NUMBER FRACTION FOR PRIME AND COMPOSITE NUMBERS

If one looks at the first twenty five integers in ascending order one has the sequence-

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

In this sequence there are certain integers , namely, -

1 2 3 5 7 11 13 17 19 23

which are divisible only by themselves and one. With the exception of 1 these represent the prime numbers. All the other numbers are referred to as composite numbers and are characterized by being divisible by numbers other than just 1 and N. One can define the degree of compositivness by adding up all the factors of a number excluding 1 and the number N itself and then divide the sum by the number N. This produces what I call the Number Fraction-

$$f(N)=\frac{[\sum_{k=1}^{m}(f_k)]-(N+1)}{N}\quad where\ f_k\ are\ all\ the\ factors\ of\ N$$

This number relates to the familiar sigma function σ(N) of number theory as-

$$f(N)=\frac{1}{N}\{\sigma(N)-(N+1)\}$$

with –

$$\sigma(N)=sum\ of\ all\ factors\ of\ N=\prod_{i=0}^{r}\frac{(p_i^{a_i+1}-1)}{(p_i-1)}$$

where p_i is the ith prime factor of N and a_i its exponent.

As an example, consider the number fraction of the number 12. Here we have f_{12}= (2+3+4+6)/12=15/12=1.250.. while N=7 has f_7=0. Using this quotient, one can characterize any integer by its f_N value. When f_N is zero we have a prime number while a fraction different from zero will indicate a composite number. The simplest way to establish the value of f_N is to use the computer operation –

with(numtheory); followed by divisors(N);

Thus N=56 produces divisors(56)= {1, 2, 4, 7, 8, 14, 28, 56} From it we have f_{56}=(2+4+7+8+14+28)/56=63/56=1.125 . We can automate these calculations for f_N by the MAPLE command-

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f:=evalf((add(i, i=divisors(N))-(N+1))/N);
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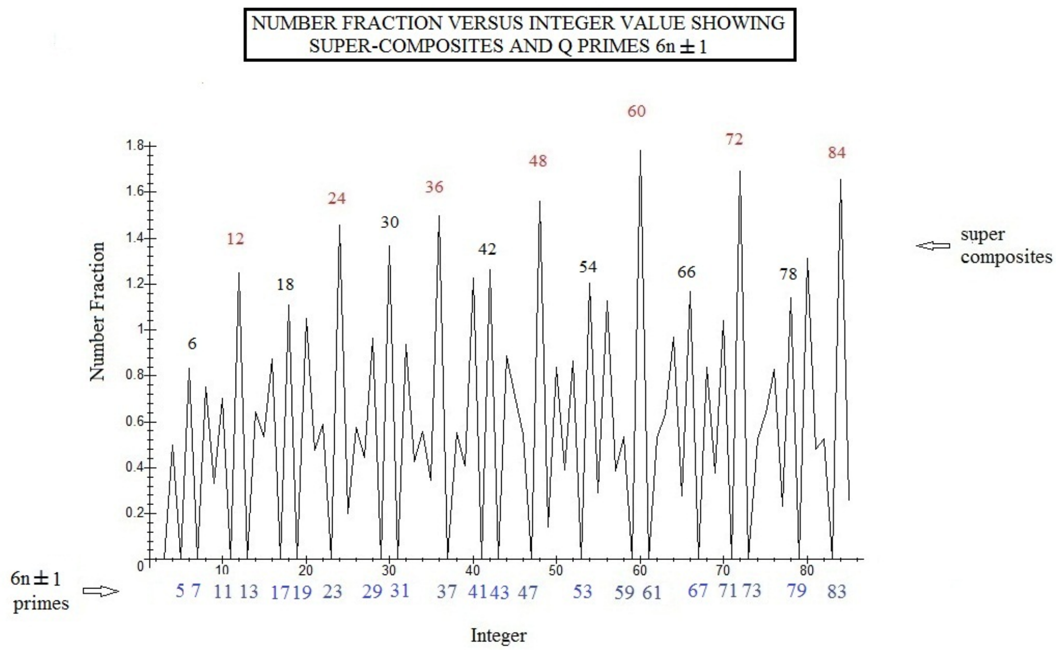
Using this last command, we have taken the first 85 integers and calculated their number fraction f_N. The result is presented in the following graph-



We observe that a number fraction of zero implies a prime number and the more f_N departs from zero the more composite its form. We use the term super-composite when f_N>1.0 or greater. These super-composites have a large number of factors and seem to be most common under conditions where N is a multiple of 6 and especially when equal to an even multiple of six. This is a very interesting observation which seems to have escaped people's attention. One notes that the number immediately preceding or following such supercomposite numbers N tend to often be prime numbers. So one can make the statement that-

$Q=6 \cdot n \pm 1$ will in many, but not all, instances be a prime number

The origin for this observation is the fact that f_N in $1<N<85$ takes on an average value about 0.6 and when f_N gets much above this value(as it does when N represents multiples of six), there is a strong statistical tendency for the next or preceding number to have $f_N=0$ and hence be a prime. We call this prime subclass the Q or 6n±1 primes. They are marked in blue in the graph above and constitute essentially all the primes in the range $5<N<85$ shown. Note that Q cannot generate the primes N=2 or N=3 but does generate all others in the range.

Let us look at some of these primes. Starting with the integer N=6·10=60, we have the divisors(60)= {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60} so that f_{60}=(2+3+4+5+6+10+12+15+20+30)/60=107/60=1.7833 ..
This value of f_N indicates we are dealing with a large super-composite and hence we suspect that Q=60-1=59 and Q=60+1=61 may be primes. This is indeed what they are. Here we actually have twin primes (or double primes) defined as two primes lying next to each other.

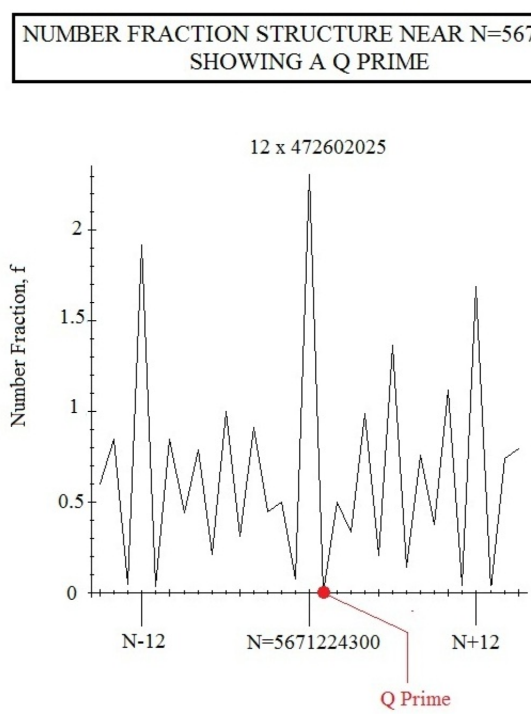
Take next the case of N=57375444=6*9562574. Here we suspect that N-1 or N+1 may be primes. A check using the isprime operation in MAPLE shows that Q=57375443 is indeed a prime number but Q=57375445 cannot be since it is obviously divisible by 5.

Since the spacing between primes increases with increasing N it is likely that having a prime lie next to an N with a large number fraction becomes less likely but should still be possible. Here are three more examples of such primes-

Q=6·1134-1=6803,   Q=6·477582-1=2865491,   and

Q=6·851345962844758-1=5108075777068547

We can also just pick a large multiple of 6 such as N= 6 x 945204050=5671224300 and then draw a graph in the neighborhood of this number. It produces the following pattern-



Note the Q prime occurring at N+1 with no other primes present over the range shown. Also note the larger values of f at the super-composite locations of N, N+6, N+12 , N-6, and N-12 and the approximate symmetry of the fs about N.

The density of the Q primes is much higher than say the Mersenne Primes $M=2^{2n+1}-1$ or Fermat primes $F=2^{\wedge}2^{n}+1$. There should be an infinite number of such Q primes since these numbers vary linearly with N and the number of integers is infinite.

One can also play around with Q=6n±1 to find sets of twin primes. All that it is necessary to do is to vary n in 6n±1 and then look at the value of n for which both 6n-1 and 6n+1 are simultaneously prime. Some of the twin primes are-

[71, 73], [599, 601] and [3539, 3541]

Just to show you how easy it is to generate the subset of twin primes based on Q=6n±1, we searched the immediate region of N equal to one billion. Using the one line program-

```
for n from -4 to 4 do {n, isprime(1000000008+n+1), isprime(1000000008+n-1)}od;
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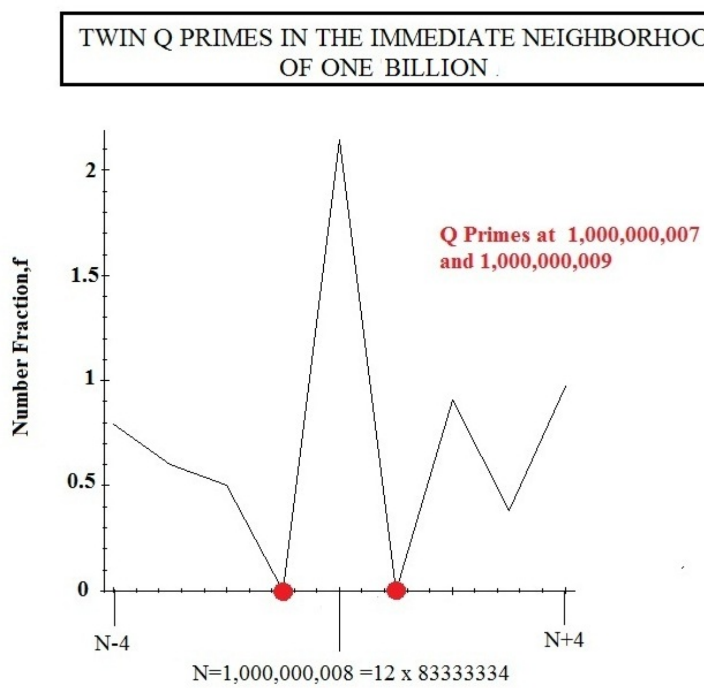
It produces the twin prime Q= 1,000,000,008±1 as shown in the accompanying graph-



The evaluation of f_N for larger N can become quite cumbersome if done by hand. Fortunately there are some analytic and numerical approaches which can be used to accelerate the process. For example, if we concentrate on the number $N=12\cdot 2^n$, we find $f_{12}=15/12$, $f_{24}=35/24$, $f_{48}=75/48$ and $f_{96}=155/96$ from this we note the denominator is just N and the numerator doubles each time plus 5. A bit of manipulation then produces the analytic result -

$$f_{12\cdot 2^n} = \frac{5}{12}[4-\frac{1}{2^n}]$$

From this we see that the number fraction for $N=12(2^n)$ is high and approaches 1.666.. as n becomes infinite.

Using our PC and the above command for finding f[N], we can quickly evaluate f_N for any integer N. For example, the super-composite N=36000 has f_N =91763/36000=2.54897. It predicts a prime number at 35999. Note that even for such a large N the number fraction is just f_N= 2.55 and so not much larger than values at lower integer values. Two interesting sidelights we have observed in this procedure are that when $N=2^n$, one finds the number fraction to be $f_N=1-(1/2^{n-1})$ so that this type of N can never be a prime number or a super-composite number. Indeed the number fraction approaches an intermediate value near $f_N=1$ as n gets large. Also we observe when f_N is very near zero then N will typically be a semi-prime $N=p\cdot q$, where p and q are primes. The number N=25895469 is such an example. It's $f_N=(p+q)/pq=0.01697$ is very close to zero. Its prime number components are found to be q=59 and p=43891. This observation may be of interest in connection with public key cryptography

Since many primes are to be found in the immediate vicinity of a super-composite, it will generally not be necessary to find the explicit value of f_N for the super-composite. One can just go ahead and test the number Q=6 (random number)±1 for primeness. Very often this will produce a prime. Here is an example of a thirty digit prime-

Q=2 · 6 · (39546218967589211096438925991)-1=474554627611070533157267111891

found by this method. The twenty-nine digit number multiplying 12 in this case was chosen at random. The fact that Q is indeed prime follows from the one line MAPLE program-

```
isprime(12*39546218967589211096438925991-1);      true
```

One can search for still higher primes of the form Q by writing down the generalized formula -

Q=6(any random number+n)±1

and then vary n until Q becomes prime. This procedure is easy to automate via the one liner-

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for n from 1 to 100 do {n, isprime(Q)}od;
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Consider a Q defined by 12 x 2^{92}-1= 59421121885698253195157962751. It is prime. So are Q=12 x 2^{456}-1, Q=12 x 2^{622}+1, and Q=12 x 2^{2022}-1.

Also, if we take a random number such as –

5396727912264986126835723387220948562253132145491842319387562678117394323 15345

Then the search using n= 50 will produce a Q which is prime when taking -1 in the ± term. Thus-

Q=51748770150476073494717983352202868064665138274703758574590210783265075213740873187784739

is a prime number. Notice that a quick way to generate large random numbers is to simply take the integers composing an irrational number such as π , e, sqrt(2) or combinations thereof out to k places and then multiply this result by 10^k. Using π out to 98 places and then applying the above search technique we are able to show within a few seconds that-

Q=376991118430775188615517205993540346103660327925012698516993351076937968754345079835364179041054567

is a prime number. We are not aware of any other technique which will generate such large primes any quicker. We also point out that the Mersenne Primes M=2^p-1 and Fermat Primes F=2^2^n are also special cases but much rarer than all Q primes. The number of Qs in a given range will match the number of primes. For example, we find 15 primes of the form Q=6(1000+n)±1 in the range 5999<N<6121 while the fundamental theorem for primes predicts the total number of primes to be-

$$N_2/\ln(N_2)-N_1/\ln(N_1)=12.40$$

So we actually have more primes than the Fundamental Theorem predicts due to the fact that N must be very large for the theorem to hold. More accurately, we can go to our MAPLE math program and find that the 783rd prime equals 5987 and the 798th prime equals 61211. The difference is 15 in exact agreement with the observed number of Q primes in this range. The number of Q primes is thus just

equal to all the primes in the same range. This leads us to our second conjecture, namely,

All prime numbers above three can be represented either as 6n+1 or 6n-1.

In terms of modular arithmetic it says all prime numbers have N mod(6)=1 or 5. We have found no exceptions to this rule looking at primes as high as 100 digit length.

Here is another large prime constructed by the present approach-

Q=19416407864998738178455042012387657412643710157669154345625383472463125553826829396486486450272693849

It should be obvious which irrational number was used in constructing this prime. It will not be as obvious, if at all possible, to find which combination of π, exp(1) and sqrt(2) I used to generate the prime number-

Q=7246204774059971399674164769845042481605466793938086306719046316822839453072431484445173803

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