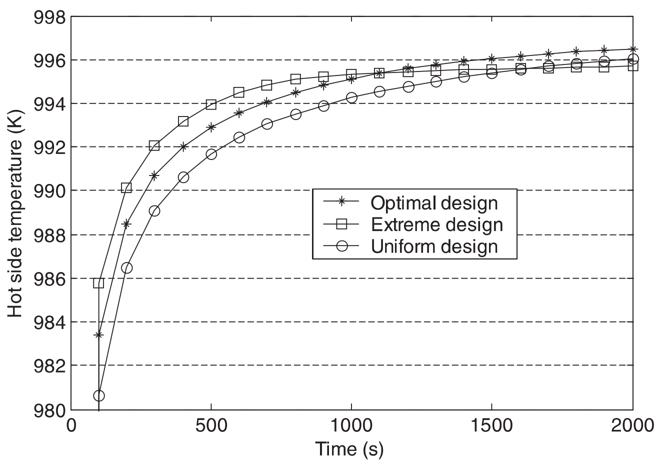

Fig. 11 History of hot-side temperatures in various two-layer designs during the heating phase

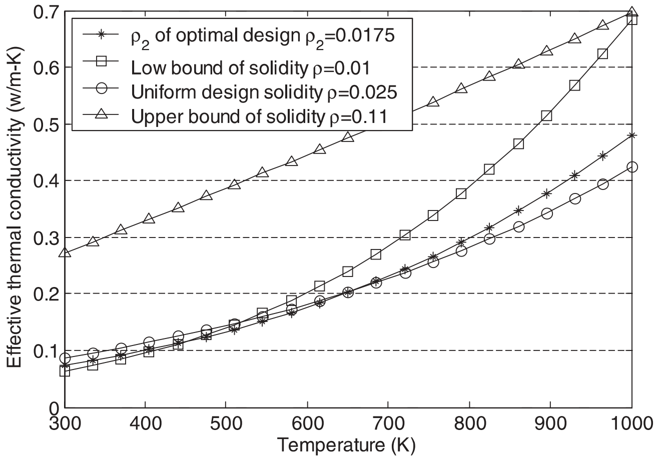

Fig. 12 Effective conductivity of foam as a function of temperature for different designs

Table 2 Comparison between optimal three-layer design and two-layer design for minimization of maximum structural temperature

	Two-layer design	Three-layer design
Cool-layer h_1 (m)	0.0162	0.0166
Cool-layer ρ_1	0.1100	0.1100
Middle-layer h_2 (m)	N.A.	0.0260
Middle-layer ρ_2	N.A.	0.0141
Hot-layer h_3 (m)	0.1838	0.1574
Hot-layer ρ_3	0.0175	0.0178
T_{str}^{max} (K)*	471.1	470.8

*T_{str}^{max} is the maximum temperature of structure.

more rapidly than the optimum design, however the optimum design eventually surpasses it. This is because the high conductivity of the low-solidity hot outer layer of the extreme design allows more of the heat to flow through the insulation instead of being stored in its outer layer. Even though the optimum and extreme insulations can reduce the heat absorbed by the whole insulation, they conduct more heat to the structure during the heating period. One reason is that their outer layers have higher conductivity, as shown in Fig. 12, and low solidity, which does not allow them to absorb much heat, and hence facilitates the heat flow to structure. The structural temperatures of the optimal and extreme design are 65 K and 90 K higher, respectively, than the uniform design at 2000 seconds, as shown in Fig. 8.

While during the cooling period the uniform insulation allows more heat to escape out than the optimal design, the difference is small, as shown in Fig. 9. The extreme design, on the other hand, has low conductivity in the outer layer once that layer cools. Thus it is more difficult for it to let out the heat stored in the inner layer (Fig. 9).

Finally we check the benefits of refining the design by increasing the number of layers to three. The total mass and total thickness remain fixed. The results for the optimal three-layer design are given in Table 2. It can be seen from Table 2 that the difference in maximum structure temperature is less than 1 K, which indicates that the optimal two-layer design captures most of the benefits of using a functionally graded foam insulation.

5 Concluding remarks

The problem of minimizing the maximum temperature of a structure insulated by a functionally graded metal foam insulation under transient heat conduction is studied. The performance of functionally graded insulation designed for steady-state conditions is compared with uniform solidity insulation. It is found that the optimal solidity profile obtained for steady-state conditions is not optimal for the transient case. The maximum temperature of a structure protected by two-layer insulation with constant solidity for each layer is minimized by varying the solidities and thicknesses of the two layers for a given thickness and mass. It is shown that the cooler inner layer has high solidity, while the hotter outer layer has low solidity, which is the reverse of the optimum design for steady-state condition. The steady-state optimal design is not a good approximation of transient optimal design. Refining the distribution to three layers has only a minimal effect on the results.

It appears that the main reason for the reversal of the solidity profile is the desirability for the outer layer to heat up fast so as to maximize the heat radiated back to the outside. The low thermal mass of the low-solidity outer layer permits a rapid rise in temperature. However, if the outer layer solidity becomes too low, the increase in conductivity during heating and decrease in conductivity during cooling counteracts some of the benefits of the low thermal mass, and that is why the optimum outer solidity is not at its lower bound.

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Appendix: Discretization scheme

The subscript i denotes the node of spatial discretization and the superscript n denotes the time step.

$$\rho_{di} C_p \frac{T_i^{n+1} - T_i^n}{\Delta t} = \frac{k_{i+1/2}\left(T_{i+1}^n - T_i^n\right)}{(\Delta x)^2} - \frac{k_{i-1/2}\left(T_i^n - T_{i-1}^n\right)}{(\Delta x)^2} \tag{A.1}$$

where C_p is the heat capacity of insulation. The conductivity at the middle of two nodes $(i, i+1)$ is given by

$$k_{i+1/2} = \frac{2k_i k_{i+1}}{k_i + k_{i+1}} \tag{A.2}$$

The structural mass m_s attached to the cool side of the insulation is thermally insulated on the inside. The finite difference equation becomes

$$q_c + m_s C_{ps} \frac{T_1^{n+1} - T_1^n}{\Delta t} + A\frac{\Delta x}{2} C_p \rho_{d1} \frac{T_1^{n+1} - T_1^n}{\Delta t} = A k_{1+1/2} \frac{T_2^n - T_1^n}{\Delta x} \tag{A.3}$$

$$T_1^{n+1} = T_1^n + \frac{\Delta t}{m_s C_{ps} + \frac{A\rho_{d1} C_p \Delta x}{2}} \times \left(A k_{1+1/2} \frac{T_2^n - T_1^n}{\Delta x} - q_c\right) \tag{A.4}$$

where A is the cross-section of the insulation, which is unity in this paper. C_{ps} is the heat capacity of structural mass and T_1 is the cool-side temperature. The heat flow from the cool side, q_c, is assumed to be zero in our study. Only radiation is included at the hot side of insulation. The emittance is assumed to be one. We neglect the convection between the insulation and outer air. The radiation boundary condition at the hot side of the TPS can be expressed as:

$$T_{hot}^{n+1} = T_{hot}^n + \frac{2}{\rho_{di} C_p \Delta x} \times \left[\Delta t\left(q_i - \varepsilon\sigma (T_{hot}^n)^4\right) - k_{hot-1/2}^n (T_{hot}^n - T_{hot-1}^n)\frac{\Delta t}{\Delta x}\right] \tag{A.5}$$

where T_{hot} is the temperature of the outmost node at hot side. At the interface between layer 1 and layer 2, we have,

$$T_i^{n+1} = T_i^n + \frac{2\Delta t}{\left(\Delta x_1 \rho_{d1} + \Delta x_2 \rho_{d2}\right) C_p} \times \left[\frac{k_{i+1/2}\left(T_{i+1}^n - T_i^n\right)}{\Delta x_2} - \frac{k_{i-1/2}\left(T_i^n - T_{i-1}^n\right)}{\Delta x_1}\right] \tag{A.6}$$

where Δx_1 and Δx_2 are the layer 1 and layer 2 discretization respectively. The finite difference scheme for the solution used 21 nodes to calculate the heat transfer and temperature distribution in the insulation and at least two elements are used to model each layer. Explicit schemes require that the maximum time step should satisfy the following condition:

$$\Delta t \le \frac{(\Delta x_1)^2}{\frac{2k_1}{\rho_1 C_p}} \qquad \Delta t \le \frac{(\Delta x_2)^2}{\frac{2k_2}{\rho_2 C_p}} \tag{A.7}$$

where k_1 and k_2 are the conductivities in layer 1 and layer 2 respectively. The maximum diffusivity $(\frac{k}{\rho C_p})$ in both layer 1 and 2 is less than $1.25 \times 10^4\ \mathrm{m^2/s}$. The smallest space discretization is 0.005 m because of our side constraints in (2). So the maximum time step we can take is 0.3125 seconds. In our numerical evaluation we take 0.25 seconds as our time step.