ADDITIONAL PROPERTIES OF THE NUMBER FRACTION

It is well known that any positive integer can be represented by the product of primes taken to specified integer powers. That is-

$$N=2^a3^b5^c7^d\ldots \quad \text{and} \quad 600=2^33^15^2$$

Knowing this fact, we are now in a position to evaluate the number fraction f(N) , discovered by us about a decade ago, for any integer using the definition-

$$f(N)=(sigma(N)-N-1)/N$$

and letting N=p_n^a , with p_n being a prime. Here the numerator on the right of the f(N) definition js just the sum of the divisors of N with N and 1 excluded. Here s.

igma is the divisor function σ of number theory.

Working out the values of f(p_n^a) we have-

$$f(p)=0$$

$$f(p^2)=1/p$$

$$f(p^3)=(1+p)/p^2$$

$$f(p^4)=(1+p+p^2)/p^3$$

from this follows the relatively simple form -

$$f(p_n^a)=\frac{1}{p^{a-1}}\sum_{k=0}^{a-2}p^k=\frac{[1-p^{1-a}]}{(p-1)}$$

The right hand term in this inequality follows from the geometric series.

Consider next the number N=2^6*3^2*5^1=2880. We know from an earlier article on this page (Aug.3, 2023) that the sigma function of N satisfies-

$$\sigma(2880)=\sigma(64)\sigma(9)\sigma(5)=9906$$

Also we have the relation-

$$\sigma(N)=1+N+Nf(N)$$

So after a little manipulation, we arrive at the number fraction result-

2880 f(2880)+2881=[64 f(64)+65][9 f(9)+10][5 f(5)+6]

But we know from the above table that f(64)=31/32, f(9)=1/3, and f(5)=0. So we find-

f(2880)=[-2881+[127][13][6]]/2880=2.4392…

We can generalize this result to-

$$\mathbf{f(N)=\{-(N+1)+\prod[p_n^a\, f(p_n^a)+p_n^a+1]\}/N}$$

Here p_n is the nth prime. It is taken to the power 'a' which varies with n. This result becomes very simple if the 'a' remains one. Let us demonstrate for N=77. Here we have –

f(77)={-78+[8][12]}/77=18/77

A more difficult evaluation occurs for N=720=2^4*3^2*5^1. Here we find-

f(720)={-721+[16*f(16)+17][9f(9)+10][5f(5)+6]}/720={-721+[31][13][6]}/720

=1697/720=2.35694…

Note that the value of f(N) , when N=pq is a semi-prime, has the relatively simple form-

f(pq)={-(pq+1)*[p+1][q+1]}/pq=(p+q)/pq

We a can use Nf(N)=p+q together with N=pq to find the value of the two primes p and q. For p we have the quadratic p^2-pNf(N)+N=0. The above case of N=77 is such a semi-prime.

We have shown that one can take any positive integer and find its number fraction f(N) using the power expansion of the number N as the known prime values taken to the ath power.

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