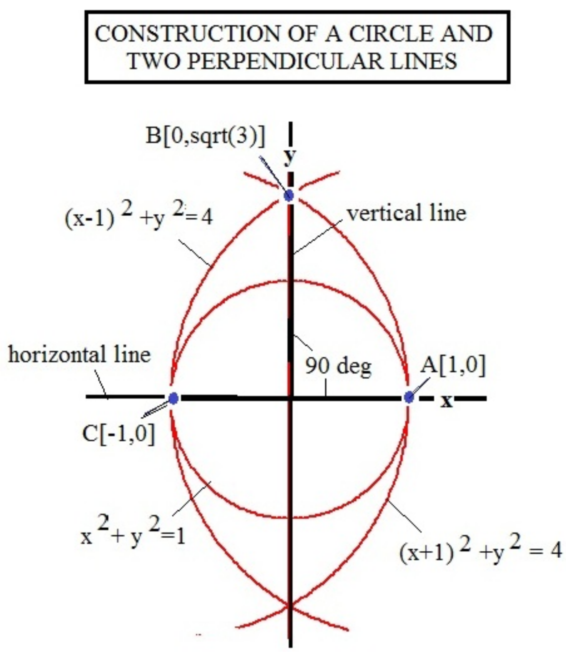
CONSTRUCTION OF TWO-DIMENSIONAL FIGURES USING ONLY RULER AND COMPASS

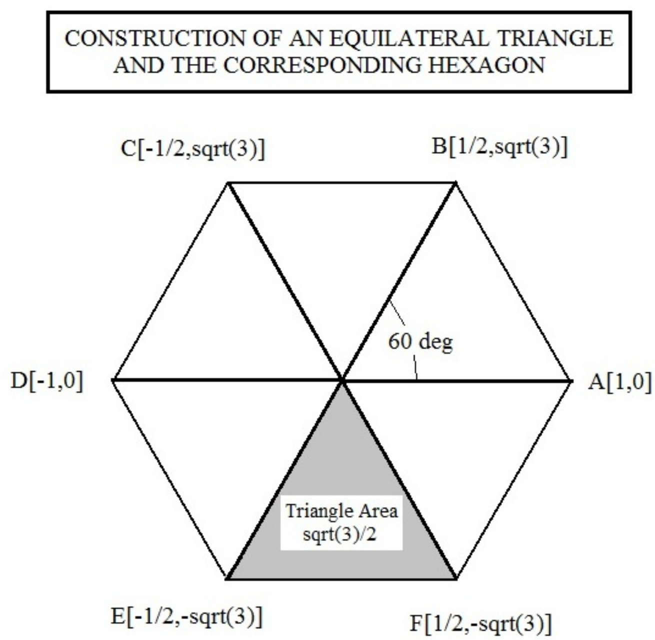
It is well known that many two-dimensional figures may be constructed using only a ruler and an adjustable compass. Prime examples are circles, triangles, and multisided polygons. We want here to show how this is done.

Our starting point is to draw a straight line on a flat horizontal surface. This line may be adjusted to point in a north-south direction as the ancient Egyptian pyramid builders probably did. Next one picks a point along this line and lets it be the center of a circle of radius R constructed by a compass or a rope of length R free to pivot about the selected point. This produces a circle. Next to construct a line perpendicular to the original line we place a compass set to radius 2R at the two original line and circle intersections and draws the two extra circles.. Where they intersect will form the point on a new perpendicular line as shown in the following figure-

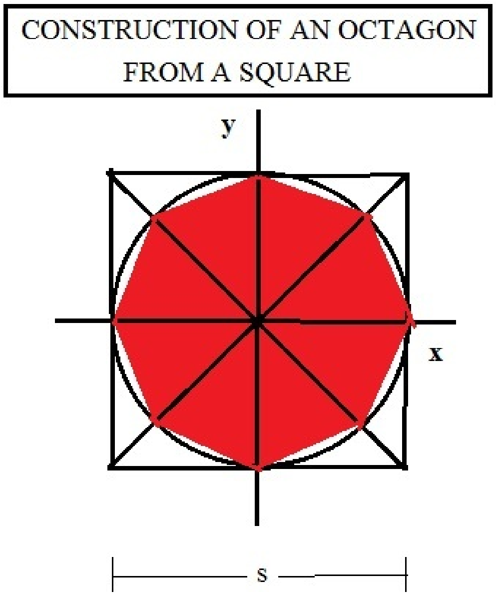


With the set of perpendicular lines, we are now ready to construct a square by marking off distances R=s along the two lines coming from the origin. Next re-positioning the compass needle at the marked off points along the perpendicular lines and drawing new circles of radius s, produces a new intersection point which will form the third vertex of a perfect square. The square will have its corners located at [x,y]=[0,0], [s,0],[s,s], and [0,s]. Bisecting this square by passing a line through either of its diagonals will produce a 45egree right triangle. A 3-4-5 right triangle is also easily generated by using the two perpendicular lines and marking the values of 3 and 4 along respective x axes. Then connecting the end points yields the 3-4-5 triangle.

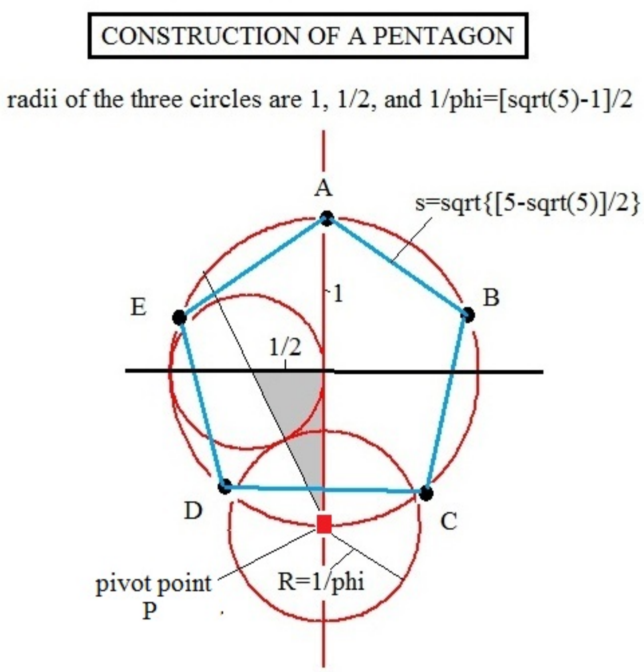
We next consider constructing a hexagon together with its six equilateral sub-triangles. This is accomplished by picking two points separated by distance s from each other along a straight line. A compass is set to R=s and circles are drawn about the end points. These circles intersect at the third vertex of and equilateral triangle. If one now continues with drawing new circles about the equilateral triangle vertexes and then connects the intersection points one will find an extra five constructed triangles to form a hexagon as shown-



Regular polygons of 3n sides where n=1,2,3,..may now be constructed by successive bisection of the sub-triangles. Likewise equal sided polygons of side number 4n can be constructed from the bisection of a square by its two diagonals. The vertexes of the 4n regular polygon all lie on a circle. Here is a schematic of an octagon construction-

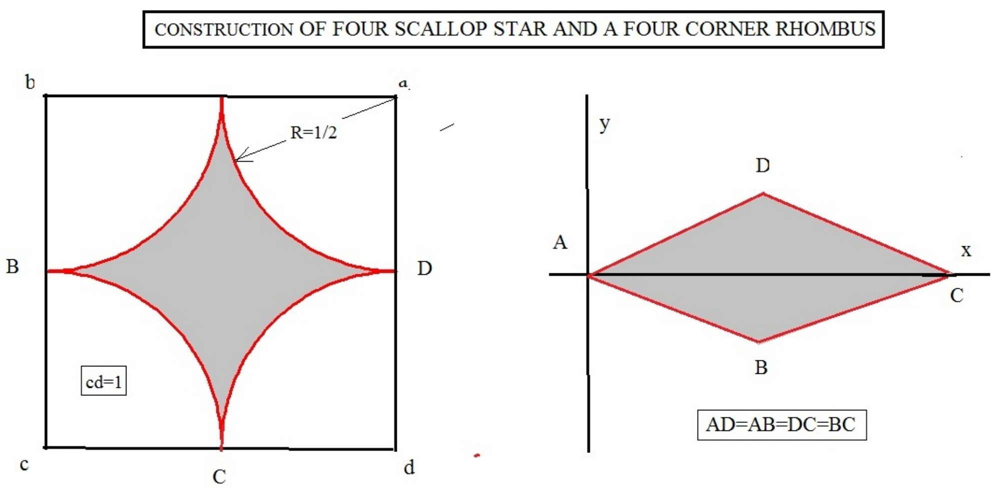


Those polygons where the number of sides are not divisible by 3 or 4 are a bit more tricky to generate and some become impossible to generate by just straight-edge and compass. Take the case of a pentagon. There we have a structure whose five vertexes lie on a circle of radius R=1 and the length of each side of the pentagon is s. To construct the pentagon we draw two additional circles of different diameter as shown-



The grey right triangle shown has a hypotenuse of length sqrt(5)/2.When extended it passes through the center of the two smaller circles. If one now draws two extra circles about a pivot point at [x,y]=[0,-1] of radii 1/phi and phi on sees they intersect at the pentagon vertexes at B,C,D, and E. Here phi is the famous golden ratio which equals [sqrt(5)+1]/2. Connecting the vertex points produces sought after pentagon.

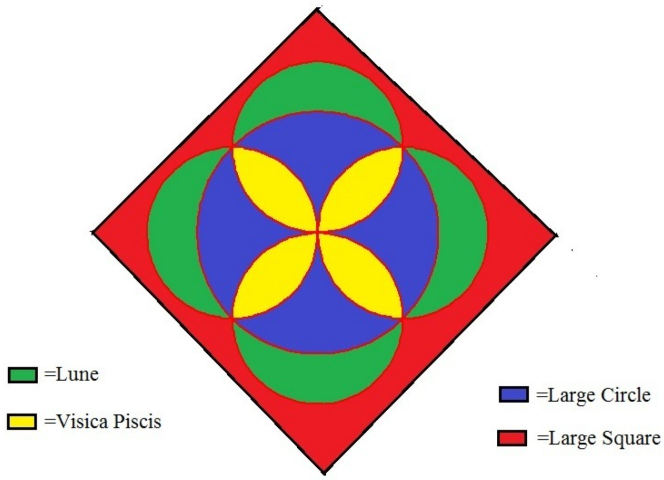
Two more, simple to construct 2D figures via straightedge and compass are the following four cusp star and the classic rhombus-



The star follows by drawing circles of radius one-half about the four corners of a square of unit side-length.. The resultant figure closely resembles , but differs from , the classic Astroid formed by rolling a circle of radius R=1/4 around the inside of R=1 circle. The Rhombus has its side lengths AD=AB=DC=BC fixed by setting the compass to these lengths and then drawing arcs from the ends of the line segments. Point C is determined by centering the compass needle at points B and D. The Rhombus with DB is adjustable.

Although there are an infinite number of additional 2D figures which can be constructed with just compass and straight edge, we finish things here by looking at a multi-figure pattern generated by five circles and one square. Here is the figure-

CONSTRUCTED 2D PATTERN USING ONLY STRAIGHT EDGE AND COMPASS



One recognizes here the four lunes and four (football shaped) visica piscis generated by the large central circle $x^2+y^2=2$ and the four smaller off- center circles $(x\pm1)^2+(y\pm1)^2=1$. Such combinations of circles and straight lines can form the basis for all kinds of computer generated art as we have already discussed in an earlier note.