GENERATION OF LARGE PRIME NUMBERS

INTRODUCTION:

The standard method for generating large semi-primes starts with a long string of integers 0 through 9 arranged in a random order and then adjusted by adding or subtracting a few more integers until the new string produces a prime number. To demonstrate the standard method consider the string of numbers-

N=58231042

This number is clearly not prime since N mod(6)=4 and hence does not have the form N mod(6) equal to 1 or 5 as demanded by our earlier defined hexagonal integer spiral. However, we can run the MAPLE search program-

```
for n from -10 to 10 do {n,58231042-3+n,isprime(58231042-3+n)}od;
```

to find n=-2 and the prime N=58231042-3-2=58231037.

It is our purpose here to discuss the details of a new way to generate large primes based upon the earlier found fact that all primes five or greater have the form 6n±1 and that the initial string is easy to obtain by looking at a string of desired length involving the product of several irrational numbers. A distinct advantage of this method is that such primes allows us to store the prime number in abbreviated form and that the number of search trials are reduced by a factor of three compared to the random number approach.

GENERATION OF THE INTIAL STRING:

One knows that there an infinite number of functions f(x) which can be expanded out as infinite series and evaluated for given values of x. Typical series for f(x) at fixed x=k are-

exp(1)=1+1/1!+1/2!+1/3!+

sin(1)= 1/1!-1/3!+1/5!-1/7!

cos(1)=1-1/2!+1/4!+1/6!+

sqrt(2)=1+1/2-1/8+1/16-

ln(2)=1-1/2+1/3-1/4+1/5-

Combining some of these constants, we find the 30 digit long string-

F=sin(1)*exp(2)/sqrt(10)= .615641692371541263389576987901

Removing the decimal point before the initial 6 produces the 30 digit long string –

N= 615641692371541263389576987901   with N mod(6)=3

To make this number a prime we add -2+6*n. This produces the 30 digit long prime-

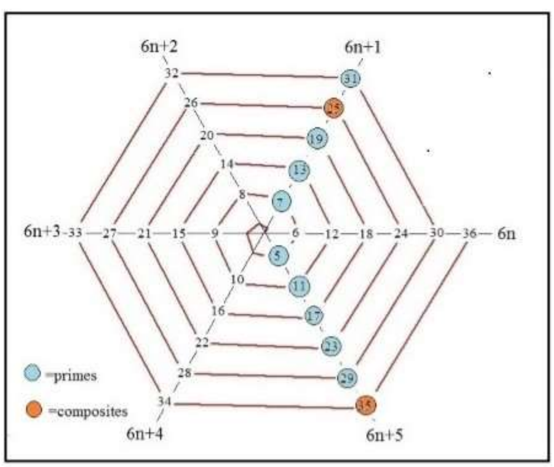
P=N-2+6(-1)= 615641692371541263389576987893

when n=-1.The term -2+6n added to the string N follows from the fact that N mod(6)=3 but all primes must lie along the radial lines 6n+1 or 6n-1 in a hexagonal integer spiral. The prime spacing along one of these two radial lines are separated by multiples of six. The size of the prime number is controlled by the number of terms used in the Taylor expansion of F. To test that the above P is indeed a prime one needs to simply evaluate isprime(P). It will confirm things by stating “true”.

ADJUSTMENT TO FINAL PRIME NUMBER FORM:

After choosing the string N of desired length based upon the product F of several chosen functions at given x, we first check out F mod(6). It can have the values 0,1,2,3,4,or 5 corresponding to one of the six radial lines shown on the following hexagonal integer spiral-

HEXAGONAL INTEGER SPIRAL



One first needs to move the string over to the prime radial lines 6n+1 or 6n+5 depending upon which form of the two possible prime forms one wants to find. Then add 6n to this to get-

P=N+m+6n

Here m is the number which brings N to either a mod(6) form of 1 or 5. So if N mod(6)=4 we need m=-3 for a 6n+1 prime or m=1 for a 6n+5 prime.

Let us demonstrate things using the fifty element string based on-

F=exp(-1)*sqrt(2)/ln(5)

It reads-

N=32325577209502409472248395634473389900503015251577

after removal of the decimal point and has N mod(6)=5. So we have 6n-1 primes given by P=N+6n and 6n+1 primes given by P=N+2+6n for 6n+1 primes.

To get the prime nearest N of the form 6n-1 (or 6n+5) we use the search program-

```
for n from -10 to 10 do (n,N+6*n,isprime(N+6*n)}od;
```

This produces the nearest prime-

P= 32325577209502409472248395634473389900503015251751

at n=29. To get the nearest prime of the form N+2+6*n, the search yields-

P= 32325577209502409472248395634473389900503015251399

at n=-30.

It should be pointed out that in picking N, the string of interest could be taken as any sub-string. Let us demonstrate for F=sqrt(3)*exp(2). It reads-

12.798220583324570798391334360603208201 20

From this number we can choose to drop all elements before the sixth following the decimal point and have the sub-string consists of twenty digits. It produces-

N=58332457079839133436 with N mod(6)=0

Here a search produces the nearest primes -

P= 58332457079839133419 and P=58332457079839133453 with the first having P mod(6)=1 and the second P mod(6)=5 (or -1)

STORAGE OF LARGE PRIMES:

One of the major advantages of using the F approach for finding large primes as opposed to the random number method is the ability to store very large primes in abbreviated form. For example, starting with a string generated by-

F=sqrt(2)*ln(3)/(cos(1)*sin(2))
=3.16239902401238146805287835538314830532249894 68859

We can pick out a sub-string starting after the 4th place to the right of the decimal point and then extending twenty-five points to the right. This yields-

N=9902401238146805287835538 with N mod(6)=0

To generate a 6n+1 prime we then have-

P= N+1+6(3)= 9902401238146805287835557 with P mod(6)=1

Combining these facts, we can then store P in the abbreviated form-

F=sqrt(2)ln(3)/(cos(1)*sin(2)) with P=F(5,25)+1+6(3)

This result says our sub-string starts with the fifth element to the right of the decimal point and extends 25 elements to the right from there. Next m=1 puts us on the 6n+1 radial line with n=3 producing P.

Another example is-

F=sqrt(3)*exp(-2) with the 6n+1 prime P=F(5,10)-2+6(1).

Evaluating things we get P=7586622541 . Note here that P mod(6)=1 and isprime(P)=true.

CONCLUSIONS:

We have shown that any string of numbers of chosen length can be converted to a prime number by adding the adjustment m+6*n. Here m is an integer 0-5 which places N onto either the 6n+1 or 6n=5 radial line in a hexagonal integer spiral and 6*n the distance one must move up one of these radial lines in order to have P=N+m+6m be a prime. Such Ps can be made relatively secure by choosing a complicated form for F, so that they can be transmitted openly in the form P([a-b],m,n] between friendly partners who know which F is being used.

U.H.Kurzweg
August 1, 2020
Gainesville, Florida