



Effect of Z-yarns on the stiffness and strength of three-dimensional woven composites

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ABSTRACT

Micro-mechanics tools investigating the in-plane elastic and strength properties of 3D woven composites are developed in this work. Particular attention is directed toward constructing detailed geometry and finite element models of an example 3D weave architecture available in the literature. The models are then modified to investigate the effect of including the Z-yarns on the changes in stiffness and strength properties. In particular, the effect of incorporating the 'Z-Crowns' is quantified with the aid of three-dimensional finite element simulations.

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1. Introduction

Traditional laminated, woven and braided composite materials are inherently susceptible to inter-layer delamination due to their micro-structural geometry and intrinsically weak inter-laminar bonds. Specifically, in hard armor applications, candidate composites should possess high areal density, superior delamination resistance and high fracture toughness. Three-dimensional (3D) woven composites are a class of materials that encompass all of the above mentioned properties. The micro-structural geometry of 3D woven composites is characterized by several layers of [0/90] laminates 'bound' together with the aid of warp weavers or Z-yarns.

In practice however, the [0/90] laminates are integrally woven with Z-yarns, resulting in the model architecture shown in Fig. 1. This process imparts favorable mechanistic characteristics such as high in-plane and through-thickness stiffness and strength, superior delamination resistance and high fracture toughness. Consequently, it becomes imperative to quantify the effect of integral Z-weaving on the stiffness, strength and fracture properties of 3D woven composites.

Several researchers [1–11] have investigated various aspects of the response of 3D woven composites in different environments. Elastic properties, strength, ductility and fatigue life of 3D woven composites [1] were experimentally determined to be highly sen-

sitive to tow waviness. It should be noted here that 3D woven fabrics are not characterized by tow undulations similar to those witnessed in plain and satin weave fabrics. Tow waviness in [1] refers to sporadic undulations of nominally straight tows. As such, tow undulations have not been neglected in Fig. 1, but rather, the presented architecture approximates an actual 3D woven system [2]. The complete stress–strain response of 3D woven composites under quasi-static loads was experimentally characterized in [2], wherein tow waviness was not evident in the reported micrographs.

Fracture of 3D woven Carbon/Epicote 828 epoxy resin composites in [3] was linked to tow/matrix debonding, tow breakage, and fiber pull-out. The studies in [3] also confirmed the observations regarding mechanical properties of 3D woven composites reported in [1]. In a more general sense, mode-I inter-laminar fracture toughness and delamination resistance of 3D woven composites [4] was shown to significantly improve with relatively small volume fractions of Z-yarns.

From a manufacturing standpoint, the experimental characterization in [5,6] brought to light severe knockdowns in the strength of dry load bearing and uni-directional composite yarns. Abrasion damage caused by the fibers sliding against each other and the loom machinery was cited as the major contributor toward the degradation in strength properties.

Performance of 3D woven composites under impact loads was the main focus of studies in [7–9]. In general, 3D woven composites appear to have better ballistic efficiency, and controlled delamination as opposed to 2D woven composites [7] and as such incorporate better energy transfer characteristics [9].

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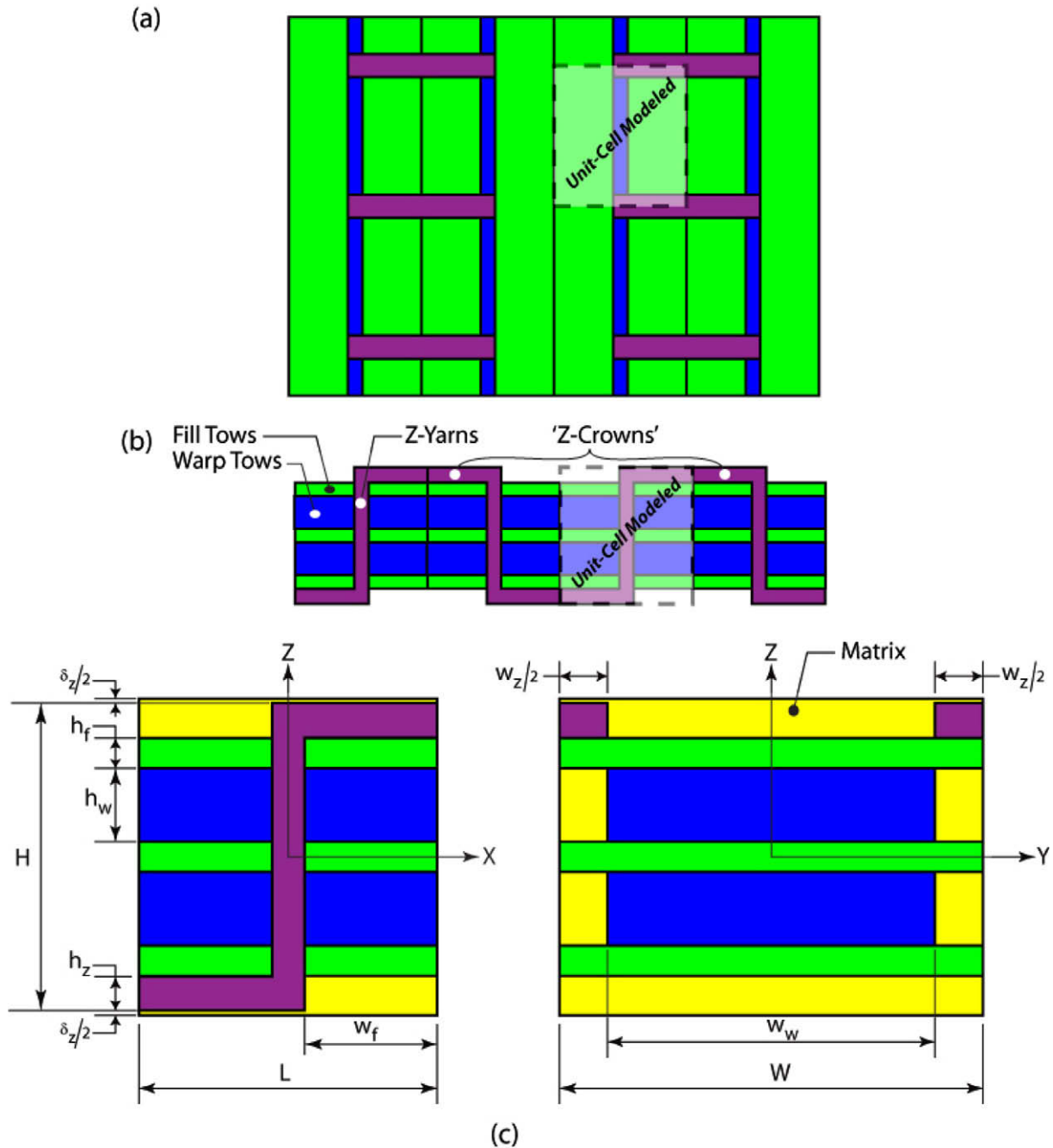


Fig. 1. Schematic representation of the weave architecture in three-dimensional (3D) woven composites. The above schematics depict geometric modifications of the weave patterns presented in [1]. (a) Plan view. (b) Front cross-sectional view exhibiting the path of the Z-yarns. (c) Details of the unit-cell modeled in this study. The two views aid in completely identifying all the geometry parameters used to develop 3D finite element models of the unit-cell.

Analytical work [2,10,11] has mainly focused on predicting the stiffness and strength of 3D woven composites. Homogenized “3D Mosaic” [2] and iso-stress/iso-strain based micro-mechanics models [10,11] tend to overestimate the properties since they are unable to account for localized events such as yarn micro-bending. Computational modeling on the other hand [8,9,11] can incorporate the three-dimensional stress state, without making any simplifying assumptions. However, in [8,9,11], the authors neglected Z-Crowns [2] (see also Fig. 1), perhaps leading to a stiffer response.

The spectrum of works cited above, while not exhaustive, suggest that tow waviness [1,3,11], abrasion damage due to weaving

[5,6], and introduction of Z-yarns [4,7] are primary factors that significantly influence the mechanics of 3D composites. Potential applications of 3D woven composites in critical environments such as vehicle armor motivate their complete in-plane and through-thickness characterization under static, dynamic, and impact loads.

However, the present study focuses on investigating the effect of Z-yarns on the in-plane elastic properties and strength of a model 3D woven composite system shown in Fig. 1 with the aid of finite element based generalized micro-mechanics tools. The generality of the tools developed herein would lend itself for application to any particular woven system. Future studies as part of

this broad research program would report on other aspects of the response of 3D woven composites mentioned above.

2. Geometric and finite element modeling of 3D woven composites

2.1. Modeling the geometry

The accurate prediction of the response of these materials under different loading environments mandates a detailed representation of their 3D weave architecture. Extension of modeling strategies developed in [12,13] to study 3D composites is not a trivial matter, simply because plain and satin weave textile composites are relatively two-dimensional (2D) when compared with material systems such as those depicted in Fig. 1. As such in this study, the commercially available ABAQUS® finite element software is employed for developing solid models and corresponding finite element meshes of the unit-cell of the material system shown in Fig. 1.

Figs. 1(a) and (b) serve to present schematic representations of the weave architecture and isolate a repeating unit-cell that captures the periodicity of this textile composite. As shown in Fig. 1(c), a coordinate system XYZ is established with its origin located at the geometric center of the repeating unit-cell, such that it constitutes the global coordinate system. Consistent with the definition of XYZ, the warp and fill tows constitute the planar 0° (longitudinal) and 90° (transverse) tows, respectively. The out-of-plane undulating fiber bundles are called warp-weavers or Z-yarns.

Micrographs of processed 3D composites indicate a thin layer of matrix material above and below the Z-Crowns [3]. As such in the model simulations, a thin sliver of additional matrix material is introduced as shown in Fig. 1(c) with $\delta_z = 0.1 \cdot h_z$. The undulating path of the Z-yarns has been assumed to be regular and the tow cross-sections have been assumed to be uniform rectangles in the present study. Particular geometry parameters employed to model the material system in Fig. 1 are reported in Table 1. Consequently, the total height of the unit-cell is given by:

$$H_{UC} = H + \delta_z \quad (1)$$

The geometric modeling strategy developed herein comprises of defining suitable volumetric partitions to represent the warp and fill tows, Z-yarns, and matrix material, within a cuboid of dimensions $L \times W \times H_{UC}$. The transversely isotropic tows and yarns, defined via partitioning, are then associated with appropriate local material directions as illustrated in Figs. 2(a) and (b). The remaining volume of the cuboid is designated as the isotropic inter-tow epoxy resin matrix depicted in Figs. 2(a) and (b).

In this study, two different configurations of the unit-cell in Fig. 1(c) are modeled. The first configuration, which includes the Z-Crowns is designated 'Z-Crowned Composite' and is shown in Fig. 2(a). In the second configuration, the Z-Crowns are neglected consistent with [2,9]; this arrangement is termed 'Uncrowned Composite' as shown in Fig. 2(b). Due to the particular geometry, the overall height of the unit-cell in this case is given by:

$$H_{UCC} = H - 2h_z \quad (2)$$

All other dimensions are the same as those reported in Table 1.

Table 1

Dimensions of the tows and yarns employed to develop 3D models of the unit-cell of the weave architecture shown in Fig. 1. The subscripts 'f', 'w', and 'z' indicate fill tows, warp tows and Z-yarns, respectively. The above dimensions were computed based on the work in [2]. All dimensions are in mm.

h_f	h_w	h_z	w_f	w_w	w_z	L	W	H
Geometry parameters								
0.292	0.637	0.292	1.514	4.120	0.910	3.320	5.028	2.734

The material system in Fig. 1 comprises of three layers of fill tows and two layers of warp tows integrally woven with Z-yarns. The volume fraction of the Z-yarns varies between 1% and 3%, and as such, the material is similar to a [0/90] stacked laminate, except for the presence of the Z-yarns and the matrix material. As such, the mechanistic limiting case of this configuration is an 'Equivalent Laminate' as shown in Fig. 2(c). The overall height of the Equivalent Laminate is $H_{LAM} = H_{UCC}$. In Eq. (2) and the above discussion, the subscripts 'UCC' and 'LAM' indicate Uncrowned Composite and Equivalent Laminate, respectively.

2.2. Finite element discretization

The solid models of the unit-cells shown in Figs. 2(a)–(c), were meshed with 3D eight-node linear continuum brick elements within ABAQUS. A structured meshing algorithm was utilized such that the individual finite elements were characterized by regular hexahedral geometry. Consequently distorted elements were altogether eschewed as part of the meshing process. The partition-feature based modeling methodology described earlier, results in merged interfaces at material boundaries. Therefore, while meshing, duplicate or coincident nodes are not generated at these interfaces. As such, additional constraints for coincident nodes and node merging operations at material boundaries are completely avoided.

2.3. Boundary conditions

Exploiting the periodicity of the woven architecture in 3D composites, appropriate periodic boundary conditions were specified on the four vertical faces of the unit-cells corresponding to $X = \pm a/2$ and $Y = \pm b/2$. The unit-cells faces located at $Z = \pm c/2$ were set free. The generalized dimensions 'a', 'b', and 'c', are reported in Fig. 2(d). In particular, the following relations are implied for 'a', 'b', and 'c':

$$\begin{aligned} a &= L \\ b &= W \\ c &= \begin{cases} H_{UC} + \delta_z \rightarrow \text{Z-Crowned Composite} \\ H_{UCC} \rightarrow \text{Uncrowned Composite} \\ H_{LAM} \rightarrow \text{Equivalent Laminate} \end{cases} \quad (3) \end{aligned}$$

Periodic boundary conditions simulating remote in-plane tension and shear are reported in Table 2. Detailed discussions regarding the derivation of these boundary conditions are presented elsewhere [14].

2.4. Constituent material properties

The transversely isotropic fill tows, warp tows and Z-yarns were intrinsically S-2 Glass/Dow Derakane 8084 Vinyl-Ester uni-directional composites, with material properties listed in Table 3. The isotropic epoxy resin matrix was characterized by elastic modulus, $E = 3.53$ GPa, Poisson's ratio, $\nu = 0.35$, and yield strength, $S_y = 125$ MPa.

3. Mechanics of 3D woven composites

The Direct Micro-Mechanics (DMM) techniques discussed in [14] are employed herein to develop generalized finite element method based approaches to predicting the in-plane effective elastic properties and strengths of 3D woven composites. Computation of the extensional stiffness [A], bending-extension coupling stiffness [B] and the bending stiffness [D] matrices is central to the

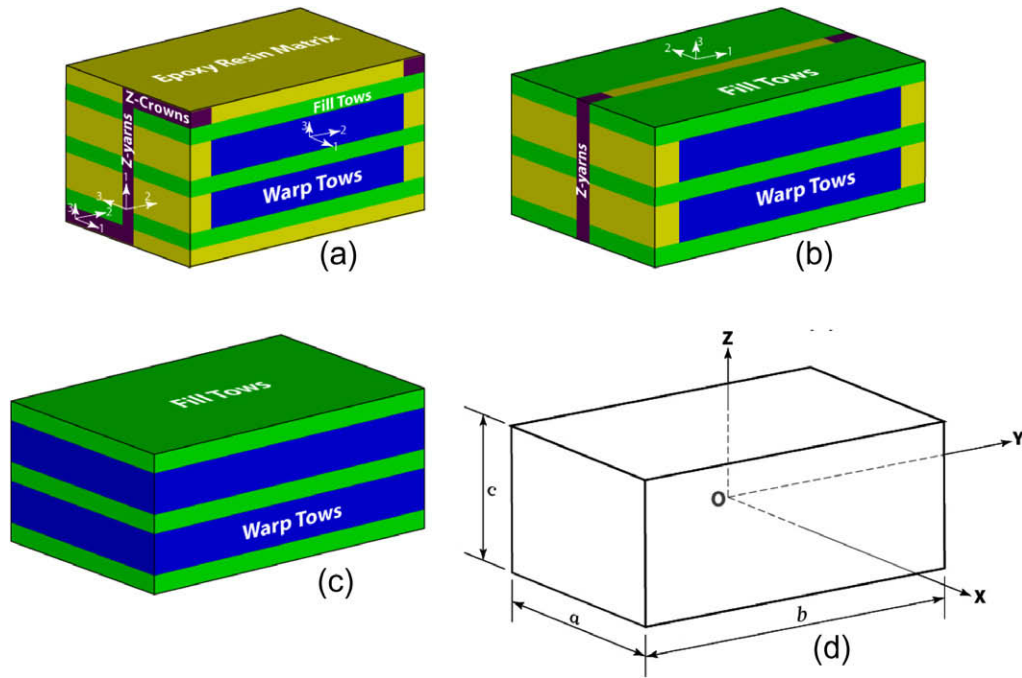


Fig. 2. 3D solid models and dimensions of the unit-cell configurations modeled in this study. (a) Z-Crowned Composite exhibiting the arrangement of warp tows, fill tows and the out-of-plane Z-yarns, along with the corresponding Principal Material Directions (PMDs). (b) Uncrowned Composite exhibiting PMDs of fill tows. (c) Equivalent Laminate. (d) Generalized unit-cell displaying the overall dimensions and the global XYZ coordinate system.

above methods. A general algorithm would be presented later toward the end of this section.

Unlike the meso-volume based 3D Mosaic [2] and analytical [10,11] models, the DMM technique allows one to capture the effects of geometric details leading to critical mechanistic events, such as yarn micro-bending. As such, this method better approximates the response of 3D woven composites. Especially while addressing non-linear rate-sensitive, impact and ballistic loading response of these advanced materials, 3D finite element models incorporating micro-structural geometric details have to be developed.

3.1. In-plane effective elastic properties of 3D woven composites

Following the discussions in [14] it could be shown that, in the absence of bending-extension coupling and applied moments, the in-plane resultant forces are given by:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = [A] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (4)$$

Furthermore, the equivalent stiffness matrix [Q] of the woven/stitched composite laminate referred to the global coordinate system could be computed as,

$$[Q] = \frac{[A]}{h} \quad (5)$$

wherein 'h' is the overall height of the laminate. Once the [Q] matrix has been determined, the effective elastic properties of the composite could be computed as:

$$\begin{Bmatrix} E_{xx} \\ E_{yy} \\ G_{xy} \\ \nu_{xy} \\ \nu_{yx} \end{Bmatrix} = \Delta \begin{Bmatrix} Q(1,1) \\ Q(2,2) \\ Q(3,3)/\Delta \\ Q(1,2)/\Delta Q(2,2) \\ Q(1,2)/\Delta^2 Q(1,1) \end{Bmatrix} \quad (6)$$

In Eq. (6):

$$\Delta = 1 - \frac{Q(1,2)^2}{Q(2,2)Q(1,1)} \quad (7)$$

3.2. Predicting the in-plane strength of 3D woven composites

The broader problem in predicting the strength of 3D woven composites focuses on computing a load factor for the composite under a general state of in-plane loading represented by $\{\sigma\} = \{\sigma_x \ \sigma_y \ \sigma_{xy}\}^T$. However, in this case, the constitutive [A] matrix is related to the resultant forces as described in Eq. (4). Therefore the above problem could be restated as involving the computation of a load factor for the composite under a given state of in-plane loading described by $[N] = [N_x \ N_y \ N_{xy}]^T$.

The approach is based on computing load factors under five unique loading conditions given by:

Table 2

Periodic boundary conditions applied on the unit-cells shown in Fig. 2 for simulating in-plane unit-strain loading cases [14].

Load case	$u(a/2, y, z)$ $-u(-a/2, y, z)$	$v(a/2, y, z)$ $-v(-a/2, y, z)$	$w(a/2, y, z)$ $-w(-a/2, y, z)$	$u(x, b/2, z)$ $-u(x, -b/2, z)$	$v(x, b/2, z)$ $-v(x, -b/2, z)$	$w(x, b/2, z)$ $-w(x, -b/2, z)$
$\epsilon_x^M = 1$	a	0	0	0	0	0
$\epsilon_y^M = 1$	0	0	0	0	b	0
$\gamma_{xy}^M = 1$	0	a/2	0	b/2	0	0

Table 3

Material properties of the transversely isotropic warp tows, fill tows and the Z-yarns [9].

E_{11}	E_{22}	E_{33}	ν_{12}	ν_{23}	ν_{13}	G_{12}	G_{23}	G_{13}
All elastic and shear moduli are reported in GPa								
53.424	10.684	10.684	0.053	0.449	0.265	4.183	3.688	4.183
S_L^+		S_T^-		S_T^+		S_T^-		S_{LT}
All strengths are reported in MPa								
1380		770		47		137		50

$$\begin{Bmatrix} N_{L(+)} \\ N_{L(-)} \\ N_{T(+)} \\ N_{T(-)} \\ N_{LT} \end{Bmatrix} = \begin{Bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{Bmatrix} \quad (8)$$

From Eq. (8), $N_{L(+)} = \{1 \ 0 \ 0\}^T$ would represent a unit tensile load in the longitudinal (L) or 0°-direction of the composite. Similarly, $N_{L(-)} = \{-1 \ 0 \ 0\}^T$ would indicate a unit compressive load in the L-direction of the composite. $N_{T(+)} = \{0 \ 1 \ 0\}^T$ and $N_{T(-)} = \{0 \ -1 \ 0\}^T$ have corresponding connotations albeit in the transverse (T) or 90°-direction of the composite. $N_{LT} = \{0 \ 0 \ 1\}^T$ indicates a unit shear load parallel to the L–T plane of the composite.

3.2.1. Computing total stress

For any given in-plane loading state [N] determined from Eq. (8), the corresponding macro-strains $\{\varepsilon\}$ are computed with the aid of Eq. (4). Employing the principle of linear superposition, and the macro-strains $\{\varepsilon\}$, the total stress vector in each element of the finite element mesh is computed as:

$$\{\sigma^{xyz}\}_{Total}^e = \varepsilon_x \{\sigma\}_{XX}^e + \varepsilon_y \{\sigma\}_{YY}^e + \gamma_{xy} \{\sigma\}_{XY}^e \quad (9)$$

The 6×1 vectors $\{\sigma\}_{XX}^e$, $\{\sigma\}_{YY}^e$ and $\{\sigma\}_{XY}^e$ in Eq. (9) correspond to the micro-stresses in each element as a result of the three individual macro-level unit-strain loading cases listed in Table 2. The subscripts 'XX', 'YY' and 'XY' refer to unit-strain in the X-direction, Y-direction and XY-shear, respectively.

For checking failure, the material code of the particular element 'e' is determined to indicate whether it resides within the fill tows, warp tows, Z-yarns or the inter-tow epoxy resin matrix. If the element 'e' resides within a tow, then consistent with [14], the total stress is transformed to the corresponding local 1–2–3 coordinate system:

$$\{\sigma^{123}\}_{Total}^e = [T] \{\sigma^{xyz}\}_{Total}^e [T]^T \quad (10)$$

In Eq. (10), [T] is the transformation matrix of direction cosines resulting from appropriate coordinate rotations, and the superscript 'T' indicates transpose of [T]. The total stress $\{\sigma^{xyz}\}_{Total}^e$ is not subject to any transformations if 'e' resides in the inter-tow epoxy resin matrix.

3.2.2. Load factor based on the Tsai–Hill failure criterion

Under plane stress conditions parallel to the 1–2 plane, with the fiber direction being aligned along the 1-direction, the Tsai–Hill failure criterion is given by (Jones [15]):

$$\frac{\sigma_{11}^2}{X^2} - \frac{\sigma_{11}\sigma_{22}}{X^2} + \frac{\sigma_{22}^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1 \quad (11)$$

In Eq. (11), $\{\sigma\} = \{\sigma_{11} \ \sigma_{22} \ \tau_{12}\}^T$ represent a particular planar loading state, and X, Y, and S are the in-plane longitudinal, transverse and shear strengths of the composite, respectively. Appropriate magnitudes of $X = X_t$ and $X = X_c$ and $Y = Y_t$ or $Y = Y_c$ should be used depending on the signs of σ_{11} and σ_{22} . In the above, subscripts 't'

and 'c' imply tension and compression, respectively. Therefore, if σ_{11} is positive, $X = X_t$ and if it is negative, $X = X_c$. Similarly, positive σ_{22} requires $Y = Y_t$ and negative σ_{22} implies $Y = Y_c$.

Let λ be the load factor for a given material under unit tensile load. Further let the planar stresses corresponding to the unit tensile load case be s_{11} , s_{22} and s_{12} . The condition $\lambda > 1$ implies that the material can withstand stresses of magnitude λs_{11} , λs_{22} and λs_{12} . If the Tsai–Hill failure criterion is used to check for failure, $\sigma_{11} = \lambda s_{11}$, $\sigma_{22} = \lambda s_{22}$ and $\tau_{12} = \lambda s_{12}$ have to be substituted in Eq. (11). As a result, the load factor is given by:

$$\lambda = \frac{1}{\sqrt{\frac{s_{11}^2}{X^2} - \frac{s_{11}s_{22}}{X^2} + \frac{s_{22}^2}{Y^2} + \frac{s_{12}^2}{S^2}}} \quad (12)$$

In this study, s_{11} , s_{22} and s_{12} appearing in Eq. (12), are determined with the aid of Eq. (10) as:

$$\begin{Bmatrix} s_{11} \\ s_{22} \\ s_{12} \end{Bmatrix} = \begin{Bmatrix} \sigma_{11}^{123} \\ \sigma_{22}^{123} \\ \sigma_{12}^{123} \end{Bmatrix}_{Total}^e \quad (13)$$

3.2.3. Load factor based on the maximum principal stress failure criterion

According to the maximum principal stress failure criterion, failure occurs if the maximum principal stress at a point in the material exceeds the yield strength of the material.

Let us assume s_1 , s_2 and s_3 are the principal stresses corresponding to the three-dimensional stress state $\{\sigma\} = \{\sigma_{xx} \ \sigma_{yy} \ \sigma_{zz} \ \tau_{xy} \ \tau_{yz} \ \tau_{zx}\}^T$ resulting from a unit tensile load. Furthermore, let S_y represent the yield strength of the material. If λ is the load factor for this load case, then the condition $\lambda > 1$ implies that the material can actually withstand stresses of the magnitude $\lambda\{\sigma\}$. As a result, the principal stresses would be λs_1 , λs_2 , λs_3 . Invoking the maximum principal stress failure criterion $\max(\lambda s_1, \lambda s_2, \lambda s_3) \geq S_y$, the load factor could be computed as:

$$\lambda = \frac{S_y}{\max(|s_1|, |s_2|, |s_3|)} \quad (14)$$

3.2.4. Strength computations based on the 'One Element' condition

Load factors for tow elements are computed from Eq. (12), whereas Eq. (14) is used to determine load factors for matrix elements. The minimum of all the load factors so determined corresponds to the strength of the composite, for each loading case. For instance,

$$S_{L(+)}^{FEA} = \frac{\min(\lambda^e)}{h} \quad (15)$$

indicates the longitudinal tensile strength of the composite corresponding to the load case $N_{L(+)} = \{100\}^T$. The subscript 'L(+)' represents tension in the longitudinal direction. The superscript 'FEA' implies that the strength is computed based on stress estimates obtained from the finite element boundary value problems listed in Table 2. Also in Eq. (15), λ^e is the element load factor and 'h' is the overall height of the composite, given by 'c' in Eq. (3).

Similarly, $S_{L(-)}^{FEA}$, $S_{T(+)}^{FEA}$, $S_{T(-)}^{FEA}$, and S_{LT}^{FEA} could be determined for the remaining four loading cases in Eq. (8). In the above, subscript '(–)' indicates compression, and superscript 'T' refers to the transverse direction.

The strength magnitudes determined above are the most conservative estimates since one element in the entire mesh dictates the response of the composite. This approach to estimating strength is designated the 'One Element' condition. Later in this study, discussions focusing on relaxing the element failure threshold for computing the planar strengths would be presented.

3.3. Computing the extensional stiffness and bending-extension coupling matrices

The crux of implementing the procedures discussed above is the computation of the extensional stiffness matrix [A]. Following the discussions in [14,16], the constitutive matrices [A], [B], and [D] for a given composite material could be computed with the aid of six fundamental macro-level unit-strain and unit-curvature finite element boundary value problems. In particular, each macro-level unit-strain and unit-curvature boundary value problem results in the determination of one column of the 6 × 6 stiffness matrix formulated from the [A], [B] and [D] matrices.

This study however, focuses on the in-plane extensional and shear response of 3D woven composites. As such, only the three periodic finite element boundary value problems listed in Table 2 would be solved. In each of these boundary value problems, all other macro-strains are set to zero. Pursuant to the discussions in [14,16], the solution of the above three boundary value problems leads to the computation of the first three columns of the 6 × 6 stiffness matrix formulated from the [A], [B] and [D] matrices. In other words, the extensional stiffness matrix [A], and the bending-extension coupling matrix [B], are fully determined.

4. Results

4.1. Deformed meshes

Fig. 3 presents the deformed mesh configurations of the Z-Crowned Composite and Uncrowned Composite unit-cell configurations. Under the application of the macro-level unit-strain $\epsilon_x = 1$, the Z-Crowned Composite exhibits significant micro-bending due to the undulating path of the Z-yarns, as shown in Fig. 3(a). In fact, through the vertical portion of the Z-yarns, the load path is aligned along the plane comprising the weakest 2–3 local material directions. Consequently, visible bulging of the Z-yarns near the top ‘Z-Crown’ is seen in Fig. 3(a). The same effect is observed near the bottom ‘Z-Crown’ as well.

However, in the case of the Uncrowned Composite shown in Fig. 3(b), the localized micro-bending is not apparent, since the Z-Crowns have been neglected. Nonetheless, perceptible bulging of the included portion of the ‘Z-yarns’ is clearly discernible on account of the alignment of the load path discussed above.

Solutions corresponding to macro-level unit-strain loading cases $\epsilon_y = 1$, and $\gamma_{xy} = 1$, do not exhibit such severe micro-bending, as reported in Fig. 3. Overall, the solutions presented in Fig. 3, appear to be mechanistically reliable.

4.2. Constituent matrices and effective elastic properties

The extensional stiffness matrix [A] and the bending-extension coupling stiffness matrix [B] were computed based on the approach discussed in the previous section. Table 4 reports on the results obtained via the finite element method and an analytical solution for a Five-Layer [0/90] Laminate resembling the Equivalent Laminate shown in Fig. 2(c). The analytical solution is based on Classical Lamination Theory as described in [15]. Comparing the solutions for the ‘Five-Layer [0/90] Laminate’ and Equivalent Laminate reinforces confidence in the approach developed in this study. All the solutions reported in Table 4 indicate that the off-diagonal terms A_{16} and A_{26} are zero implying that these materials do not exhibit any shear-extension coupling. Furthermore, owing to the geometric symmetry of these materials with respect to the mid-plane, the bending-extension coupling matrix [B] turns out to be [0].

Results for the Z-Crowned Composite and the Uncrowned Composite in Table 4 bring to light the detrimental effect of Z-yarns on the stiffness properties of 3D composites. Comparing the properties of the Equivalent Laminate in Table 4 and results from [2,9] reproduced in Table 5, it is yet again confirmed that introduction of Z-yarns results in stiffness knockdown. As further seen from Table 5, it is evident that the results of the present study agree well with published data.

4.3. Load factors and in-plane strengths

In-plane strengths for the Z-Crowned Composite, Uncrowned Composite and the Equivalent Laminate, corresponding to the five loading cases in Eq. (8) were computed as discussed in a previous section, and are reported in Fig. 4. Additionally in Fig. 4, the in-plane strength of the ‘Five-Layer [0/90] Laminate’ computed based on Classical Lamination Theory [15] under different loading cases is also reported. For each loading case, the first, second, third and fourth bars represent the corresponding strengths of the ‘Five-Layer [0/90] Laminate’, Equivalent Laminate, Uncrowned Compos-

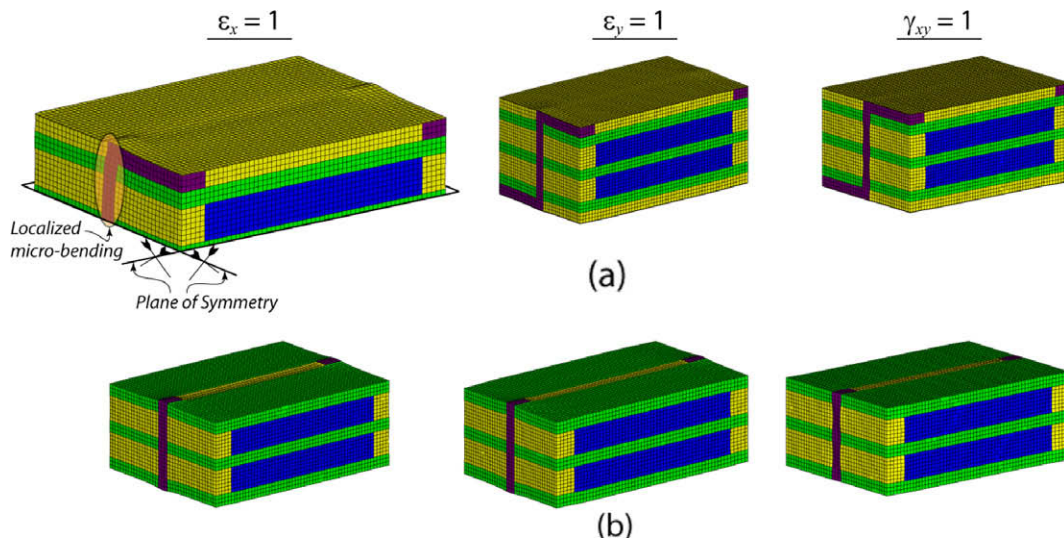


Fig. 3. Deformed configurations of the unit-cells of the materials modeled in this study. The first, second and third column of meshes correspond to deformed configurations under macro-level unit-strains $\epsilon_x = 1$, $\epsilon_y = 1$, $\gamma_{xy} = 1$, respectively. (a) Z-Crowned Composite. For $\epsilon_x = 1$, merely the top half of the unit-cell is shown in order to highlight the severe micro-bending of the Z-yarns. (b) Uncrowned Composite.

Table 4
The extensional stiffness matrices and effective elastic properties of 3D woven composites. Owing to the symmetry of the weave architecture about the mid-plane $[B] \approx [0]$. Clearly the addition of Z-yarns results in significant knockdowns in the properties.

Weave architecture	[A] (MPa m)	E_{xx} (GPa)	E_{yy} (GPa)	ν_{xy}	ν_{yx}	G_{xy} (GPa)
<i>Finite element solutions</i>						
Z-Crowned Composite	$\begin{bmatrix} 69.61 & 2.57 & 0.00 \\ 2.57 & 57.61 & 0.00 \\ 0.00 & 0.00 & 8.61 \end{bmatrix}$	25.15	20.81	0.0447	0.0370	3.11
Uncrowned Composite	$\begin{bmatrix} 65.66 & 1.75 & 0.00 \\ 1.75 & 54.91 & 0.00 \\ 0.00 & 0.00 & 7.60 \end{bmatrix}$	30.51	25.52	0.0318	0.0266	3.35
Equivalent Laminate	$\begin{bmatrix} 77.60 & 1.22 & 0.00 \\ 1.22 & 60.50 & 0.00 \\ 0.00 & 0.00 & 8.99 \end{bmatrix}$	36.10	28.13	0.0202	0.0157	4.183
<i>Analytical solution</i>						
Five-Layer [0/90] Laminate	$\begin{bmatrix} 77.47 & 1.21 & 0.00 \\ 1.21 & 60.45 & 0.00 \\ 0.00 & 0.00 & 8.99 \end{bmatrix}$	36.02	28.10	0.0202	0.0157	4.183

ite, and the Z-Crowned Composite, respectively. Excellent agreement between the strengths computed for the 'Five-Layer [0/90] Laminate' and the Equivalent Laminate, inspire confidence in the methods developed in this study. Consistent with the stiffness properties of these material systems, the in-plane strengths also suffer significant knockdowns as a result of integral weaving with Z-yarns.

In order to relax the One Element failure condition, elements in the finite element mesh were grouped together on the basis of load factors, regardless of the material code. Distributions of the load factors for the first 15% of the elements in the Z-Crowned Composite and the Uncrowned Composite unit-cell finite element meshes are presented in Fig. 5. The load factors for the first 15% elements in the Equivalent Laminate finite element mesh acquired unique constant magnitudes for individual loading cases, on account of its geometric and mechanistic homogeneity. The distributions in Fig. 5 show that the load factors stabilize after about 1% of the elements are allowed to fail. Moreover, allowing either 5% or 10% elements to fail would be too liberal from the perspective of determining strength for a particular load case.

Based on load factors corresponding to 1%, 5% and 10% element failure, new strengths were computed for the Z-Crowned Composite and the Uncrowned Composite, for all the five loading cases, and are presented in Fig. 6. The dashed horizontal lines in Fig. 6 represent the strengths for the Equivalent Laminate, which as expected remain unaltered. To maintain clarity of the results presented in Figs. 4 and 6, the strength magnitudes of all the material systems corresponding to the One Element and 1% element failure criterion are listed in Table 6.

Changing the allowable element failure threshold, clearly affects the predicted strengths for the Z-Crowned Composite, and the Uncrowned Composite, as shown in Figs. 6(a) and (b), respec-

Table 5
Comparison of longitudinal (E_{xx}) and transverse (E_{yy}) elastic moduli reported in [2,9] with corresponding results obtained in the present study. The predictions obtained from the current analyses are in reasonably good agreement with published data within the realm of numerical uncertainty.

Model	E_{xx} (GPa)	E_{yy} (GPa)	$E_{xx}^{Experimental}$ (GPa)	$E_{yy}^{Experimental}$ (GPa)
Ref. [2]	27.31	25.70	24.68	20.75
Ref. [9]	27.90	26.20	NA	
Uncrowned Composite	30.51	25.52		
Z-Crowned Composite	25.15	20.81		

tively. The percentage increase in the strengths of the Z-Crowned Composite, and the Uncrowned Composite for all five loading cases reported in Table 6, correspond to changing the element failure threshold from the One Element to the 1% element failure criterion. The longitudinal compressive strength of the Z-Crowned Composite is predicted to experience the most significant increase. On the other hand, maximum increase in strength for the Uncrowned Composite occurs under the transverse compression loading case. Further increases in allowable element failure thresholds to 5% and 10% element failure, do serve to push the predicted strengths toward those of the Equivalent Laminate, but these criteria are considered to be very liberal.

Pursuant to the above discussion, 1% allowable element failure threshold is now set as the criterion to choose the load factor for all the three material configurations, under any given state of loading. Strength knockdowns resulting from the One Element and 1% allowable element failure threshold are presented in Fig. 7. The first bar in Figs. 7(a) and (b) corresponds to the Equivalent Laminate whereas the second and third bars represent in-plane strengths for the Uncrowned Composite and the Z-Crowned Composite, respectively. As the element failure threshold is increased, individual load factors decrease and the corresponding strength increases. Consequently, the percentage knockdown in strength decreases.

4.4. Failure loci

The in-plane strengths computed for the Z-Crowned Composite, Uncrowned Composite and the Equivalent Laminate, under the One Element and 1% element failure condition are reported in Table 6. Failure envelopes are then plotted based on the Tsai–Wu, Tsai–Hill, and Maximum Stress Theory criteria, using the strength values corresponding to the 1% element failure condition in Table 6.

Failure envelopes for the Z-Crowned Composite, Uncrowned Composite and the Equivalent Laminate, could be plotted for generalized load states $\{\sigma\} = \{\sigma_{xx} \sigma_{yy} \tau_{xy}\}$, by suitably varying the Euler angles ϕ and θ , shown in Fig. 8. In particular, σ_{xx} and σ_{yy} are related as $\sigma_{yy} = \sigma_{xx} \tan(\phi)$ and failure loci are plotted by varying ϕ in the range $0 \leq \phi \leq 2\pi$. While using the Tsai–Hill failure criterion, X and Y (see Eq. (11)) are assigned appropriate magnitudes depending of the value of ϕ . For example, if $\frac{\pi}{2} \leq \phi \leq \pi$, then $X = -S_{L(-)}^{FEA}$ and $Y = -S_{T(+)}^{FEA}$. Similarly, the appropriate sign of σ_{xx} is chosen depending on the magnitude of ϕ . The Tsai–Wu failure envelope on the other hand, is plotted consistent with the discussion in [17].

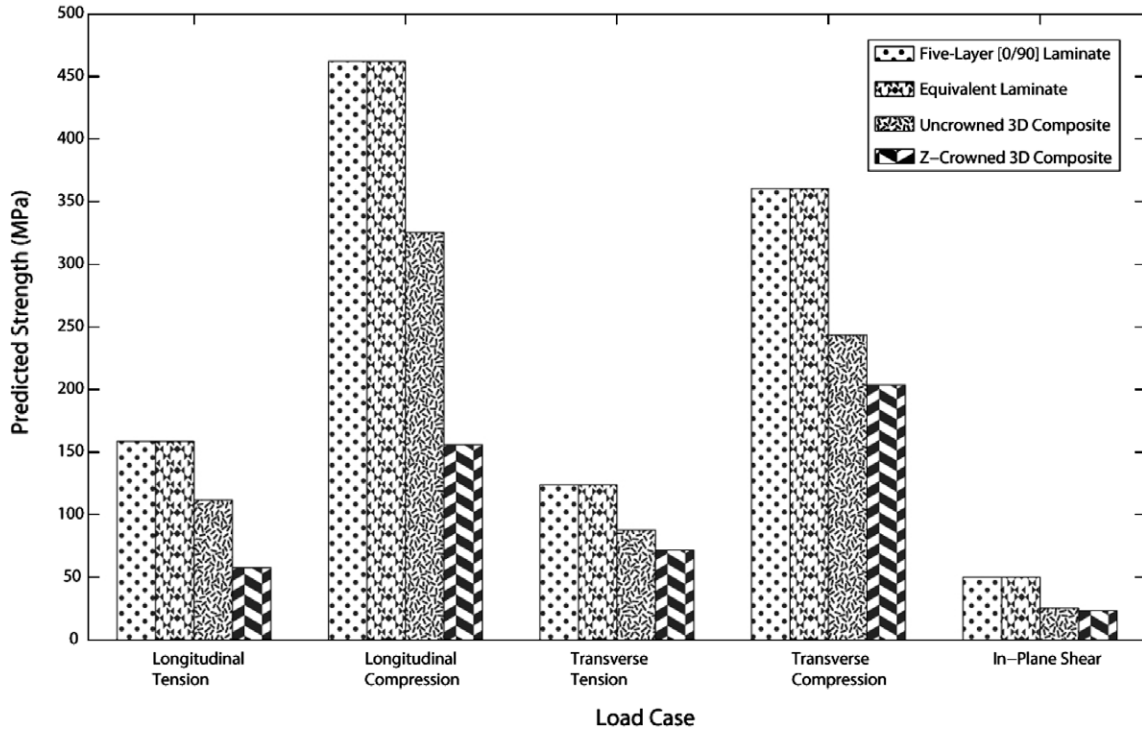


Fig. 4. In-plane strengths of the ‘Five-Layer [0/90] Laminate’, Equivalent Laminate, Uncrowned Composite and the Z-Crowned Composite, under different loading cases. The strength of the ‘Five-Layer [0/90] Laminate’ was predicted based on Classical Lamination Theory, whereas, the strengths of the other material configurations were predicted in accordance with the One Element failure threshold, for individual loading cases.

However, for the sake of brevity, failure loci shown in Fig. 9, correspond to the case wherein $\theta = 0$, implying planar loading states given by $\{\sigma\} = \{\sigma_{xx} \ \sigma_{yy} \ 0\}$. Intersection of the individual failure envelopes with the coordinate axes represents longitudinal and transverse strengths for corresponding material configurations. Specifically, intersections in positive domains represent tensile

strengths, whereas those on the negative domains of the coordinate axes correspond to compressive strengths, respectively. As seen from Fig. 9, introduction of the Z-yarns results in significant knockdowns in the in-plane strengths. Furthermore in Figs. 9(a)–(d), the small ellipse is placed merely to indicate the location of the origin.

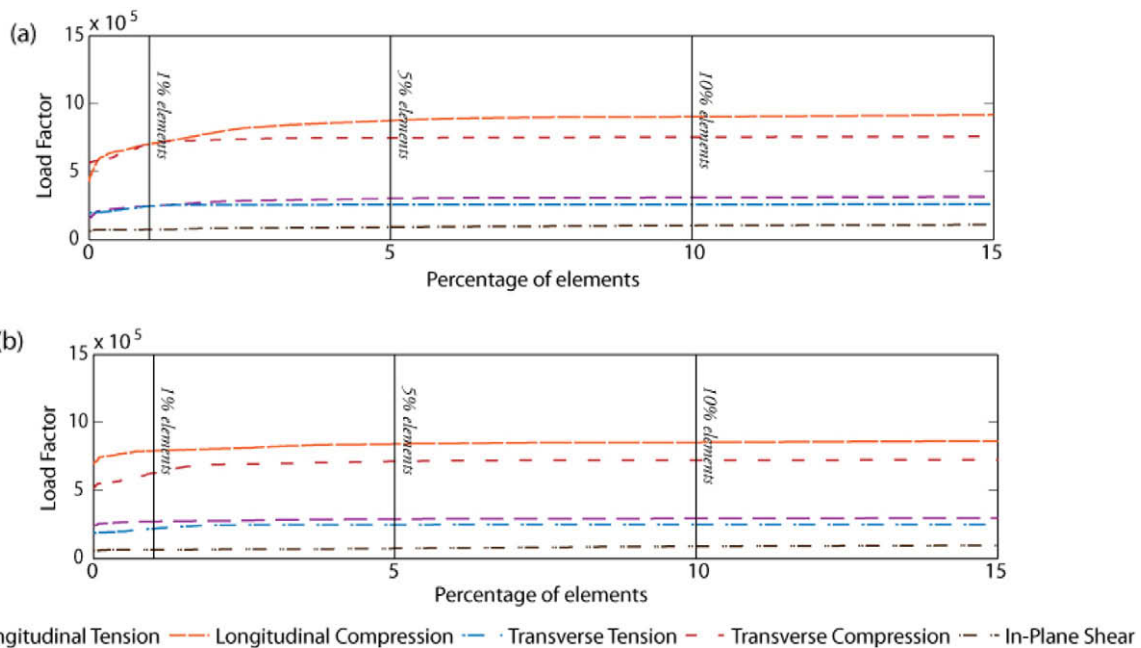


Fig. 5. Detailed inspection of the load factors of 15% of the elements in individual finite element meshes sorted in ascending order. (a) Z-Crowned Composite. (b) Uncrowned Composite.

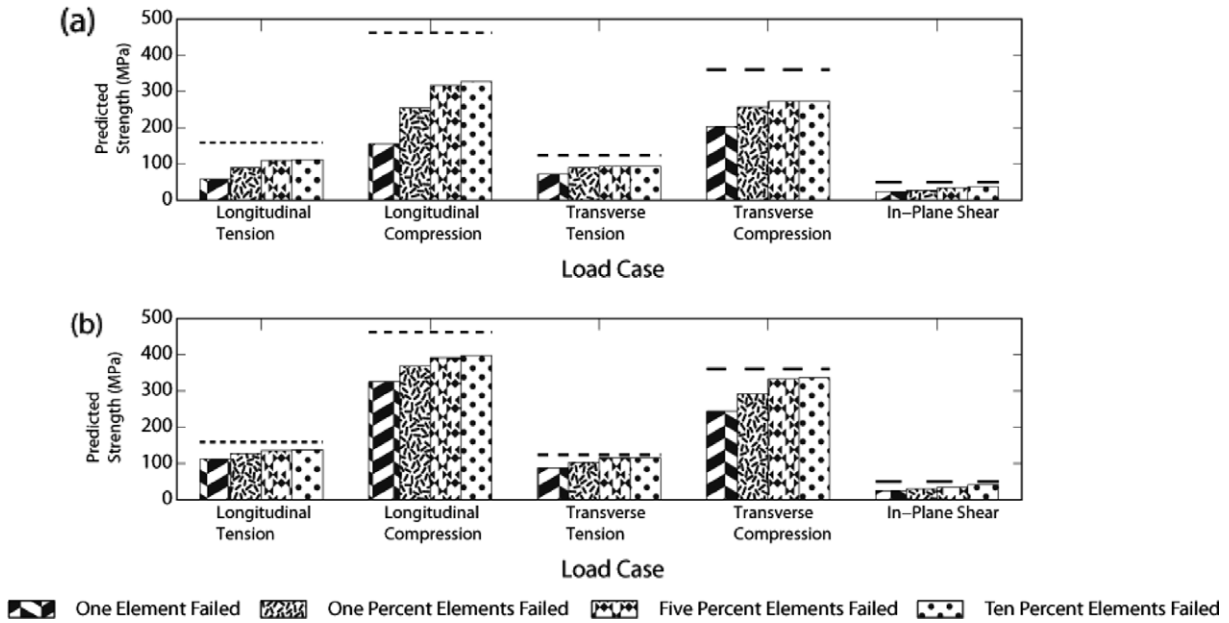


Fig. 6. Variation in the planar strengths of the three material configurations modeled in this study. The dashed horizontal lines represent the strengths of the Equivalent Laminate for individual loading cases. (a) Z-Crowned Composite. (b) Uncrowned Composite. These results were obtained based on choosing load factors corresponding to different levels of allowable element failure as shown in Fig. 5.

Table 6 Predicted in-plane longitudinal, transverse and shear strengths for the three material configurations investigated in this study. The results indicate significant increases in predicted strengths for the Uncrowned Composite and the Z-Crowned Composite as the failure threshold is increased from the One Element level to the 1% allowable element failure condition. The predicted strengths for the Equivalent Laminate do not change with failure threshold as discussed previously.

Failure threshold	$S_{L(+)}^{FEA}$ (MPa)	$S_{L(-)}^{FEA}$ (MPa)	$S_{T(+)}^{FEA}$ (MPa)	$S_{T(-)}^{FEA}$ (MPa)	S_{LT}^{FEA} (MPa)
<i>Z-Crowned Composite</i>					
One Element	57.51	155.73	71.58	203.61	23.24
1% Elements	89.86	254.50	89.82	257.17	27.40
% Increase	56.25	63.42	25.48	26.30	17.90
<i>Uncrowned Composite</i>					
One Element	111.70	325.40	87.65	243.36	25.35
1% Elements	126.86	368.29	102.06	292.49	29.62
% Increase	13.57	13.18	16.44	20.18	16.84
<i>Equivalent Laminate</i>					
One Element	158.52	462.00	123.70	360.44	50.00
1% Elements	158.52	462.00	123.70	360.44	50.00

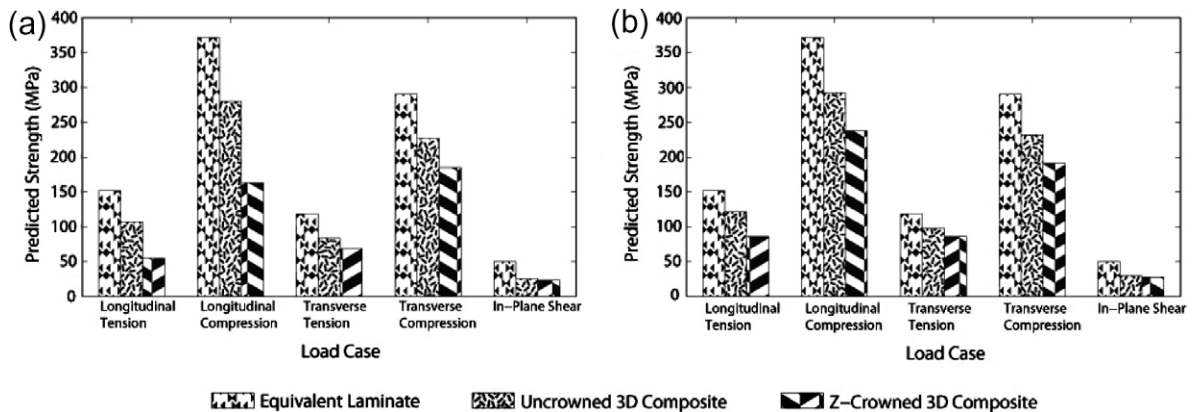


Fig. 7. Strength knockdowns in the Equivalent Laminate, Uncrowned Composite and the Z-Crowned Composite based on different levels of allowable element failure. (a) One Element failure condition wherein the entire composite is assumed to have failed when the first element failure is encountered – most conservative threshold. (b) One percent of the elements are allowed to fail.

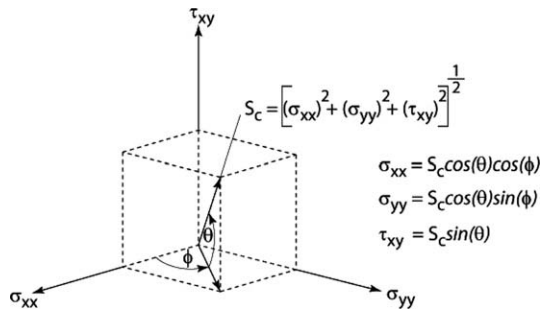


Fig. 8. Representation of combined in-plane loading as functions of the Euler loading angles ϕ and θ .

5. Discussion

Previous studies on these and other materials belonging to the family of woven composites [2–9,14,16], have shown that their mechanical behavior is intrinsically tied to the weave architecture. Finite element method based mechanistic studies therefore, require robust geometry models with the ability to capture the micro-structural architecture in great detail and sensitivity to adapt to changes in the same. The models developed herein while robust from the perspective of yielding reliable solutions, need to be refined to better approximate the path of the Z-yarns through the thickness of the composite.

One of the mechanistic effects of Z-yarns manifests itself as pronounced micro-bending discussed earlier. These micro-bending effects, while significant, are however, highly localized. In the bulk of

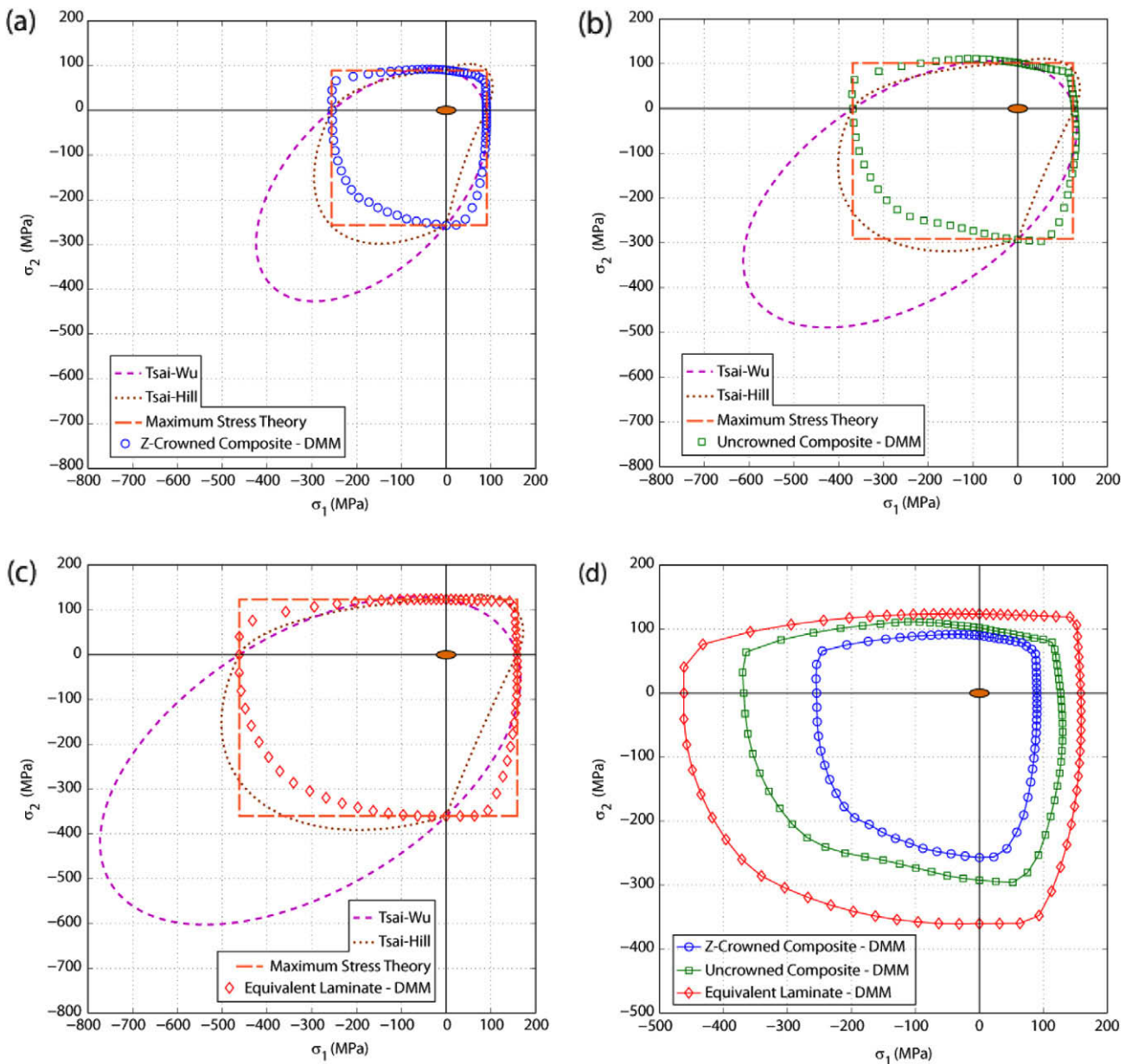


Fig. 9. Failure loci and Direct Micro-Mechanics (DMM) based individual failure data points. (a) Z-Crowned Composite. (b) Uncrowned Composite. (c) Equivalent Laminate. (d) The DMM failure loci for the Z-Crowned Composite, Uncrowned Composite, and the Equivalent Laminate. With the introduction of the Z-yarns, the failure envelopes shrink progressively, indicating severe knockdowns in strengths.

the unit-cell, these local effects are negligible. Though the associated strain and stress gradients are localized, significant knock-downs in the effective properties are predicted as reported in Tables 4 and 6. Therefore including the effects of Z-Crowns becomes imperative even in micro-mechanical analyses. By extrapolation then, Z-Crowns should be included in scenarios wherein inter-laminar delamination is expected. This might be the case in impact loading situations, since the very purpose of introducing Z-yarns in integrally woven 3D fabric composites is to contain delamination.

The strength results obtained as part of this study are compared with those reported in [2] and presented in Table 7. The 'Ref [2] – Theoretical' strength $S_{L(+)}$ is greater than the Uncrowned Composite strength $S_{L(+)}$ by 16% based on the results in Table 7.

However, the 'Ref [2] – Experimental' strength $S_{L(+)}$ is greater than the Z-Crowned Composite strength $S_{L(+)}$ by 36%. These variations could be attributed to differences in micro-structural geometry and boundary conditions imposed in the current study to simulate uni-axial tensile response.

Specifically, in case of the Uncrowned Composite, the volume fraction of warp and fill tows, and the Z-yarns was determined to be 48.65%, 37.16%, and 1.57%, respectively. However, for the Z-Crowned Composite the above volume fractions were 37.86%, 28.91% and 3.28%, respectively. These significant variations in volume fractions could be ascribed to differences in the actual and modeled micro-structural geometry – an observation consistent with [2]. Since the volume fractions for the Uncrowned Composite are closer to those reported in [2], the strengths presented in Table 7 for this case are in better agreement with the corresponding magnitudes from [2]. The discrepancy in the results obtained for the Z-Crowned Composite whose modeled architecture is much closer to real materials indicates that one should endeavor to construct better geometry models rather than neglect the Z-Crowns.

As seen from the results in Fig. 9, the Tsai–Wu, Tsai–Hill and Maximum Stress Theory phenomenological failure criteria are unable to completely capture the behavior of 3D woven composites. These results indicate that 3D woven composites respond more like a structure rather than a material, when subjected to mechanical loading. Consequently, DMM techniques along with constrained optimization methods [18] to minimize the enclosed area of a failure ellipse or volume of a failure ellipsoid, are perhaps better suited to predict the strengths of 3D woven composites under a generalized loading state.

6. Conclusions

This study focused on investigating the effect of Z-yarns on the in-plane stiffness and strength of 3D woven composites. The tow materials were modeled as transversely isotropic, whereas the matrix material was regarded as an isotropic entity. Failure of the tows was assumed to be governed by the Tsai–Hill criterion, while the maximum principal stress condition was invoked to check for failure of the matrix material.

If only the through-thickness segments of the Z-yarns are included (Uncrowned Composite), the elastic moduli in the longitu-

Table 8

The knockdown in the predicted strength of the 3D woven composite studied in this work. The individual strength magnitudes of the Uncrowned Composite and Z-Crowned Composite for comparison with the Equivalent Laminate are based on the 1% element failure condition reported in Table 6. The 'Ref. [2] – Theoretical' and 'Ref. [2] – Experimental' strength magnitudes for comparison with the Equivalent Laminate are given in Table 7.

Model	$S_{L(+)}$	$S_{L(-)}$	$S_{T(+)}$	$S_{T(-)}$	S_{LT}
<i>Percentage knockdown</i>					
Ref. [2] – Theoretical	5%	NA	NA	NA	NA
Ref. [2] – Experimental	12%	NA	NA	NA	NA
Uncrowned Composite	20%	20%	18%	19%	41%
Z-Crowned Composite	43.31%	45%	28%	29%	45%

dinal (E_{xx}) and transverse (E_{yy}) directions are knocked down by approximately 16% and 10%, respectively. The knockdowns are computed with respect to the Equivalent Laminate which is a comparable 2D cross-ply laminate comprised of stacked [0/90] unidirectional laminas. However, if both the 0°-oriented planar and through-thickness segments of Z-yarns are included (Z-Crowned Composite) then the above properties are knocked down approximately 30% and 24%, respectively. Similarly, the knockdown in the in-plane strengths of the material system investigated in this study is compared with data available in [2] and reported in Table 8. As expected, the most severe knockdowns are predicted for the Z-Crowned Composite.

The Equivalent Laminate, Uncrowned Composite and the Z-Crowned Composite were found to be stiffer in the longitudinal direction as compared with the transverse direction, as indicated by the result $E_{xx} > E_{yy}$ in Table 4. Consistently, the strengths in the longitudinal and transverse directions were characterized by the same relationship since $S_{L(+)}^{FEA} > S_{T(+)}^{FEA}$ and $S_{L(-)}^{FEA} > S_{T(-)}^{FEA}$ in Table 6. As expected, the Equivalent Laminate is much stronger in shear as compared with both Uncrowned Composite and the Z-Crowned Composite, as reported in Table 6.

The mechanics approaches developed in this study are applicable to a broad range of woven composites [14,16]. This assertion is further attested by consistent stiffness results reported in Tables 4 and 5 and strength results presented in Fig. 4. Consistency of the above results stems from the fact that the finite element results for the Equivalent Laminate morphology are in excellent agreement with the comparable analytical 'Five-Layer [0/90] Laminate' layup.

On account of the solutions reported in Figs. 3 and 4, Tables 4 and 5, it is reasonable to conclude that these models and corresponding solutions are mechanistically reliable.

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Table 7

Comparison of longitudinal and transverse tensile strengths of the 3D woven composites studied in [2] and the present work. The results from [2] reported herein are based on the initial failure strain $\epsilon_x^f = 0.554\%$.

Model	$S_{L(+)}$ (MPa)	$S_{T(+)}$ (MPa)
Ref. [2] – Theoretical	151.00	NA
Ref. [2] – Experimental	140.00	NA
Uncrowned Composite	126.86	102.06
Z-Crowned Composite	89.86	89.82

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