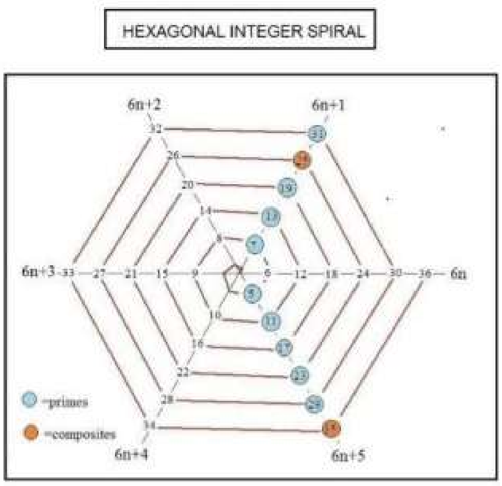
WHAT IS A SIMPLE WAY TO CONSTRUCT A HEXAGONAL INTEGER SPIRAL?

In recent years we have made extensive use of the Hexagonal Integer Spiral to present both composite and prime integers in a way which separates nicely prime from composite numbers. Our early work in this area goes back about a decade when we first ran into the Ulam Spiral. Mathematicians for years had tried to explain the semi-random manner of how prime numbers appear in an Ulam Spiral without much success. We were the first to recognize that by morphing the Ulam Spiral, a much simpler form separating composite from prime numbers appears. Indeed we showed that by placing integers along radial lines and the vertexes of a hexagonal integer spiral, all primes are forced to lie along just two radial lines $6n\pm1$. The typical picture one gets looks as follows-



From the graph one sees that <u>all primes greater than three have the form $6n\pm 1$.</u> Numbers lying along 6n+1 have N mod(6)=1 and ones lying along 6n-1 have N mod(6)=5. Consider the example of-

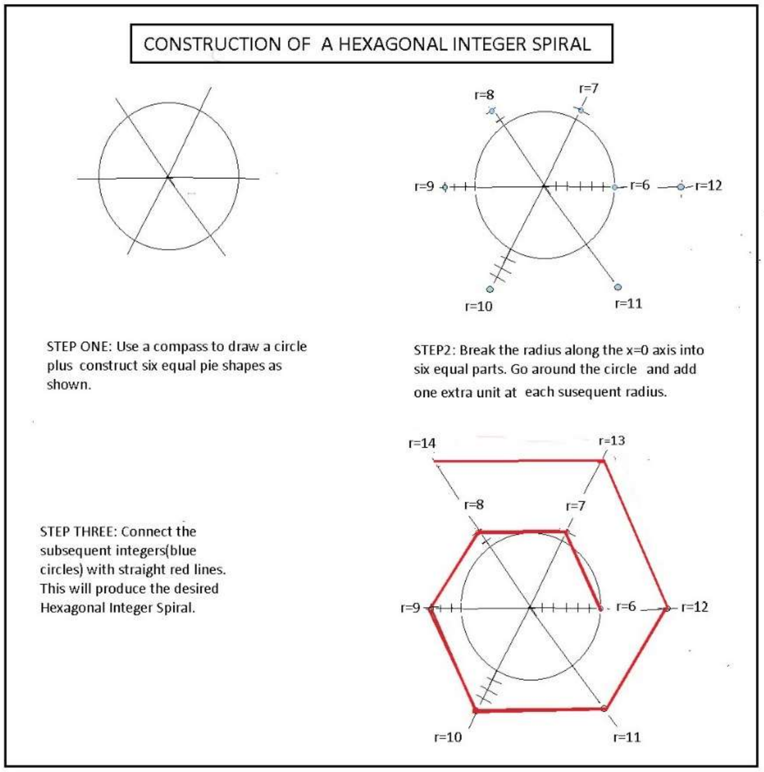
N:= 3810243110911; with N mod(6)=1

This number falls along the radial line 6n+1, so it may be composite or prime. To test for primeness we use our earlier employed identity-

$$\sigma(N) = 1+N$$

, for N prime. Here this is satisfied so N is a prime. Numerous articles have resulted from the above graph, including new insights into twin primes and super-composites. We wish here to go back and develop an alternate and much simpler way to construct a Hexagonal Integer Spiral.

One begins with a simple circle of radius six and divides this circle into 6 equal pie shapes. Next one breaks the radial line along the x axis into six equal length segments. Going counterclockwise around the circle one adds one of the segments to the radial distance yielding a r=7 at the next radial line. This is continued with a second element added to the third radial distance r=8 and so on until once around the entire circle. We mark these values of r by small blue circles. Finally one connects neighboring small circles by straight red lines to yield the desired Hexagonal Integer Spiral. The full procedure is summarized in the following graph-



Note that no direct need for an Archimedes Spiral was required. The points represented by small blue circles are found at polar coordinate positions-

$$[r,\theta]=[N, N\pi/3]$$

Thus N=37 =6*6+1 has $\theta=12\pi+\pi/3$. This means N lies along the 6n+1 radial line at the 6^{th} turn of the spiral. It happens to be a prime number since sigma(37)=37+1.

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