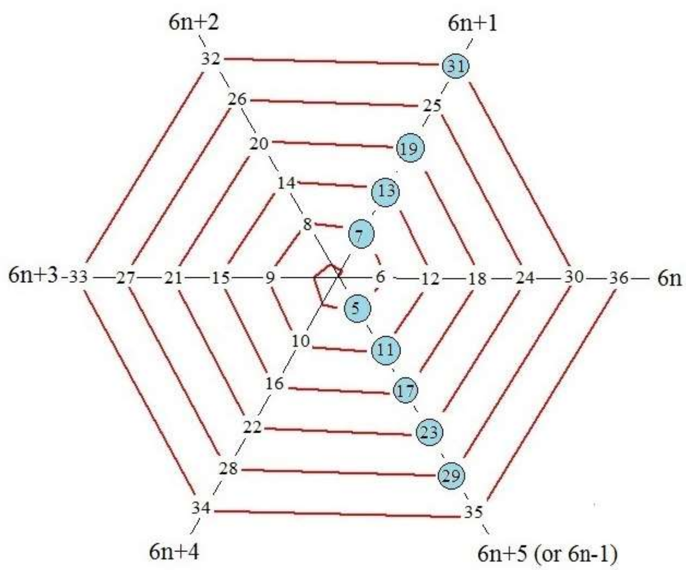
FINDING LARGE TWIN PRIMES USING FINITE LENGTH PRODUCTS OF IRRATIONAL NUMBERS

INTRODUCTION:

We have shown in numerous earlier pages on this web page that all primes five or greater have the form $6n\pm1$ without exception. This fact allows one to graph this infinite set of primes (greater than three) along just two radial lines crossing appropriate vertexes of a hexagonal integer spiral as shown-



The blue circles represent primes. They lie strictly along the radial lines 6n+1 and 6n-1 and cross the hexagonal integer spiral at appropriate vertexes. Since there are also some points along these two radial lines which are composites (such as 25 and 35), a more accurate statement is that-

A necessary but not sufficient condition that a number five or greater is a prime is that it has the form 6n± 1

Another interesting observation following directly from the above diagram is that the positive x axis contains only composite numbers of the form 6n=6,12,18,24,30,...If one now finds 6n+1 and 6n-1 to be primes, one has what are known as a twin prime. Examples of twin primes are [5,7] [11,13],[17,19],[29,31],… . We want in this article to show how large twin primes can be created using finite length approximations of products of irrational numbers.

DEFINITION OF TWIN PRIMES:

A twin prime consists of two primes p=6n-1 and q=6n+1 differing from each other by two units and having 6n as its mean value. For smaller values of n up say 20 one can readily find twin primes by running the computer program-

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for n from 1 to 20 do ({n,isprime(6*n+1), isprime(6*n-1)})od;
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This produces twin primes for-

n=1,2,3,5,7,10,12,17,18

and yields the twin primes-

[5,7],[11,13],[17,19],[29,31],[41,43] ,[59,61],[71,73],[101,103],[107,109].

To find a twin prime in the neighborhood of N=1260, one can use the above formula re-written as-

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for n=-10 to 10 do ({1260+6*n,isprime(1260+6*n+1),isprime(1260+6*n -1)})od;
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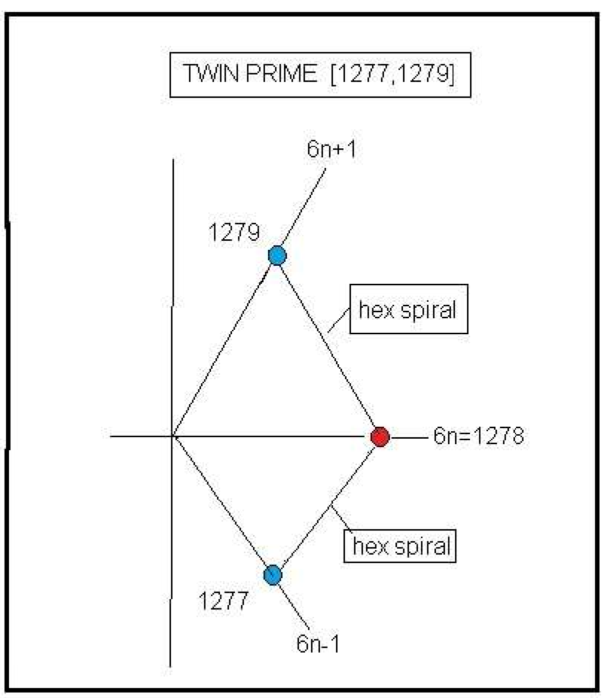
It yields the twin primes-

[1230,1278,1290,1302,1320] ±1

Thus the closest twin prime to 1260 is [1277,1279]. Here is a view of 6(213)±1 in a hexagonal integer diagram-



Another observation which follows from the original hexagonal integer spiral above is that there can be no triple primes p, q, and r differing from each other by two units each.

LARGE TWIN PRIMES:

The spacing between twin primes, like that for primes, tend to increase with increasing n. Thus for very large n the search for twin primes can be quite time consuming. To get partially around this difficulty one can always use the finite product of irrational numbers N of chosen digit length and then adjust n to make 6(N+n)±1 a prime number. That is ,we have the twin prime-

[6(N+n)-1,6(N+n)+1]

with the number of digits taken for N determining the size of the twin prime.

Let us demonstrate by letting N= evalf(sqrt(2),20) with the decimal point removed. That is we start with the 20 digit long number-

N:= 14142135623730950488

Next applying the search program-

for n from -10 to 10 do({6*(N+n), isprime(6*(N+n)-1), isprime(6*(N+1)+1)})od;

we find n=2 and M=6*(N+n)= 84852813742385702940. So we have the 20 digit long twin prime-

[84852813742385702941, 84852813742385702939]

An even larger twin prime can be generated by expanding the number N=sqrt(5)*Pi^2/36 to 60 digits and then removing the decimal point. This produces the large number –

N=613030731996302925287329145627225718455075062372349708283087

Searching about the number 6(N+n) we find a twin prime for n=-374.It reads-

3678184391977817551723974873763354310730450374234098249696278±1

This number has never seen the light of day before. Without knowledge of the number N there is no way one could come up with this last sixty digit long twin prime. Even larger twin primes can be found by having the finite length series for products of irrational numbers taken to more than sixty places. This will, however, require much longer searches for n to make 6(N+n)±1 primes**.**

CONCLUDING REMARKS:

We have shown that one can use large Ns using longer expansions of irrational numbers to generate even larger twin primes. Such primes as well as standard prime numbers can be stored as a code based on a chosen N expansion. This suggests the use of irrational number products as a convenient way to store and transmit such numbers openly in cryptography. For example, N=exp(2)*sqrt(15)/(Pi^2*ln(2)) expanded 80 digits and eliminating the decimal point , reads-

N=41832073019584420992209942932918033606941824653754323775362094039485106516070472

Next carrying out a search over the range -50<n<50 produces a prime for n=-25. The new eighty digit long prime number is-

p=250992438117506525953259657597508201641650947922525942652172564236910639096422683

To actually find its corresponding twin prime would take quite a bit more searching. However to find another prime q using a different N would be relatively easy. Hence public keys involving semi-primes N=pq in cryptography could be stored by just transmitting these two primes disguised as different Ns.

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September 24, 2022
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