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Dimensionality Reduction Approach for Response Surface Approximations: Application to Thermal Design

Christian Gogu,* Raphael T. Haftka,† Satish K. Bapanapalli,‡ and Bhavani V. Sankar§
University of Florida, Gainesville, Florida 32611-6250

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Response surface approximations are a common engineering tool, often constructed based on finite element simulations. For some design problems, the finite element models can involve a large number of parameters. However, it is advantageous to construct the response surface approximations as a function of the smallest possible number of variables. The purpose of this paper is to demonstrate that a significant reduction in the number of variables needed for a response surface approximation is possible through physical reasoning, dimensional analysis, and global sensitivity analysis. This approach is demonstrated for a transient thermal problem, but we also show how it can be applied to any finite-element-based surrogate model construction. The thermal problem considered is the design of an integrated thermal protection system for spacecraft reentry for which a response surface approximation of the maximum bottom face temperature is needed. The finite element model used to evaluate the maximum temperature depended on 15 parameters of interest for the design. A small number of assumptions simplified the thermal equations, allowing easy nondimensionalization, which together with a global sensitivity analysis showed that the maximum temperature mainly depends on only two nondimensional parameters. These were selected to be the variables of the response surface approximation for maximum temperature, which was constructed using simulations from the original nonsimplified finite element model. The major error in the two-dimensional response surface approximation was found to be due to the fact that the two nondimensional variables account for only part (albeit the major part) of the dependence on the original 15 variables. This error was checked and reasonable agreement was found. The two-dimensional nature of the response surface approximations allowed graphical representation, which we used for material selection from among hundreds of possible materials for the design optimization of an integrated thermal protection system panel.

Nomenclature

A	=	cross-sectional area, m^2
Bi	=	nondimensional convection coefficient (or Biot number)
C	=	specific heat, $J/(kg \cdot K)$
d	=	thickness, m
f	=	final reduced number of parameters
h	=	convection coefficient, $W/(m^2 \cdot K)$
k	=	thermal conductivity, $W/(m^2 \cdot K)$
L	=	height of the sandwich panel, m
p	=	half-unit cell length, m
Q, Q_i	=	heat flux and total incident heat flux, respectively, W/m^2
q	=	number of nondimensional variables in problem Σ
r	=	reduced number of parameters obtained through physical reasoning
S, S^*	=	general and simplified finite element problem considered, respectively
s	=	number of variables of interest in the problem S
T, T_i	=	temperature and initial temperature, respectively, K
$T_{\text{Max BFS}}$	=	maximum bottom face sheet temperature, K
t	=	time relative to the transient thermal problem, s
t_{end}	=	duration of the reentry simulation, s
V	=	volume, m^3
w	=	dimensional parameters of interest in problem S
x	=	position through the thickness of the panel, m
Y, Y'	=	response sought to the problem S and its response surface approximation, respectively
β	=	nondimensional thermal diffusivity (or Fourier number)
Γ	=	nondimensional temperature
γ	=	nondimensional heat capacity parameter
ε	=	emissivity, $W/(m^2 \cdot K^4)$
θ	=	angle of corrugations, deg
κ	=	nondimensional radiation coefficient
ξ	=	nondimensional position through the panel thickness
ρ	=	density, kg/m^3
Σ	=	nondimensionalized version of problem S^*
τ	=	nondimensional time
φ	=	nondimensional heat flux
ψ	=	nondimensional response of the problem Σ
ω	=	nondimensional parameters of interest in problem Σ

Subscripts

B	=	bottom face sheet
C	=	homogenized core
S	=	Saffil foam
T	=	top face sheet
W	=	web

I. Introduction

DIMENSIONAL analysis is a several-hundred-years-old concept going as far back as Galilei [1]. This concept has its roots in a very simple idea: the solution to a physical problem has to be independent of the units used. This means that the equations modeling a problem can always be written in a nondimensional form.

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*Research Assistant, Department of Mechanical and Aerospace Engineering, Post Office Box 116250; also Ph.D. Student, Centre Sciences des Matériaux et des Structures, 158 Cours Fauriel, École Nationale Supérieure des Mines de Saint Etienne, 42023 St. Etienne Cedex 2, France; christian.gogu@gmail.com

†Distinguished Professor, Department of Mechanical and Aerospace Engineering, Post Office Box 116250. Fellow AIAA.

‡Research Assistant, Department of Mechanical and Aerospace Engineering, Post Office Box 116250; currently Scientist, Pacific Northwest National Lab, Post Office Box 999, Richland, Washington 99354.

§Ebaugh Professor, Department of Mechanical and Aerospace Engineering, Post Office Box 116250. Associate Fellow AIAA.

In the process, nondimensional parameters are constructed, which, when done appropriately, are the minimum number of variables required to formulate the problem.

These basic concepts turned out to be very powerful and, throughout the past century, dimensional analysis has been extremely successful in solving scientific and engineering problems and presenting the results in a compact form. The first theoretical foundations of dimensional analysis were set by Vaschy [2] and Buckingham [3] at the end of the 19th century. Since then, and up to the 1960s, nondimensional solutions have been a major form of transmitting knowledge among scientists and engineers, often in the form of graphs in nondimensionalized variables. Then, the advent of widely available numerical simulation software and hardware made it easier to obtain solutions to physical problems without going through nondimensionalization. This led to a reduced interest in dimensional analysis, except for reduced scale modeling and areas in which nondimensional parameters have a strong physical interpretation and allow us to differentiate between regimes of different numerical solution techniques (Mach number, Reynolds number, etc.).

With the increase in computational power, numerical simulation techniques such as finite element analysis (FEA) became not only feasible for single engineering design analyses but also for design optimization, often in conjunction with the use of surrogate models. Surrogate models, however, suffer from the "curse of dimensionality," that is, the number of experiments needed for a surrogate for a given accuracy increases exponentially with the number of dimensions of the problem.

This issue can be generally attacked in two ways. One is by reducing the cost of a single analysis, thus allowing a large number of analyses to be run for the response surface approximation (RSA) construction. The other is by reducing the number of design variables, thus reducing the dimensionality of the problem. Many techniques, often referred to as model reduction, have been developed to deal with these problems, including static and dynamic condensation, modal coordinate reduction, the Ritz vector method, component mode synthesis, proper orthogonal decomposition, and balanced realization reduction. For an overview of these techniques, the reader can refer to Qu [4].

A simple, yet relatively little-used way of reducing the dimensionality of the surrogates or response surface approximations is by applying dimensional analysis to the equations of the physical problem that the finite element (FE) model describes. Kaufman et al. [5], Vignaux and Scott [6], and Lacey and Steele [7] showed that better accuracy of the RSA can be obtained by using nondimensional variables. This is mainly because, for the same number of numerical simulations, the generally much-fewer nondimensional variables allow a fit with a higher-order polynomial. Vignaux and Scott [6] illustrated such a method using statistical data from a survey, whereas Lacey and Steele [7] applied the method to several engineering case studies, including an FE-based example.

In [8], Venter et al. illustrated how dimensional analysis can be used to reduce the number of variables of an RSA constructed from FEAs modeling a mechanical problem of a plate with an abrupt change in thickness. The dimensional analysis was done directly on the governing equations and the boundary conditions that the FEA solved, reducing the number of variables from nine to seven.

Dimensional analysis can be used to reduce the number of variables in any FE-based model. Indeed, FEA models an underlying set of explicit equations (ordinary or partial differential equations, boundary conditions, and initial conditions). These equations, whether coming from mechanical, thermal, fluids, or other problems, can be nondimensionalized in a systematic way using the Vaschy–Buckingham theorem (or Pi theorem) [2,3]. Systematic nondimensionalization techniques are also described in [9,10].

Although dimensional analysis is a natural tool to reduce the number of variables through which a problem has to be expressed, an even-higher reduction can be obtained if it is combined with other analytical and numerical techniques. The aim of this paper is to show that, through a combination of physical reasoning, dimensional analysis, and global sensitivity analysis, a drastic reduction in the number of variables needed for an RSA is possible.

The basic idea is that, even after nondimensionalization, it is still possible to end up with nondimensional parameters that only have a marginal influence on the quantity of interest for the design problem considered. Determining and removing these parameters can further reduce the total number of variables. This can be done at two moments. Before nondimensionalization, physical reasoning can allow the formulation of a set of assumptions that simplify the equations of the problem. After nondimensionalization, a global sensitivity analysis, for example, Sobol [11], can be used to remove any remaining parameters with negligible effects.

In Sec. II, we present the general methodology for reducing the number of variables in a response surface approximation. In the rest of the paper, we apply the method to solve a transient thermal problem of spacecraft atmospheric reentry, wherein the maximum temperature attained is critical. In Sec. III, we describe the thermal problem of atmospheric reentry and, in Sec. IV, the corresponding FE model used in the analysis. A dimensional analysis on a simplified problem in conjunction with a global sensitivity analysis is used in Sec. V to reduce the number of variables. The RSA is constructed in Sec. VI using the accurate FE model and the ability of the RSA to account for all the variables of interest to the problem tested. In Sec. VII, we discuss the advantages of the procedure in terms of computational cost. Finally, in Sec. VIII, we give a brief overview of how the RSA was used to carry out a material comparison and selection for the design and optimization of an integrated thermal protection system (ITPS). Section IX presents concluding remarks.

II. Methodology for the Reduction of the Number of Variables in a Response Surface Approximation

We consider the general problem in which we are interested in the response Y of an FE problem denoted as S. The response Y potentially depends on s parameters of interest, denoted as $\boldsymbol{w}^s = \{w_1, \ldots, w_s\}$. We consider the case in which an RSA of Y is needed. If s is high (>10), then it can be beneficial to seek to construct the RSA in a lower-dimension space. Indeed, an RSA in a lower-dimension space reduces the computational cost (number of simulations required) for a fixed accuracy or improves the accuracy for a fixed computational cost. A low dimension is also preferable, especially if the RSA is later used for optimization.

To construct the RSA as a function of a small number of parameters, we use the following procedure involving three major steps.

1) Using preliminary physical reasoning, we can often determine that only r out of the s initial parameters ($r \leq s$) significantly affect the response Y. Indeed, in many engineering problems it is known based on empirical, theoretical, or numerical evidence that some parameters have little effect on the response for the particular problem considered. Different choices for the numerical model or the use of homogenization can also allow simplification of the problem. The simplified problem involving only $\boldsymbol{w}^r = \{w_1, \ldots, w_r\}$ is denoted as S^*.

Sometimes a designer might not have enough domain expertise to formulate all the simplifying assumptions through physical reasoning. If little or nothing is known in advance that can help simplify the problem, this step can then be aided by a global sensitivity analysis (GSA) as described by Sobol [11]. GSA is a variance-based technique, quantifying the part of the variance of the response explained by each parameter and thus determining the parameters that have negligible effects. However, the GSA can only be carried out if the computational cost does not become prohibitive. If nothing works, it is always possible to go directly to step 2.

The aim of step 1, when successful, is to define the simplified problem S^*, which will facilitate the next step, the nondimensionalization.

2) In this step, we further reduce the number of variables by determining the nondimensional parameters characterizing the problem. The dimensional problem S^* can indeed be transformed into the nondimensional problem Σ using the Vaschy–Buckingham theorem [2,3]. A systematic nondimensionalization technique is

provided in [9,10]. We can then express the nondimensional response ψ of the problem Σ as a function of the nondimensional parameters $\boldsymbol{\omega}^q = \{\omega_1, \ldots, \omega_q\}$. According to the Vaschy–Buckingham theorem, $q \leq r$.

Note that the problem Σ is equivalent to S^*, and so no additional approximation is involved in this step. However, because ψ is a solution to Σ, which is equivalent to S^*, it will only provide an approximate solution to the initial problem S.

3) Out of the $\boldsymbol{\omega}^q$ nondimensional parameters that we have determined in step 2, not all will necessarily have a significant influence on the response ψ. To determine and remove parameters with negligible influence, we carry out in this step a GSA (cf. Sobol [11]). After such parameters have been removed, we can write ψ approximately as a function of $\boldsymbol{\omega}^f = \{\omega_1, \ldots, \omega_p\}$, with $f \leq q$.

At the conclusion of the process, we have $f \leq q \leq r \leq s$. The possibility of obtaining equality everywhere, while theoretically possible, is extremely unlikely for an actual engineering problem, and it is hoped that we achieved f significantly smaller than s after these three steps.

At this point we have determined that the approximate nondimensional response ψ approximately depends only on the parameters $\boldsymbol{\omega}^f$. However, our final aim is to construct a response surface approximation of Y, the actual response, and not of ψ, which is the response of the approximate problem Σ. Accordingly, we chose to construct an RSA Y' of Y, but as a function of the reduced number of nondimensional parameters $\boldsymbol{\omega}^f$.

That is, even though we made simplifying assumptions and a GSA to determine $\boldsymbol{\omega}^f$, we will construct the RSA function of these $\boldsymbol{\omega}^f$ parameters using simulations of Y, coming from the initial nonsimplified FE model of S. This allows part of the error induced by constructing the RSA function of $\boldsymbol{\omega}^f$ instead of $\boldsymbol{w}^s$ to be compensated by fitting to the actual nonsimplified FE simulations of Y.

The sampling for the RSA simulations is done in the $\boldsymbol{\omega}^f$ space. The RSA $Y' = f(\boldsymbol{\omega}^f)$ is then constructed, and the quality of its fit can be analyzed using classical techniques (prediction sum of squares (PRESS) error, for example, [12,13]). Note, however, that these analyses provide mainly the quality of the fit in the reduced nondimensional variables $\boldsymbol{\omega}^f$ but not in the initial variables $\boldsymbol{w}^s$. To remedy this, an additional validation step can be carried out. A number of additional points are sampled in the initial, high-dimensional $\boldsymbol{w}^s$ space. The FE response Y is calculated at these points and compared with the prediction of the reduced nondimensional RSA Y' to make sure the accuracy of the RSA Y' is acceptable.

In the rest of this paper, we show how we applied this procedure to a transient thermal problem of spacecraft atmospheric reentry, for which a response surface approximation of the maximum temperature was required. Note that the application problem presented is a one-dimensional heat transfer problem. However, the general method described can be applied as well to two- or three-dimensional finite element problems. Steps 1 and 3 are not affected much by moving from one- to three-dimensional models, other than maybe through increased computational cost. Nondimensionalizing the governing equations of the problem in step 2 may be slightly more complex. However, although nondimensionalization is simple enough to be often applied by hand to one-dimensional problems, there are systematic nondimensionalization techniques [9,10] that can be applied to any governing equations and boundary conditions.

A final note concerns the application of the nondimensional RSA to a design optimization framework. Because the RSA is in terms of nondimensional parameters, these could be chosen as variables for the optimization algorithm. This is, however, often a bad choice, because it is often difficult to move from a point (the optimum, for example) in the nondimensional variables space to the corresponding design point in the physical, dimensional variables space. A better choice in this case is to do the optimization in terms of the dimensional variables. A typical function evaluation step in the optimization routine would then look like the following: dimensional variables point at which the response is required → calculate the corresponding nondimensional variables for this point → calculate the response at this point using the nondimensional RSA. Although this may leave a large number of design variables, that is usually affordable because surrogate-based function evaluations are inexpensive.

III. Integrated Thermal Protection System Thermal Problem of Atmospheric Reentry

An ITPS is a proposed spacecraft system that differs from a traditional TPS in that it provides not only thermal insulation to the vehicle during atmospheric reentry but it carries structural loads at the same time. Thus, the thermal protection function is integrated with the structural function of the spacecraft. Our study involves an ITPS based on a corrugated core sandwich panel construction. The design of such an ITPS involves both thermal and structural constraints. In the present paper, we focus on the thermal constraint represented by the maximum temperature of the bottom face sheet (BFS) of the ITPS panel. The combined thermostructural approach was presented in separate articles [14,15].

An RSA of the maximum BFS temperature was needed to reduce computational time. The RSA is used to carry out material selection for the ITPS panel.

To calculate the maximum BFS temperature, we constructed an FE model using the commercial FE software Abaqus® [16]. The corrugated core sandwich panel design as well as the thermal problem of atmospheric reentry is shown in Fig. 1. The ITPS panel is subject to an incident heat flux assumed to vary, as shown in Fig. 2. This heat flux is typical of a reusable launch vehicle.

Radiation is also modeled on the top face sheet (TFS), whereas the BFS is assumed to be perfectly insulated, which is a worst-case assumption, because if heat could be removed from the BFS the maximum temperature would decrease, becoming less critical. The core of the sandwich panel is assumed to be filled with Saffil foam insulation, whereas we explore different materials for the three main sections, the TFS, BFS, and web (cf. Figure 1), to determine the combinations of materials that will result in low BFS temperatures.

IV. Finite Element Model of the Thermal Problem

The FE thermal problem is modeled as a one-dimensional heat transfer analysis, as shown in Fig. 3. The core of the sandwich panel has been homogenized using the rule of mixtures formulas:

$$\rho_C = \frac{\rho_W V_W + \rho_S V_S}{V_C} = \frac{\rho_W d_W + \rho_S(p \sin\theta - d_W)}{p \sin\theta} \tag{1}$$

$$C_C = \frac{C_W \rho_W V_W + C_S \rho_S V_S}{\rho_C V_C} = \frac{\rho_W C_W d_W + \rho_S C_S (p \sin\theta - d_W)}{\rho_W d_W + \rho_S(p \sin\theta - d_W)} \tag{2}$$

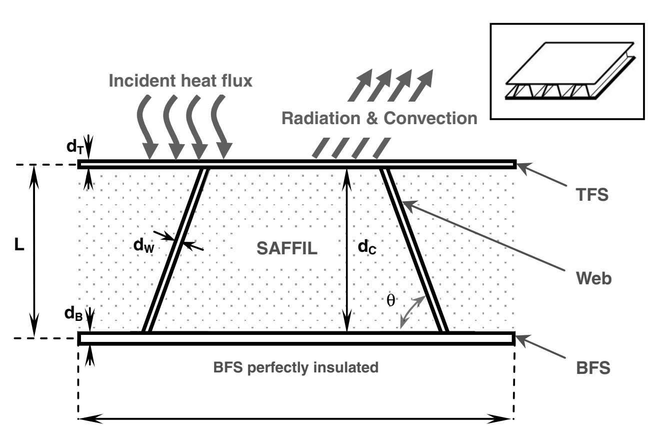

Fig. 1 Corrugated core sandwich panel depicting the thermal boundary conditions and the geometric parameters.

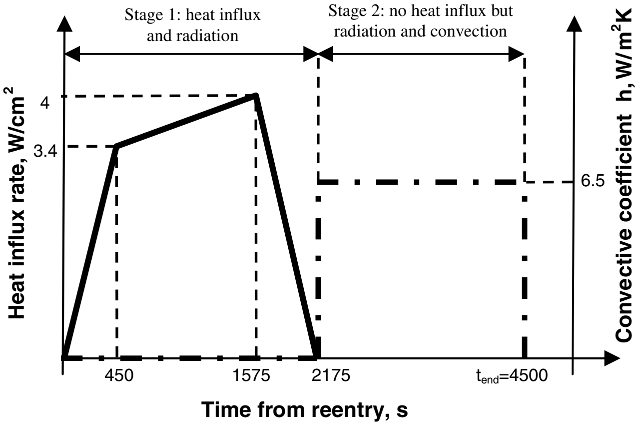

Fig. 2 Incident heat flux (solid line) and convection (dashed dotted line) profile on the TFS surface as a function of reentry time.

$$k_C = \frac{k_W A_W + k_S A_S}{A_C} = \frac{k_W d_W + k_S(p \sin\theta - d_W)}{p \sin\theta} \qquad (3)$$

It has been shown in [14] that a one-dimensional FE model can accurately predict the temperature at the bottom face sheet of the sandwich panel. The maximum difference in the BFS temperature prediction between the one- and two-dimensional model is typically less than 8 K. For this preliminary design phase of the ITPS, this difference is acceptable. Radiation, convection, and the incident heat flux (as shown in Fig. 1 and 2) were modeled in the Abaqus® one-dimensional model using four steps (three for stage 1 of Fig. 2 and one for stage 2). Fifty-four three-node heat transfer link elements were used in the transient analyses.

For this one-dimensional thermal model, the governing equations and boundary conditions are as follows.

Heat conduction equation:

$$\frac{\partial}{\partial x}\left(k(x,T)\frac{\partial T(x,t)}{\partial x}\right) = \rho(x,T)C(x,T)\frac{\partial T(x,t)}{\partial t} \qquad (4)$$

Initial condition:

$$T(x, t = 0) = T_i \qquad (5)$$

Boundary conditions:

$$Q(x = 0, t) = -k_T \frac{\partial T(x,t)}{\partial x}\bigg|_{x=0} = Q_i(t) - \varepsilon T(0,t)^4 - h(t)T(0,t) \qquad (6)$$

$$Q(x = L, t) = 0 \qquad (7)$$

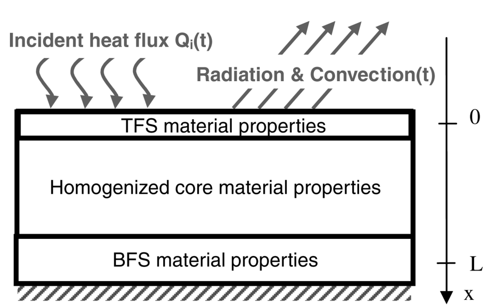

Fig. 3 One-dimensional heat transfer model representation using homogenization (not to scale).

Note that $q_i(t)$ is the heat influx and $h(t)$ is the convection coefficient at the TFS, which vary with time of reentry as shown in Fig. 2. Most of the material properties are temperature dependent and, due to the different materials in the different ITPS sections, most material properties also depend on the position x. The temperature and spatial dependency make nondimensionalization of the previous equations cumbersome. Furthermore, these dependencies increase the number of nondimensional parameters needed, which is contrary to our goal. Accordingly, the thermal problem is studied in the next section under several assumptions that allow easier nondimensionalization of the equations as well as a reduction in the number of variables.

V. Determining the Minimum Number of Parameters for the Temperature Response Surface

A. Simplifying the Problem

Our goal for the ITPS study is to determine which materials are the best for use in the ITPS panel based on the maximum BFS temperature. Considering that the expected range of this temperature when the materials are varied is about 250 K, an approximation of the temperature with an accuracy of the order of 12.5 K (5%) is considered acceptable for the purpose of material selection.

The thermal model presented in the previous section involves 13 material parameters (the specific heat C_i, conductivities k_i, and densities ρ_i of the TFS, BFS, web, and Saffil, as well as the emissivity ε of the TFS), of which most are temperature dependent. Some of these parameters are considered fixed, including ε as well as all the foam parameters (Saffil has been determined in previous studies [17,18] to be the best-suited foam for use in similar thermal protection systems). Note that the emissivity ε is defined as the relative emissivity times the Stefan–Boltzmann constant. The relative emissivity of the TFS depends more on surface treatments than on the nature of the TFS material (thus, a typical value for this kind of application of 0.8 was used [18,19]). Fixing these parameters leaves nine material variables. Describing the temperature dependency of the material properties would increase this number further.

On top of the nine material parameters, we also have six geometric design variables (cf. Figure 1) that we use to find the optimal geometry for each material combination. In total, we have 15 variables of interest for the maximum BFS temperature determination.

To reduce the number of design variables, the equations were studied under several simplifying assumptions that removed parameters that have a negligible role on the maximum BFS temperature. These assumptions have been established and checked on a Nextel(TFS)–zirconia(web)–aluminum(BFS) ITPS material combination with the dimensions given in Table 1. The assumptions are as follows:

1) The three thermal properties of the TFS (C_T, k_T, and ρ_T) have a negligible impact on the maximum BFS temperature, mainly due to the small thickness of the TFS (about 2.2 mm compared with a total ITPS thickness of about 120 mm). This assumption allowed removing C_T, k_T, ρ_T, and d_T from the relevant parameters influencing the BFS temperature.

2) The temperature is approximately constant through the BFS, because the BFS thickness is small (typically 5 mm thick compared with a total ITPS thickness of 120 mm) and its conductivity is about 1 order of magnitude higher than that of the homogenized core. This assumption allowed the removal of k_B and the simplification of the boundary condition at the BFS.

3) The temperature dependence of the material properties have been simplified as follows. In the FE model, temperature dependence has been included for all materials, but the largest dependence was for the Saffil foam. Hence, in the simplified problem, the TFS, web, and BFS materials were assigned constant properties provided by the CES 2005 material database.¶ For Saffil, the material properties were

¶CES Selector Edupack 2005 material database and selection software by Granta Design, 2005.

Table 1 Dimensions of the ITPS used among other to establish the simplifying assumptions. These dimensions were optimal for an Inconel(TFS)–Ti6Al4V(Web)–Al(BFS) ITPS (cf. [14])

Parameter	d_T, mm	d_B, mm	d_W, mm	θ, deg	L, mm	p, mm
Value	2.1	5.3	3.1	87	120	117

assigned values at a representative temperature chosen so as to minimize the difference between the maximum BFS temperature when using the constant values and the one when using temperature-dependent values for an ITPS design with the dimensions given in Table 1 and a Nextel(TFS)–zirconia(web)–aluminum(BFS) material combination. The effects of varying the materials were then found to be small enough to use this constant value for the range of materials we consider (more detailed results are provided at the end of Sec. VI).

These assumptions reduced the number of relevant material parameters from 15 to 10 and also simplified the problem so that it can be easily nondimensionalized, as will be shown next.

B. Nondimensionalizing the Thermal Problem

Under the previous simplifying assumptions, the thermal problem is equivalent to the one shown in Fig. 4 and its equations can be rewritten as follows.

Heat conduction equation:

$$k_C \frac{\partial^2 T(x,t)}{\partial x^2} = \rho_C C_C \frac{\partial T(x,t)}{\partial t} \quad \text{for } 0 < t < t_{\text{end}} \tag{8}$$

Initial condition:

$$T(x, t=0) = T_i \tag{9}$$

Boundary conditions:

$$Q_{\text{out}} = -k_C \frac{\partial T(x,t)}{\partial x}\bigg|_{x=d_C} = \rho_B C_B d_B \frac{\partial T(x,t)}{\partial t}\bigg|_{x=d_C} \tag{10}$$

$$Q_{\text{in}} = -k_C \frac{\partial T(x,t)}{\partial x}\bigg|_{x=0} = Q_i(t) - \varepsilon T(0,t)^4 - h(t)T(0,t) \tag{11}$$

To nondimensionalize these equations, we use the Vaschy–Buckingham theorem (or Pi theorem) [2,3], which also provides the minimum number of nondimensional variables. The theorem states that we have to count the total number of variables and the corresponding number of dimensional groups. The variables and the groups are listed in Table 2.

We have a total of 12 variables in four independent dimensional groups, namely, length, time, temperature, and power (in meters, seconds, Kelvin, and watts). From the Vaschy–Buckingham theorem, we know that we can have a minimum of $12 - 4 = 8$ nondimensional variables, which are provided in Table 3.

In terms of these nondimensional variables, the simplified thermal problem can be written in the following nondimensional form.

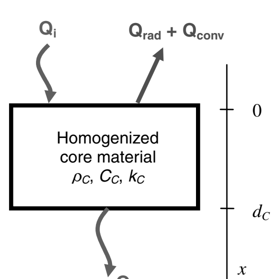

Fig. 4 Simplified thermal problem for dimensional analysis.

Heat conduction equation:

$$\beta \frac{\partial^2 \Gamma}{\partial \xi^2} = \frac{\partial \Gamma}{\partial \tau} \quad \text{for } 0 < \tau < 1 \tag{20}$$

Initial condition:

$$\Gamma(\xi, \tau = 0) = 1 \tag{21}$$

Boundary conditions:

$$-\beta \frac{\partial \Gamma}{\partial \xi}\bigg|_{\xi=1} = \gamma \frac{\partial \Gamma}{\partial \tau}\bigg|_{\xi=1} \tag{22}$$

$$-\frac{\partial \Gamma}{\partial \xi}\bigg|_{\xi=0} = \varphi(\tau) - \kappa \cdot \Gamma(0,\tau)^4 - Bi(\tau) \cdot \Gamma(0,\tau) \tag{23}$$

The complete set of nondimensional variables needed for the problem is given in Eqs. (12–19) in Table 3. Note that the problem's equations were nondimensionalized only to determine a reduced number of nondimensional parameters, which will be used as variables of the response surface approximation. The nondimensional formulation is not used for solving the problem, nor do the finite element simulations use this formulation directly.

The nondimensional temperature Γ can be expressed as a function of the nondimensional distance ξ and the nondimensional time τ, as well as a function of five other nondimensional parameters. Because at the maximum BFS temperature we are at a fixed location and we are not interested in the time at which this maximum occurs, the nondimensional distance ξ and the nondimensional time τ are not needed for the maximum BFS temperature RSA.

The physical interpretation of the remaining five nondimensional parameters in Eqs. (15–19) is the following. β, the Fourier number or a nondimensional thermal diffusivity, is the ratio of the rate of heat conduction and the rate of heat storage (thermal energy storage) of the homogenized core. γ is the ratio of the heat capacity of the BFS and the heat capacity of the homogenized core, and κ the ratio between the rate of radiation and the rate of heat conduction. φ is the ratio of the incident heat flux and the rate of heat conduction, or can be seen as a nondimensional heat flux. Finally, Bi, the Biot number, is the ratio of the rate of convection and the rate of heat conduction.

We can note that the three nondimensional parameters κ, φ, and Bi are all proportional to d_C/k, whereas all the other parameters defining κ, φ, and Bi are fixed in our study. Indeed, we are only interested in varying the materials and the geometry, but the initial temperature T_i, the emissivity ε, the incident heat flux profile $Q_i(t)$, and the convection film coefficient profile $h(t)$ are all fixed in the present study (cf. Figure 2 for the profiles of $Q_i(t)$ and $h(t)$ used). This means that for our purpose we can consider only one of these three nondimensional parameters: κ, for example.

Summing up, simplifying assumptions together with dimensional analysis allowed us to determine that we can construct a response surface approximation of the maximum BFS temperature function of the three parameters, β, γ, and κ. An initial third-degree polynomial response surface in these three parameters was constructed from 40 finite element simulations using Latin hypercube sampling (LHS) and used for a global sensitivity analysis, according to Sobol's approach [11]. We found that variable β accounts for 35.7% of the model variance and variable γ accounts for 64.1% of the model variance, whereas variable κ accounts for only 0.06% of the model variance (corresponding to 9.3×10^{-2} K). Considering the 12.5 K accuracy requirement we set ourselves for the material selection, this

Table 2 Dimensional groups for the thermal problem

Variable	T	T_i	x	d_C	t	t_{end}
Unit	K	K	m	m	s	s
Variable	k_C	$\rho_C C_C$	$\rho_B C_B d_B$	Q_i	ε	h
Unit	$\frac{\text{W}}{\text{m}\cdot\text{K}}$	$\frac{\text{W}\cdot\text{s}}{\text{m}^3\cdot\text{K}}$	$\frac{\text{W}\cdot\text{s}}{\text{m}^2\cdot\text{K}}$	$\frac{\text{W}}{\text{m}^2}$	$\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}$	$\frac{\text{W}}{\text{m}^2\cdot\text{K}}$

Table 3 Nondimensional parameters of the problem

$\frac{T}{T_i} = \Gamma$	(12)	$\frac{x}{d_C} = \xi$	(13)
$\frac{t}{t_{\text{end}}} = \tau$	(14)	$\frac{k_C t_{\text{end}}}{d_C^2 \rho_C C_C} = \beta$	(15)
$\frac{d_B \rho_B C_B}{d_C \rho_C C_C} = \gamma$	(16)	$\frac{d_C \varepsilon T_i^3}{k_C} = \kappa$	(17)
$\frac{d_C Q_i(t)}{k_C T_i} = \varphi(\tau)$	(18)	$\frac{h(t) d_C}{k_C} = Bi(\tau)$	(19)

means that we can effectively model the maximum BFS temperature with only the two variables β and γ. Note that the GSA was carried out using an approximation of the response. This is reasonable because in the next section we check and validate the validity of the approach in more detail.

From a physical point of view, the fact that κ has a negligible role can be explained as follows. κ is proportional to d_C/k_C, which is also present in β. That means that, if we want to change κ while keeping β constant, we need to also modify t_{end} (which is the only other variable in β that does not appear in either γ or κ). If we increase κ by decreasing k_C, we need to also increase t_{end} by a certain amount to keep β constant. Decreasing k_C has the effect of lowering the BFS temperature, whereas increasing t_{end} has the effect of making it higher. From the global sensitivity analysis, it turns out that these two effects cancel out, which explains why κ has very little impact. Note that both d_C and k_C are relevant to the problem and that neither of them could be neglected. Parameter κ, which is proportional to d_C/k_C, turns out, however, to have a negligible impact based on the GSA results. This is because both d_C and k_C appear in the remaining two nondimensional parameters, β and γ.

A summary of the procedure used to reduce the number of variables needed for constructing the RSA from 15 to 2 is given in Fig. 5.

The reduction obtained is higher than what could have been obtained by applying any single technique. The use of only simplifying assumptions based on physical reasoning allows a reduction from 15 to 10 variables. GSA alone on the initial problem, which can be seen as an initial screening of the variables, would have allowed a reduction from 15 to 9 variables. Such a GSA showed that parameters d_T, k_T, ρ_T, C_T, k_B, and θ have negligible effects. Applying the GSA to the original problem is, indeed, almost equivalent to the simplifying assumptions we arrived at based on the physical understanding of the problem, which is not surprising. Depending on the complexity of the temperature dependency of the material properties, dimensional analysis alone could have achieved a maximum reduction from 15 to 8 (this reduction is for the case in which no temperature dependency is considered). Thus, we can see that none of the three techniques alone (simplifying assumptions, nondimensionalization, GSA) could have achieved the reduction from 15 to 2 obtained by combining all three.

VI. Maximum Bottom Face Sheet Temperature Response Surface Approximation

An RSA in the two nondimensional parameters β and γ was then constructed. We chose to construct a third-degree polynomial response surface (PRS) in β and γ. These two variables account for the thermal material properties (C_i, k_i, ρ_i) as well as for the geometric parameters of the ITPS panel (d_i, p, L, θ), as shown in Eqs. (24) and (25). These equations were obtained by substituting the expressions of ρ_C, C_C, and k_C from Eqs. (1–3) back into the nondimensional parameters β and γ of Eqs. (15) and (16).

$$\beta = \frac{[k_W d_W + k_S(p\sin\theta - d_W)]\cdot t_{\text{end}}}{(L - 0.5d_T - 0.5d_B)^2 \cdot [\rho_W C_W d_W + \rho_S C_S(p\sin\theta - d_W)]} \tag{24}$$

$$\gamma = \frac{d_B \rho_B C_B p \sin\theta}{(L - 0.5d_T - 0.5d_B)\cdot[\rho_W C_W d_W + \rho_S C_S(p\sin\theta - d_W)]} \tag{25}$$

Note that a given (β, γ) point can be obtained by a multitude of combinations of the 15 individual variables (the nine materials properties and six geometric parameters discussed in Sec. V.A). If the simplifying assumptions we made are exact, then all the different parameter combinations that lead to an identical (β, γ) couple should have the same maximum BFS temperature. If, on the other hand, the effect of the simplifying assumptions is large, then the difference in the BFS temperature for different combinations of the same (β, γ) couple would be large as well. Indeed, even if two points have the

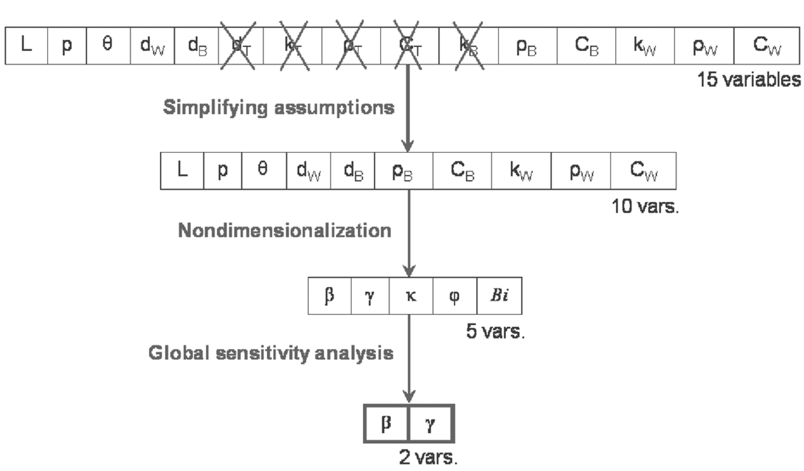

Fig. 5 Summary of the dimension reduction procedure for the ITPS problem.

Table 4 Lower and upper bounds used for sampling in the 15 variables space (all units are SI)

	k_W	ρ_W	C_W	k_B	ρ_B	C_B	k_T	ρ_T	C_T	d_T	d_B	d_W	θ	L	p
LB	2	2500	500	2	1550	900	3	2000	500	0.001	0.0046	0.00224	80	0.102	0.099
UB	7	6000	950	50	3000	1820	50	5000	1700	0.0037	0.007	0.00416	90	0.150	0.150

Table 5 Ranges of the nondimensional design variables

Variable	β	γ
Range	0.1–0.5	0.6–2.4

same (β, γ) values, they might not have the same values of the other parameters that we ignored through simplifying assumptions or global sensitivity analysis.

The design of experiments for constructing the RSA involved sampling 855 LHS points in the initial 15-dimensional space with the bounds provided in Table 4. β and γ were calculated for each of these points, and a subset of 20 out of the 855 points was selected according to the D-optimality criterion in the (β, γ) space within the bounds given in Table 5. FE analyses, described in Sec. IV and involving none of the simplifying assumptions done for nondimensionalization, were run at these 20 points and the third-degree PRS was constructed in (β, γ). This RSA is represented in Fig. 6. One of the advantages of having only two variables in the RSA is an easy graphical representation of the results. This graphical representation possibility will be later used for the material selection.

The RMS error in the approximation was 1.09 K, the cross-validation PRESS error 1.96 K, and the R^2 0.99989 (the range of the RSA is about 250 K). These are satisfactory error measures for our application, but they poorly account for the total error involved in using this RSA. Indeed, part of the total error is due to the fact that the FE results, obtained without the simplifying assumptions, will not be the same for different combinations of the dimensional parameters corresponding to an identical (β, γ) couple. This error is poorly accounted for with only 20 points. Instead, to check this error we randomly sampled 100 out of the 855 LHS points in the 15-dimensional space. We calculated the maximum BFS temperature at these 100 points using FE analyses and compared it with the two-dimensional variable RSA predictions. We obtained the following errors: the RMS error was 2.74 K, the mean of the absolute error was 2.1 K, and the standard deviation of the absolute error was 1.70 K. These errors are well within the 12.5 K accuracy requirement we set ourselves for the material selection.

Note that an alternative would have been to construct the RSA directly using the $20 + 100$ points and look only at the corresponding PRESS error, which leads to similar results.

If we wanted to know how much of the error is due to each of the techniques used (simplifying assumptions, dimensional analysis, GSA), we can note the following. Dimensional analysis alone never involves any error because the nondimensional equations are strictly equivalent to the initial ones. GSA turned out, in our case, to give very good results. Indeed, we found that 99.94% of the variance of the response could be explained by two variables. Because we showed that three out of the five variables are equivalent to each other and account for this very small part of the variance, the error of going from five to two variables is likely to be very small in our case. This means that most of the error in the RSA is explained by the simplifying assumptions.

To gain more insight into where the maximum errors occur and which simplifying assumptions have the most impact, antioptimization [20,21] of the error in the RSA was carried out. The antioptimization process looks to find the places (i.e., materials and geometries) with the highest error in the RSA and, by looking at the designs corresponding to the antioptimum, we can understand what causes these errors. Antioptimization was carried out in [15]. It showed that the RSA has poor accuracy when the geometry is very different from the geometry for which the representative temperature of assumption 3 (cf. Sec. V.A) was established. For these unusual geometries, the representative temperature shifts due to the temperature dependence of the core; this shift is not accounted for by the RSA, which explains the poor accuracy for these geometries. To further improve the accuracy of the RSA for a large range of geometries, we would have to add nondimensional parameters that account for the temperature dependence. However, for the geometry that we will use for the RSA in the next section, the maximum absolute error among eight test points corresponding to actual material combinations was 7.6 K with a mean of 1.87 K (see Fig. 7). The figure shows the absolute error Δ of the response surface estimates compared with the FE predictions for eight different material combinations (format for the material combination names: web material–BFS material). The TFS material is an alumino–silicate–Nextel 720 composite laminate except in the reference all-titanium design. The maximum BFS temperature from the RSA is superposed as a contour plot, with the bottom (330 K) and the top (540 K) contour lines labeled.

For varying geometries as well as varying materials, the antioptimization carried out in [15] showed that the worst-case error is 9.05 K. Note that we compare the RSA predictions with the FE results, which also have limited accuracy. A convergence study on the finite element model showed that the discretization error for the

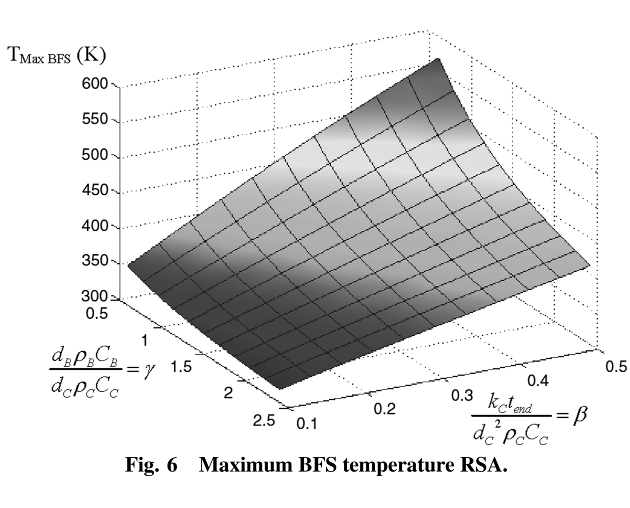

Fig. 6 Maximum BFS temperature RSA.

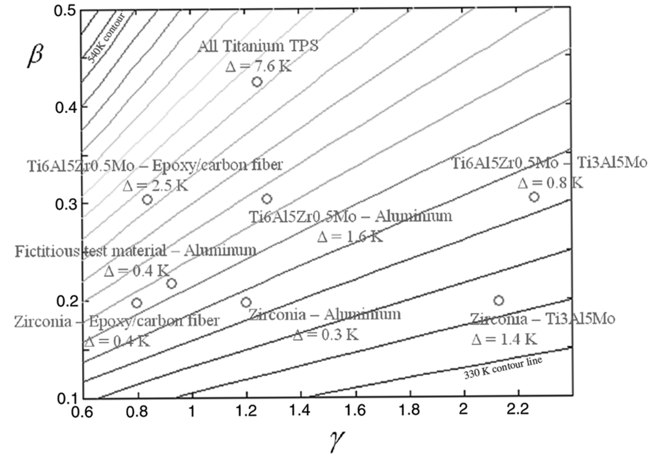

Fig. 7 Absolute error Δ of the response surface estimates compared with the FE predictions.

BFS temperature is less than 0.007 K. Accordingly, considering that the errors in the RSA are of the order of 10 K, we can assume the finite element simulations are exact, meaning that all the error is considered to come from the use of the dimension reduction method.

These errors are within the 12.5 K requirement we set ourselves for the material selection.

VII. Response Surface Approximation Construction Computational Cost

The possibility of graphical representation of the two nondimensional variables RSA is of great benefit in our case. Most of the problems, however, can not be reduced to only two or three nondimensional variables. In these other cases, constructing the RSA in nondimensional variables can still benefit the computational cost.

In our case, we used 40 FE simulations for the global sensitivity analysis in three-dimensional space, 20 simulations for constructing the third-degree PRS in (β, γ) and 108 simulations to verify the accuracy of the RSA. In total, we used 168 simulations.

Constructing the maximum BFS temperature RSA in the 15 initial variables leads to following results. A linear PRS in the 15 variables required 32 FE simulations and led to an RSA with an RMS error of 9.16 K, a PRESS error of 12.9 K, and an R^2 of 0.969 (recall the range of the RSA is about 250 K). A second-degree PRS in the 15 variables required 272 FE simulations and led to an RSA with an RMS error of 1.23 K, a PRESS error of 1.78 K, and an R^2 of 0.99989.

We can note that constructing the third-degree PRS in the two nondimensional variables had an overall computational cost about 40% lower than constructing a second-degree PRS in the initial 15 variables, while the error was maintained at an acceptable level for our application.

Note also that in most problems a second-degree PRS is the minimum usable, linear PRS being very rarely acceptable. Often, third-degree PRS are even required to achieve acceptable error measures. For third-degree PRS, the computational cost difference between using all the variables or using the reduced number of nondimensional variables can become very significant. For the thermal problem for example, a third-degree PRS in the 15 dimensions would have required 1632 experiments.

VIII. Applying the Response Surface Approximation for Comparison of Materials for the Integrated Thermal Protection System

The graphical representation possibility of the two-dimensional RSA was used next for comparison of alternate materials for the ITPS sections. For this part, the dimensions of the ITPS are once again fixed to the values in Table 1.

We used the CES 2005 material database software. Constraints on properties such as maximum service temperature, fracture toughness, and Young's modulus were imposed during the search in the database to avoid unreasonable materials.

To compare the web materials, the BFS material was fixed to an aluminum alloy 2024 and the potential web materials were plotted in the (β, γ) plane with the RSA of the maximum BFS temperature superposed as a contour plot, as shown in Fig. 8. Note that in this figure numerous materials are grouped under generic names (denoted as an asterisk), such as stainless steels, nickel chromium alloys, or cobalt superalloys.

Figure 8 shows that materials such as alumino–silicate/Nextel 720 composites or zirconia ceramics provide a significant reduction in the maximum BFS temperature compared with metals such as Ti alloys or nickel chromium alloys, which were considered in previous designs (cf. [14]). Because we seek materials leading to a low

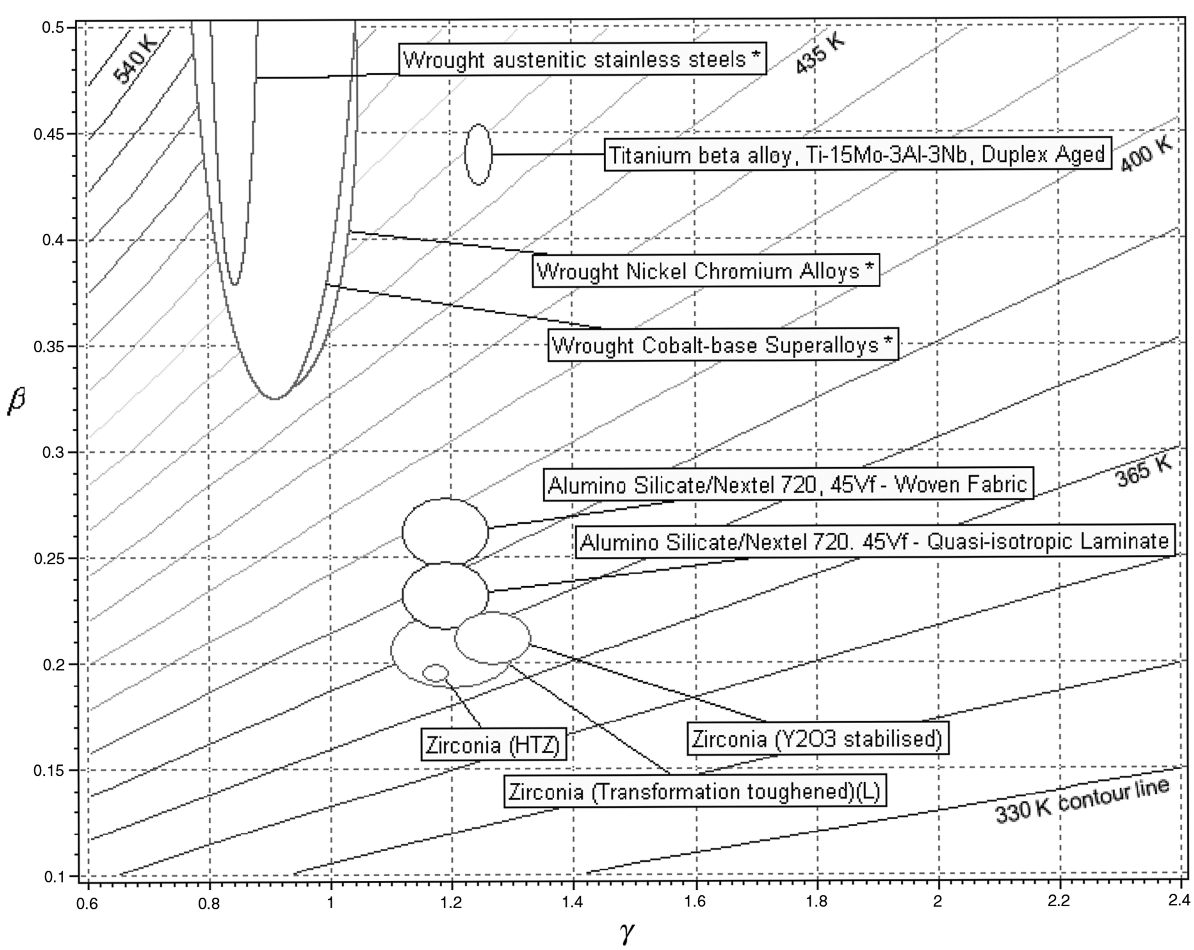

Fig. 8 Thermal comparison of materials suitable for the web using the contour plot of the maximum BFS temperature RSA.

maximum BFS temperature, these materials were selected for further study as good potential candidates for the web of the ITPS panel.

The same material selection procedure was applied for BFS materials. The complete results are given in [15]. In summary, the material selection process based on the RSA constructed here identified a small number of good potential material candidates for the different sections of the ITPS. These materials were used in an optimization routine developed for the ITPS. The optimization procedure used is the one presented in [14], which seeks to minimize the mass of an ITPS panel by finding the optimal geometry parameters for a given material combination. Applying it to the different material combinations found through this material selection process allowed us to obtain both the best-suited material combination and the optimal corresponding geometry for the ITPS panel.

IX. Conclusions

The present paper illustrates the use of combined physical reasoning, dimensional analysis, and global sensitivity analysis to significantly reduce the number of variables in RSA.

Nondimensionalizing the exact equations involved in the finite element analysis, while reducing the number of variables, can be relatively cumbersome and lead to a still relatively high number of nondimensional parameters. Some of these parameters might only have marginal influence on the quantity of interest. In this case, the process can be aided by simplifying assumptions and a global sensitivity analysis that can help further reduce the number of nondimensional parameters by keeping only those that control most of the variation of the quantity of interest. It is important to note that removing variables that have a small impact on the problem can have relatively small detrimental effects on the accuracy of the RSA, because the RSA is fitted to the finite element simulations obtained without simplifying assumptions, thus allowing for error compensation.

The presented approach is general and can be applied to any finite-element-based response surface construction. It was used here with success on a transient thermal heat transfer problem for an ITPS. Dimensional analysis in combination with several simplifying assumptions and a global sensitivity analysis allowed the reduction of the number of parameters of the response surface approximation of the maximum temperature from 15 to only 2 while maintaining reasonable accuracy of the RSA.

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J. Samareh
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