NUMBER FRACTIONS,TWIN PRIMES, SUPER-COMPOSITES, AND SEMI-PRIMES

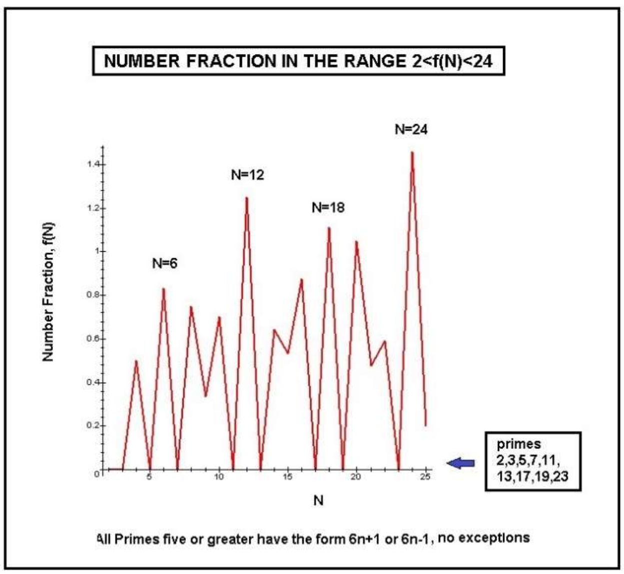
Several years ago while studying Number Theory, we came up with a new fraction defined as-

$$f(N)=\{sum\ of\ all\ divisors\ of\ N\ minus\ (N+1)\}/N$$

In terms of the sigma function σ(N), which represents the sum of all integer divisors of N, we have the new unique point function-

$$f(N)=\frac{\sigma(N)-N-1}{N}$$

which we have termed the **number fraction**. This integer ratio has the unique property that f(N) vanishes whenever N is a prime and slowly increases in locally averaged value as N increases. A plot of the first 25 of these follows-



Notice the vanishing of f(N) at all primes. Furthermore the f(N)=0 occur only at N=6n±1, whenever N=5 or greater. No exceptions to this rule have been found inside or outside this range. So, for example, the Mersenne Prime –

$$N= 2^{61}-1= 2305843009213693951= 6(384307168202282325)+1$$

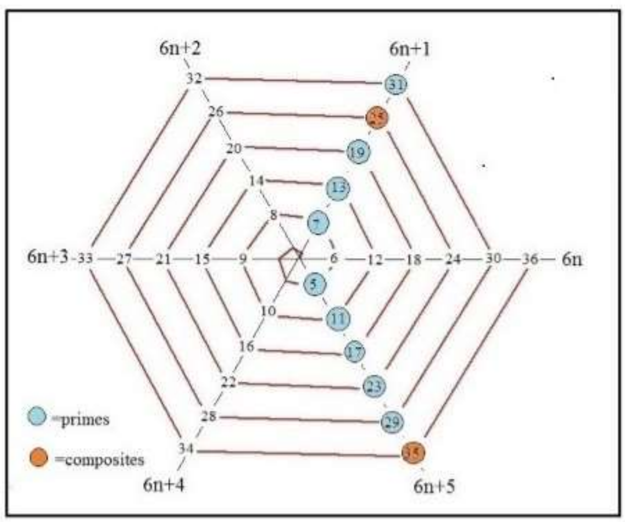
Since the vanishing of f(N) indicates a prime, we can also say that –

σ(N)=N+1

is a necessary and sufficient condition for a number to be prime. So N=375981413 has σ(N)= 375981414 and hence N is a prime.

A most interesting result which may be drawn from the discussion above is that one can draw a new type of integer spiral where the integers are located at the intersections of an Archimedes Spiral r=(3/π)θ and six radial lines 6n, 6n+1, 6n+2,6n+3, 6n+4, 6n+5.The Archimedes spiral between neighboring integers is replaced by straight lines. The resultant pattern is shown-

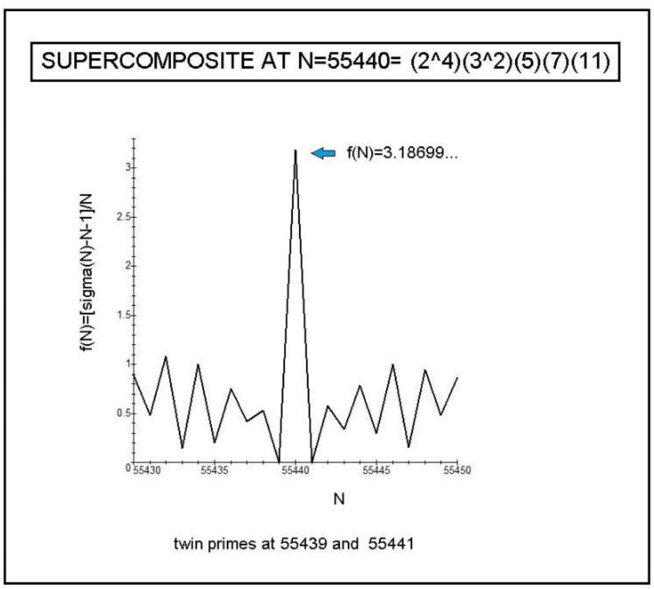
HEXAGONAL INTEGER SPIRAL



We have called this pattern the Hexagonal Integer Spiral. Unlike for the standard Ulam Spiral where primes appear to be scattered quasi-randomly, the present spiral allows primes above N=3 (shown in light blue) to only fall along the two radial lines 6n±1. This fact allows the effort of factoring large semi-primes to be reduced by a factor of three. In Number Theory language, all primes five or greater must have N(mod6)=1 or 5. Semi-primes N=pq must also fall along the same radial lines 6n+1 or 6n-1. They will be found in the gaps between the primes . Thus the number N=35 is a semi-prime with 35(mod6)=5.

Next we look at twin primes. These are prime numbers which differ from each other by two units. An example is [17,19]. The average value of these two numbers is 18=6(3). It turns out that the average value of all twin primes must be multiples of six without exception. In addition 6n±1 must be primes. Such twin primes are easy to spot on the left of the above spiral figure. Two extra twin primes shown in the spiral are [11,13] and [29,31]. Their average is 6x2 =12 and 6x5=30. Note that a 6x4=24 average fails to produce a twin prime since 25 is a composite. It is believed that there are an infinite number of twin primes when all positive averages 6n are considered. At the same time, it is not possible to produce Triple Primes since a third prime is seen to be impossible with the required two separations between each of three primes.

Next we look at case where f(N) has large values above unity. Examples from the above number fraction graph show these include N=12,18,and 24. We call such numbers with f(N) greater than one super-composites. They seem to be characterized by factors with high powers for the low prime number factors. Here follows an example of a super-composite-



Note the towering over its neighbors. In this particular case we also have twin primes as direct neighbors. This happens infrequently. The factoring of N here involves larger powers of the lower primes 2 and 3. That seems to be the case in general. That is, super-composites have the form-

N= (2^a)(3^b)(5^c)….. with a>b>c

This makes sense since the lower the primes being used the more the number of divisors of N will be possible. Another super-composite N is created by (2^12)(3^8)(5) where N=134369280. Its immediate neighbors are not primes since the number fraction remains finite but very small at N±1.

A final point to note from our Number Theory studies is that the semi-prime N=pq are given by the formula-

$$[p,q]=S\mp sqrt(S^2-N) \quad with\ S=\frac{[\sigma(N)-N-1]}{2}=\frac{Nf(N)}{2}$$

Since σ(N) is given by most advanced mathematics programs to at least 40 digits, semi-primes of this size can be factored in split seconds. Here is one such example-

N= 4605610681630941075574302042190945330849=

5279126753098214761 7 x 87241903766148436097

In this case σ(N)= 4959950895840793250180164495937543607888 and f(N)=[σ(N)-N-1]/N.

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