DETERMINING WHETHER A NUMBER IS PRIME OR COMPOSITE

It is well known that an integer is a prime if its only divisors are one and N. It is a composite if it is also devisable by additional integers. Thus N=5720371 is a prime while N=368159 is a composite. One of the easiest ways to distinguish between the two types of numbers is to carry out an evaluation of sigma(N)-1, where sigma(N) is the summation point function of number theory. We want here to derive a few other ways to distinguish primes from composite numbers.

We start by looking at the ratio-

$$\sigma(p^3)/\sigma(p^2)=(1+p+p^2+p^3)/(1+p+p^2)=1+p^3/(\sigma(p^2)$$

, where p are primes. Multiplying this expression by $\sigma(p^2)$ produces the result-

$$0=\sigma(p^3)-\sigma(p^2)-p^3$$

If one now replaces p by any positive integer n, we get the related point function-

$$H=\sigma(n^3)-\sigma(n^2)-n^3$$

It has zero value only if n is a prime but not otherwise. Hence we have a new criterion for a number being prime, namely that H vanishes. Here is a short table confirming when n is a prime-

n	H	N	H
1	---	11	0
2	0	12	2949
3	0	13	0
4	32	14	2857
5	0	15	2462
6	293	16	3584
7	0	17	0
8	384	18	9716
9	243	19	0
10	1123	20	10851

The primes and the corresponding H are marked in red. Let us try the prime criterion for a couple of large numbers. First take-

n=2^32+1=4294967297

Here we get in a split second that H=12380582790969867616436 2246. So the number is composite. Next take –

n=6209613847

Here we get H=0 so the number is a prime.

Another way to detect whether a number is prime or composite is to start with the numberfraction for powers of primes. This function is defined as-

f(p^n)=$(\sigma(p^n) - p^n - 1)/p\hat{}n$

Next take the ratio –

f(p^3)/f(p^2)=(p+1)/p

This can be rewritten as –

1=1/pf(p^2)

So that the right hand side for primes will be one. Relaxing the n=p condition allows us to re-write things as-

F=1/(nf(n^2))

, with F=1 occurring when n=p and F less than unity for composite numbers. Here F]=1 meaning p=41 is a prime.

Besides the simple existing criterion for primeness being sigma σ(N)-1=N, we have found two other rules which may be applied to any positive integer to determine whether N is a prime or composite, They yield primes if-

H=sigma(n^3)-sigma(n^2)-n^3=0

and/or

F=1/[nf(n^2)]=1

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