Evaluation of Y(n)=$\prod_{k=1}^{n} k$ ^$(n+1-k)$

Recently , while evaluating the function f(n)=sigma(n)-n-1)/n, we came up with a new point function J(n) defined as-

J(n)=$\prod_{k=1}^{n} ithprime(k)$^$ithprime(n+1-k)$

In studying this rather complicated expansion, we now realize that there is a much simpler related expression-

Y(n)=$\prod_{k=1}^{n} k$^$(n+1-k)$

Evaluating this new function for n=1, 2, 3, 4 and 5 produces-

Y(1)=1

Y(2)=2

Y(3)=12

Y(4)=288

Y(5)= 34560

It is seen to be a rapidly increasing sequence of n with Y(inf.)=infinity.We wish in this note to examine the most important properties of this point function.

One notices first of all that-

Y(n)=n!Y(n-1)

so that Y(6)=6!34560=2488329. We also have that-

Y(n)=$\prod_{k=1}^{n} k$!=1!*2!*3!*…*(n-1)!*n!

This form for expressing Y(N) is much simpler since it requires no exponents other than one. With this last form we can construct the following interesting pattern-

$$6!\,5!\,4!\,3!\,2!\,1! = 2488329$$

$$5!\,4!\,3!\,2!\,1! = 34560$$

$$4!\,3!\,2!\,1! = 288$$

$$3!\,2!\,1! = 12$$

$$2!\,1! = 2$$

$$1! = 1$$

The factorial form of Y(n) above at first glance looks completely different from the exponential form we started with. To further prove that they are identical we have constructed the following four column table-

n	$\prod_{k=1}^{n} k\hat{}(n+1-k)$	Y(n)	$\prod_{k=1}^{n} k!$
1	1^1	1	1=1
2	1^2x2^1=2	2	1x2=2
3	1^3x2^2x3^1=12	12	1x2x6=12
4	1^4x2^3x3^2x4^1=288	288	1x2x6x24=288
5	1^5x2^4x3^3x2^2x1^1=34560	34560	1x2x6x24x120=34560

A related number product function is-

$$T(n)=\prod_{k=1}^{n}(k!)\hat{}2$$

For n=1,2,3,4,5 it reads-

T(1)=1

T(2)=4

T(3)=144

T(4)=82944

T(5)=1194393600

From this we see at once that T(n)=Y(n)^2.

As another related function we can also look at-

$$R(n)= \sum_{k=1}^{n} k!/\prod_{k=1}^{n} k!$$

The first few R(n) look as follows-

R(1)=1

R(2)=3/2

R(3)=2/3

R(4)=1/9

R(5)=17/3840

This ratio point function goes to zero as n becomes infinite.

U.H.Kurzweg
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Happy New Year