CONSTRUCTION OF HEXAGONAL INTEGER SPIRALS

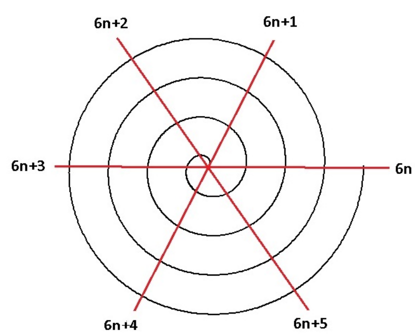
About a decade ago I found a new way to locate all positive integers as points at the vertexes of a hexagonal integer spiral. Not only did this lead to an excellent new way to separate primes from composites(unlike what is not possible with a standard Ulam Spiral) it also led to new insights on twin-primes and the location of super-composites characterized by large number fractions. We want in this note to show how one can construct such a hexagonal integer spiral and discuss more of its important attributes.

Here is the procedure. We start with a standard Archimedes spiral-

$$r= (3/\pi)\theta$$

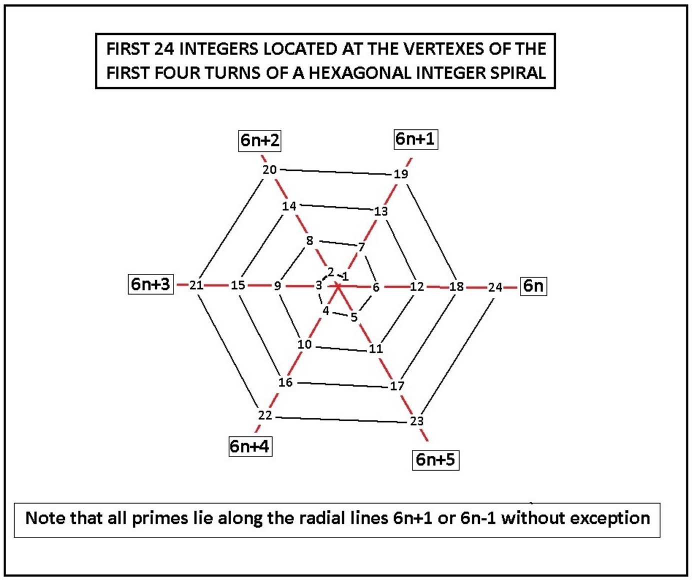
which, when taken out four turns and intersected by six equally spaced radial lines, looks as follows-

ARCHIMEDES SPIRAL r=(3/Pi) THETA PLUS SIX RADIAL LINES



Angle between neighboring radial lines equals Pi/3

The crossing points between the Archimedes Spiral and the radial lines will be the location of the positive integers. Also since we are only interested in integers , the spiral can be modfied by a new hexagonal integer spiral obtained by connecting neighboring integers by straight lines. This roduces the hexagonal integer spiral shown-



This is a HEXAGONAL INTEGER SPIRAL showing the integers sitting at following vertexes of the spiral. We could also have called it the spidernet spiral as it mimics what spiders do in building a spider web. They begin by placing a few radial web fibers and then walk around in a spiral manner laying the rest of the sticky fiber to form the spider web. The following shows such a net-

ACTUAL SPIDER WEB SHOWING RADIAL LINES AND SPIRAL WEB PATTERN

There are several new mathematical observations following from the hexagonal integer pattern. In brief they are-

(1)-Each turn of the spiral increases the integer count by a factor of six. Thus the number N=35679 has N mod(6)=3 so it lies on the 6n+3 line at the 5946th turn of the spiral.

(2)-Primes greater than three are found only along the lines 6n+1 and 6n-1. However since some of the numbers along these two radial lines may be composites we have the restricted statement-

A necessary but not sufficient condition for N>3 to be prime is that it equals 6n+1 or 6n-1

(3)-Twin primes differ from each other by two units. Hence their mean value must always be a multiple of six. An example is [p,q]=[347 ,349]. These are twin primes with the mean equal to 348=6(58). We also have for the mean value that 348 mod(6)=0.

(4)-To test for primeness of integers along 6n±1, we can use the test-

$$\sigma(N)=1+N \quad \text{or} \quad f(N)=0$$

, where f(N)=[σ(N) –N-1]/N is the number fraction and σ(N) is the sigma function of number theory . Most advanced computer programs give the value of σ(N) for Ns of up to about twenty digit length. Lets do a prime test on the number-

N=3695509=6(6157918)+1 which has N mod(6)=1

Here σ(N)=3695510. Hence 3695510=1+3695509. This confirms that N is a prime.

(5)-We define super-composites as those numbers where the number fraction f >2. These are typically found next to prime numbers where f=0. An example of such a super-composite is-

N=2^15*3^8*5^3=26873856000 with a number fraction of f=σ[(N)-N-1]/N=2.7437526

The number next to it yields f(N-1)=0 and hence is a prime lying along the radial line 6n-1.

(6)-A semi-prime is defined as as N=pq with-

[p,q]=S∓sqrt[S^2-N] with S=(p+q)/2=[σ(N)-N-1]/2=Nf(N)/2

Typically any semi-prime will lie along either 6n+1 or 6n-1 with a value for f(N) only slightly above zero.

The semi-prime N=77 has σ(77)=96 and S=9. So the factors [p,q] become [7,11]. We have N mod(6)=-1 and f(77)=18/77.

Hopefully the above six points concerning Integer Spirals will help clarify the importance of the use of these new spirals in discussing additional upcoming unsolved problems in number theory.

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