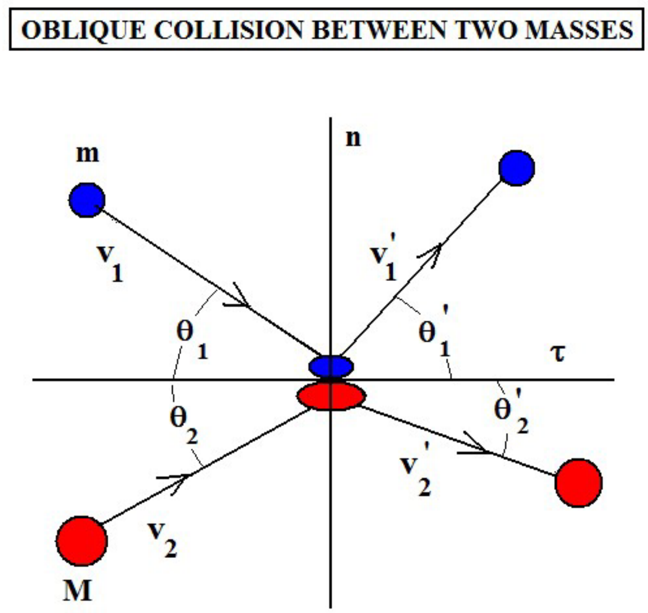
OBLIQUE COLLISION BETWEEN TWO SPHERES

Consider two spherical balls of mass m and M on a collision path, colliding, and then continuing on as shown-



Notice that we are here taking about oblique collisions so that the balls are excluded from making head to head collisions. From the conservation of momentum in the normal direction 'n' one has-

$$-mv_1\sin(\theta_1)+Mv_2\sin(\theta_2)=mv_1'\sin(\theta_1')-Mv_2'\sin(\theta_2')$$

And, since the momentum of the individual masses does not change from before to after the collision, one finds along the collision plane 'τ' that-

$$v_1\cos(\theta_1)=v_1'\cos(\theta_1') \quad and \quad v_2\cos(\theta_2)=v_2'\cos(\theta_2')$$

To make this set of equations solvable one requires an extra condition, namely, the coefficient of restitution-

$$e=\frac{|\,velocity\ of\ \ sep\ along\ n\ after\ collision\,|}{|\,velocity\ of\ \ appr\ along\ n\ before\ collision\,|}=\frac{v_2'\sin(\theta_2')+v_1'\sin(\theta_1')}{v_1\sin(\theta_1)+v_2\sin(\theta_2)}$$

One can now solve for the four unknowns v_1', v_2', θ_1', and θ_2'. The value of e ranges between e=1 for an elastic collision to e=0 for a plastic collision. For billiard balls e is close to unity while for spheres of putty one has essentially e=0.

CASE 1: Consider a ball of mass m hitting a floor at angle θ_1 with respect to the floor. Here M is infinite and $v_2=v_2'=0$. In this case the four equations are easy to solve and yield-

$$v_1' = ev_1\sin(\theta_1)\sqrt{1+[e\tan(\theta_1)]^2} \quad and \quad \theta_1' = \arctan[e\tan(\theta_1)]$$

As expected, for an elastic collision(e=1) , we find $v_1'=v_1$ and have a specular reflection with $\theta_1' = \theta_1$. For the e=0 limit, the ball just sticks to the floor at impact.

CASE 2: Here we let m=M and assume e=1 with $v_1=v_2=v$ and $\theta_1= \theta_2=\theta$. This time the equations read-

$$v\sin(\theta) = v_2'\cos(\theta_2') \quad and \quad v\cos(\theta) = v_2'\cos(\theta_2')$$

with the solutions $v_1'=v_2'=v$ and $\theta_1'= \theta_2'=\theta$.

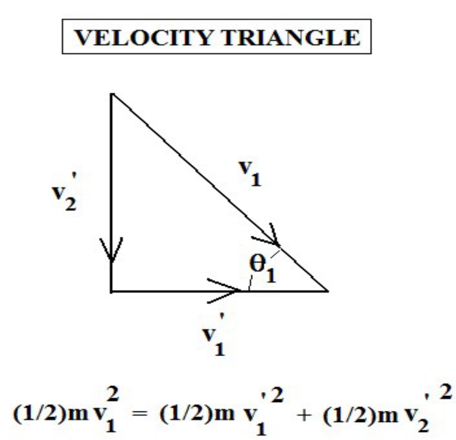
CASE 3: Here we take M=m, e=1 and assume M to be initially stationary. In this elastic case the above equations reduce to-

$$\theta_2' = (\pi/2)\,,\; v_2' = v_1\sin(\theta_1)\,,\; 0 = 2v_1'\sin(\theta_1')$$

Thus we have that the original stationary mass will move down along the negative n axis with a velocity $v_2'=v_1\sin(\theta_1)$ while original moving mass goes along the positive τ axis with velocity $v_1'= v_1\cos(\theta_1)$ after collision. Note that in this elastic case the kinetic energy of the system before and after collision remain the same. That is-

$$T = \frac{1}{2}mv_1^{\;2} = \frac{1}{2}m\left[(v_2')^2 + (v_1')^2\right]$$

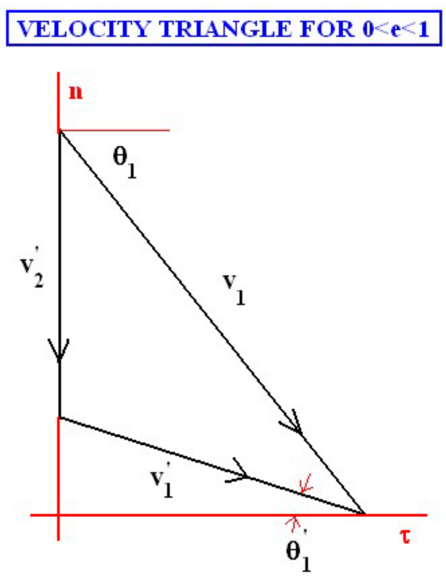
We can also summarize this last result by the following velocity triangle-



For collisions where 0<e<1 there will always be a loss in kinetic energy during the collision and thus a right triangle of the above type will need to be modified. Indeed, one finds for CASE 3 when 0<e<1, that the ratio of kinetic energy after to before collision becomes-

$$\frac{T_{after}}{T_{before}} = 1 - \frac{1}{2}(1-e^2)\sin(\theta_1)^2 < 1$$

with v_2' moving at a slower speed along the negative n axis while v_1' moves at an angle below the τ axis. This time the velocity triangle becomes oblique as shown here-



The corresponding solutions from the above equations, plus application of the law of sines and cosines, are-

$$\theta_2' = \frac{\pi}{2}, \quad \tan(\theta_1') = \frac{(1-e)}{2}\tan(\theta_1), \quad v_2' = \frac{(1+e)}{2} v_1 \sin(\theta_1) \quad \textit{and}$$

$$v_1' = v_1 \cos(\theta_1)\sqrt{1 + [\frac{(1-e)}{2}]^2 [\tan(\theta_1)]^2}$$