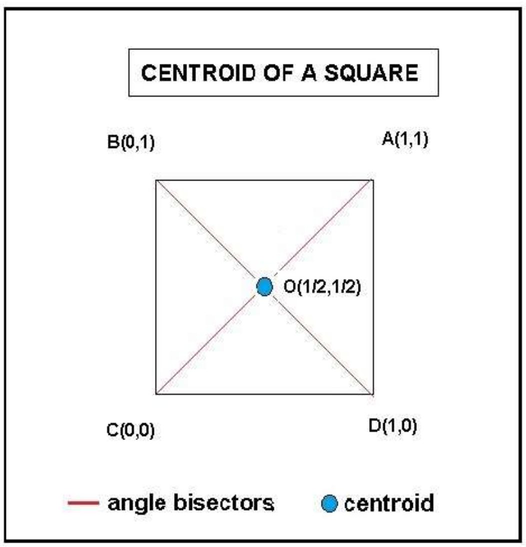
CENTROIDS OF IRREGULAR POLYGONS

It is well known that the centroid of any lamina is given by the integral expressions-

$$\bar{x}=\text{int}(x\ dA)/A \quad \text{and} \quad \bar{y}=\text{int}(y\ dA)/A$$

, where A is the area of the entire lamina. In many instances, where the lamina boundaries are straight lines forming irregular polygons, one can just use a simplified formula involving only the coordinates of the N vertexes of a polygon to find this centroid without the use of calculus. For example, by bisecting four angles of a square with unit side-length, we get the picture-



The centroid of the square lies where the diagonals cross. Its coordinates are(1/2,1/2). Denoting the four corners as (0,0),(1,0),(1,1) and (0,1), we can also get the same result by adding up the x and y values to get -

$$\bar{x}=(0+1+1+0)/4=1/2 \quad \text{and} \quad \bar{y}=(0+0+1+1)/4=1/2$$

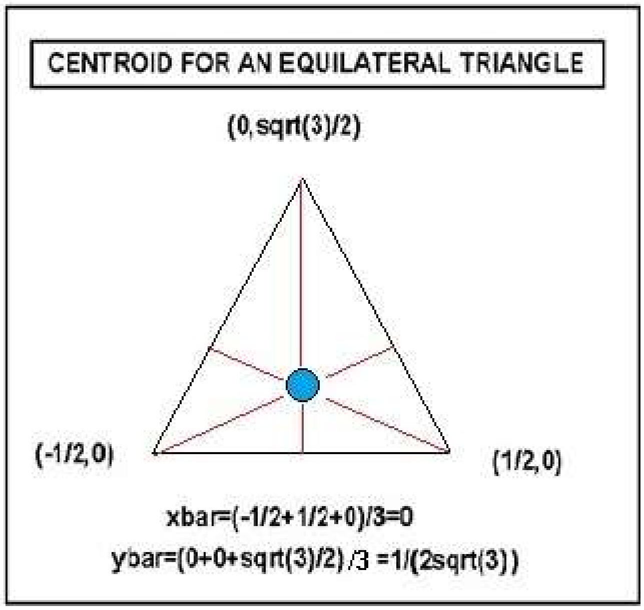
The later follows from the fact that breaking a polygon of N sides into N equal sized sub-areas $A_o=A/N$ and having these sub-areas center on the vertex points, we have the centroid of the lamina given by-

$$\bar{x}=\text{sum}(x_nA_o,n=1..N)/A=\text{sum}(x_n,n=1..N)/N$$

and-

$\overline{y}$=sum($y_n A_o$,n-1..N)/A=sum(y_n,n=1..N)/N

Another lamina for quick centroid evaluation is the equilateral triangle of unity length sides. The three diagonals emanating from the triangle vertices are shown in red. They intersect at the centroid $(\overline{x},\overline{y}) = \left(0,\frac{sqrt(3)}{2}\right)$ as shown in blue-



Here $\overline{x}$=(-1/2+ 1/2+0)/3=0 and $\overline{y}$=(0+0+sqrt(3)/2)/3=1/(2sqrt(3) .

The above two examples have shown that sometimes it is not necessary to use calculus to find the centroid of certain lamina. This appears to be especially true for any irregular polygon where $\overline{x}$ and $\overline{y}$ can be quickly given by the summation of xs and ys. It is our purpose here to find such centroids for N sided irregular polygons.

Let us begin with the right triangle having vertex coordinates (0,0), (3,0) and (3,4). Here we get at once, without the use of calculus, that

$\overline{x}$=(0+3+3)/3=2 and $\overline{y}$=(0+0+4)/3=4/3

We can also get the centroid via the longer way of using calculus. There we find the identical result-

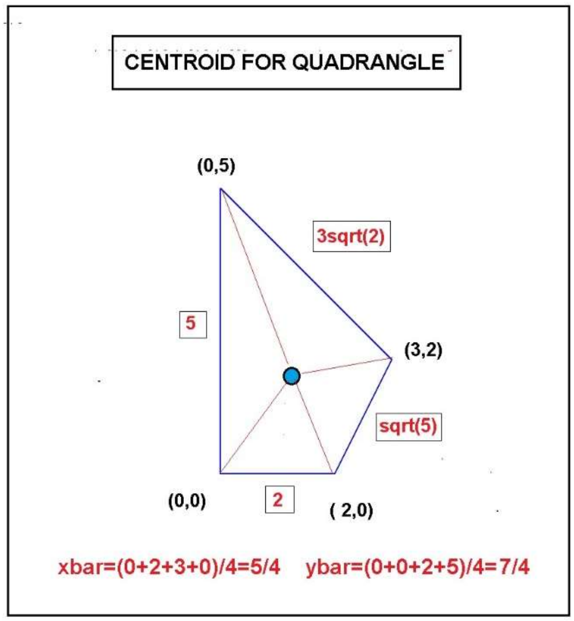
$\overline{x}$=(2/9)$\int_0^3 x^2 dx = 2$ and $\overline{y}$=(1/2)$\int_0^4 y\left(1-\frac{y}{4}\right)dy$=4/3

Note the division by N for N sided polygons. So N=3 for triangles and N=5 for pentagons.

Take next the case of the irregular four sided quadrangle whose vertexes are given by (0,0),(2,0),(3,2),and (0,5). Here we find the centroid at-

$$\overline{x} = [0 + 2 + 3 + 0]/4 = 5/4 \quad and \qquad \overline{y}=[0+0+2+5]/4 = 7/4$$

The quadrangle figure showing the centroid at (5/4,7/4) follows-



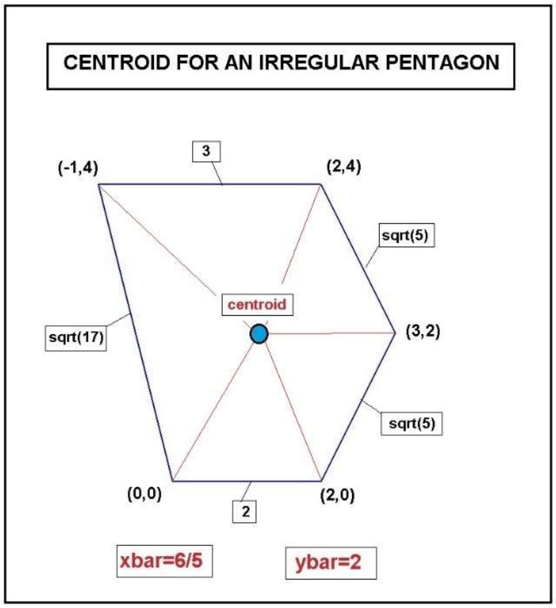
Continuing on, we consider the five sided irregular polygon with vertex coordinates at-

(x,y)={(0,0),(2,0),(3,2),(2,4), and (-1,4)}

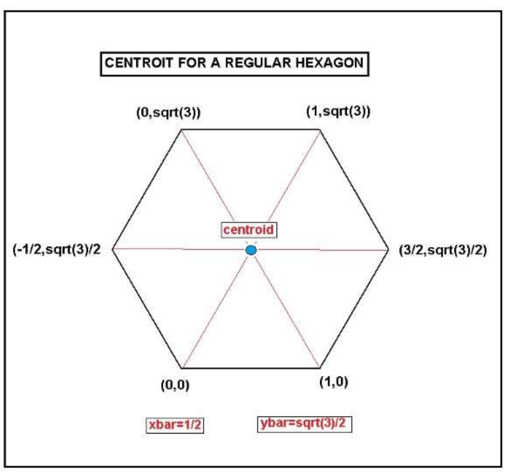
Using these vertex values, we find-

$\overline{x}$=(0+2+3+2+-1)/5=6/5 and $\overline{y}$=(0+0+2+4+4)/5=2

Here is a picture of this pentagon and the location of its centroid.



Note to get this same result for the centroid using calculus would have required considerable more work involving multiple integrals. The centroid for regular polygons (like the above discussed square) will always be found at the polygon center. To confirm this point look at the following regular hexagon and its centroid-



Here the values for the centroid are found at the polygon center-

$\overline{x}$=(0+1+3/2+1+0-1/2)/6=1/2 and

$\overline{y}$ =(0+0+sqrt(3)/2+sqrt(3)+sqrt(3)+sqrt(3)/2)/6=sqrt(3)/2 .

U.H.Kurzweg
May 30, 2022
Gainesville, Florida