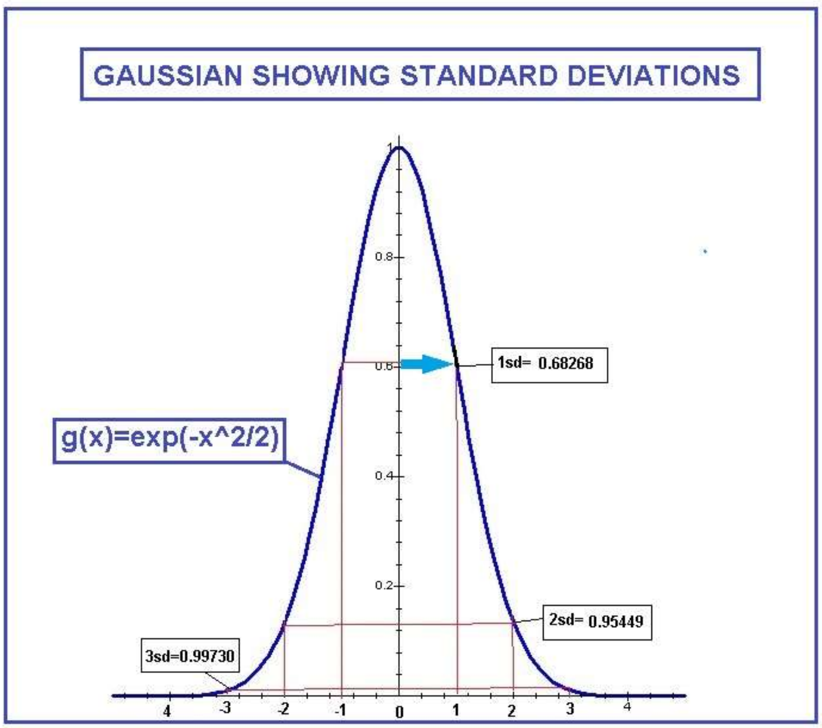
THE GAUSSIAN AND ITS ROLE IN MEASURING IQS

An interesting function used extensively in statistics is the Gaussian. In its simplest form it reads-

G(x,A,m,c)=A exp(-(x-m)^2/(2c^2)

, where A is its maximum value occurring at x=m and c is the standard deviation which determines the width of the function. A plot of this function for A=1, m=0, and c=1 follows-



Note that it is a bell shaped curve symmetric about the vertical line x=0 and has height A=1. Its second derivative vanishes at the inflection points x=$\pm$1. Here the standard deviation c equals exactly one. It is our purpose here to look at some additional properties of g and in particular talk about the area under parts of this curve to show that it can be used to record an individual's IQ in terms of standard deviations from the average 100 IQ . Also some definite integrals containing the Gaussian function will be evaluated.

Let us begin by taking a few derivatives of the general form of g with respect to x. We have the result-

g=A exp(-(x-m)^2/(2*c^2))

g'=A [-(x-m)/(c^2)]exp(-(x-m)^2/2c^2)

and

g"=(A/c^2) {-1+(x-m)^2/(c^2)}exp(-(x-m)^2/2c^2)

From the first derivative we see that g has zero slope at x=m and x=$\pm\infty$. The second derivative vanishes when (x-m)^2=c^2. From this we see that the first standard deviation equals-

c=x-m   or   c=m-x

We indicate the width of the first deviation by the blue arrow in the above figure.

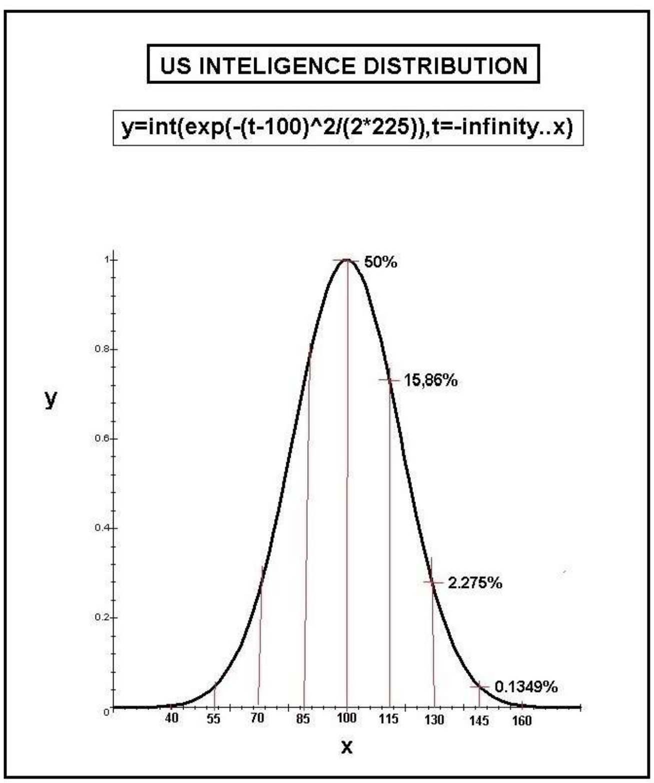
Next we figure out the total area under the Gaussian extending from minus infinity to the value x. This area is-

Area(x)=$\int_{t=-\infty}^{x} \exp((-(t-m)^2/(2c^2)dt$=c sqrt(π/2){1+erf((x-m)/(sqrt(2)c))}

This result, involving the error function, makes sense since the ratio-

AREA(x=0)/AREA(x=infinity)=sqrt(π/2)/sqrt(2π)= 1/2

With this formula for AREA(x) we are now ready to discuss IQ distributions. We know from early research work (Stanfort-Binet,Wechsler) that US adult IQ distribution follows a Gaussian shape with a mean of m=100 and a standard deviation of c=15. Upon plotting this distribution we get the following picture-



We see the symmetry about the average IQ at 100 and also some of the values at one, two and three standard deviations away from this. It says, for example, that individuals with IQs of 130 make up only about two percent of the adult US population. The average PhD in physics or mathematics has an IQ of around 140 although the spread can often be quite large. These days much of the more offensive language used by earlier researchers to describe individuals falling below an IQ 100 has been removed from the repertoire. Also, in recent years, there have been many questions raised about the value of IQ tests in determining promotions etc. There also has arisen the realization that high IQ scores are not the only thing important for student success in school and in later life. For instance abilities in sports, music , artistic and acting abilities are not measured by standard IQ tests yet they can be the very factors determining a favorable life path.

As a final application involving the Gaussian, consider some definite integrals involving

exp-(a*x^2). The first of these is –

$$J(a)=\int_{x=-\infty}^{\infty} \exp(-ax^2)\, dx$$

. To solve it we first set u=ax^2.This produces

$$J(a)=[1/\mathrm{sqrt}(a)\int_{x=o}^{+\infty} u^{-\left(\frac{1}{2}\right)} \exp(-u)\, du$$

This integral is just the Laplace transform of u^(-1/2) which reads Γ(1/2) if s is set to one. Hence we find –

J(a)=Γ(1/2)/sqrt(a)=sqrt(π/a)

Another definite integral involving the Gaussian is-

$$K(a,b)=\int_{x=0}^{\infty} (bx) \exp(-ax\text{^}2)\, dx$$

Again the substitution u=ax^2 produces the simplification-

$$K(a,b)=b/(2a)\int_{x=0}^{\infty} \exp(-u)=b/(2a)$$

A final integral involving g is-

$$L=\int_{x=0}^{\infty} x^n \exp(-x^2)\, dx$$

Leting u=x^2 produces-

$$L=(1/2)\int_{x=0}^{\infty} u^{\left(\frac{n-1}{2}\right)} exp(-u)\, du=(1/2)\Gamma((n+1)/2)$$

The last step involved taking a Laplace transform and setting the Laplace variable s to one,

U.H.Kurzweg
August 7, 2023
Gainesville, Florida