CONSTRUCTING NUMBER SANDWICHES

I our studies in number theory over the last couple of decades we have come up with several new concepts including the hexagonal integer spiral for prime number representations, a new prime number density formula valid over 23 orders of magnitude, and discovery of a new parameter termed the numberfraction . This number fraction is defined as-

$$f(N)=[sigma(N)-N-1]/N$$

, where sigma(N) presents the sum of all divisors of N. It has the interesting property that f(N)=0 , whenever N is a prime. We want in this article to present some further discussions concerning f(N) and in particular show how this function may be used to construct number sandwiches having the form-

[(6n-1)… (6n)…(6n+1)]   or   [f(6n-1) --- f(6n) --- f(6n+1)

where 6n-1 and 6n+1 represent prime numbers and f(6n) represents a super-composite characterized by having a large number of divisors and , hence, have f(6n)>1. Typical examples of sandwiches are [179-180-181] and [6089-6090-.6091] with the corresponding f(N) values of [0, 2.0277777, 0] and [0-1.837274-0], respectively. Note that the central terms in the f expansions are both greater than one. This fact allows us to call such numbers super-composites. These number sandwiches are defined as-

[prime-supercomposite-prime]

The actual construction of a number sandwich can be accomplished in two ways. The first is the easiest and starts with finding two primes differing from each other by two . Such prime pairs represent twin primes, They are discovered by graphing f(N) about a chosen value of N. Thus $p_1=5$ and $p_2=7$ would produce a super-composite with f(6)=(2+3)/6=5/6=0.8333 provided f(6)was greater than one. Here the value of f(6) is a little less than one and hence by definition does not quite satisfy the form of a super-composite. However , if we take $p_1=6n+1$ and $p_2=6n-1$ we have that N=6n will have the value –

$$f(6n)=[sigma(6n)-1-6n]/6n$$

subject to f(6n-1)=f(6n+1)=0. Starting with n=2, we find the first 10 number sandwiches up to n=25 to be-

n	f(6n)	sandwich
2	5/4=1.2500	11-12-13
3	10/9=1.1111	17-18-19
5	41/30=1.3666	29-30-31
7	53/42=1.2619	41-42-43
10	107/60=1.7833	59-60-61
12	61/36=1.6944	71-72-73
17	113/102=1.1078	101-102-103
18	19/12=1.5833	107-108-109
23	149/138=1.0797	137-138-139
25	221/150=1.4733	149-150-151

You will notice that the central number of the above sandwiches all are divisible by six and have f(6n)greater than one. This is to be expected for primes greater than three and in a hexagonal integer representation means that if one of the primes lies along the 6n+1 radial line the other must lie along the 6n-1 radial line. One expects that there are an infinite number of sandwiches with the central number being a super-composite with f(6n)>1. As we will show via examples below, the larger 6n gets the more prevalent the value f(6n) will be over its neighbors.

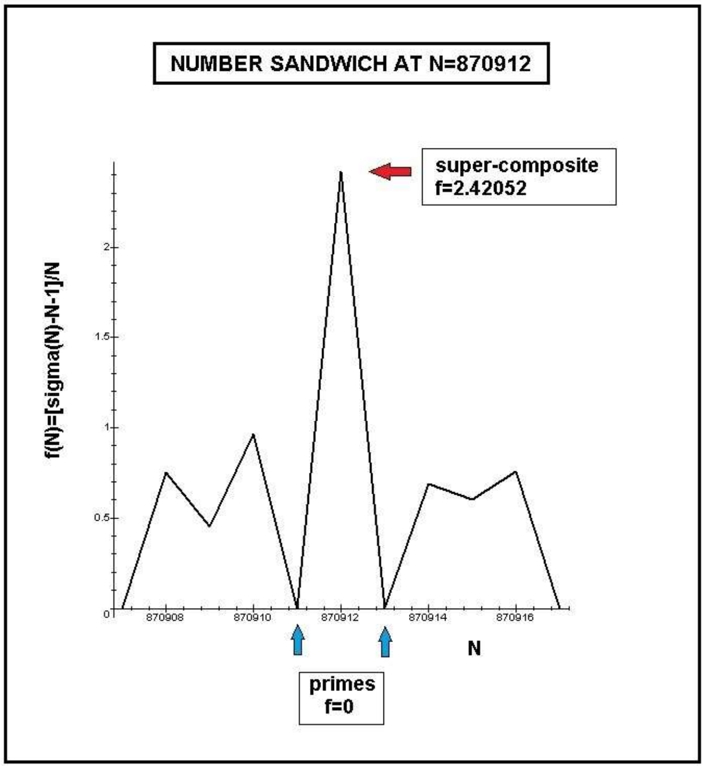
Let us demonstrate finding a number sandwich by the first approach. We begin by generating a computer plot of f(N) versus N about 870900 over the range 870900 to 870900+20. Inspecting the graph, we find the twin primes p_1=870911 and p_2=870913. Also the super-composite is s=870912=6(145152). This number is, as required, divisible by six and has the number fraction value of f=2.42052.. .>1. The number sandwich here becomes-

[870911-870912-870913]

in terms of integers and-

[0- 2.42052 - 0]

in terms of number fractions. A graph of this number sandwich looks as follows-



You can see how the super-composite towers above its neighbors.

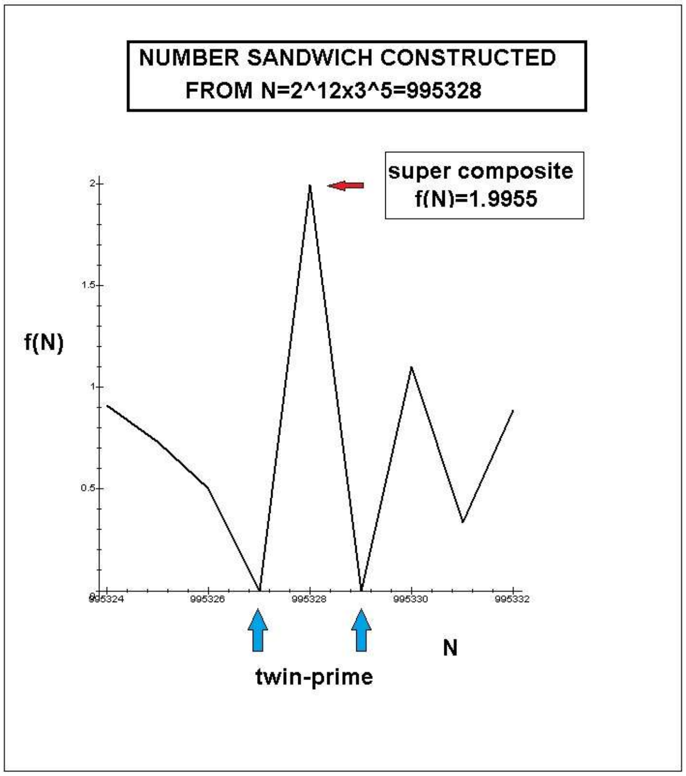
A second way to construct a number sandwich is to start with a large composite number divisible by six and then search over a range of N to see if the immediate neighbors are twin primes. To generate a large super-composite we use the product form-

$$N=2^a \cdot 3^b \cdot 5^c \cdot 7^d \ldots$$

with integers a>b>c>d. This assures that f(N) will be a large number as the number of divisors for this form are large. The number N must also be divisible by 6 to insure that f(N-1)=f(N+1)=0. Lets consider the super-composite -

N=(2^12)(3^5)=995328

,where f(N)=1.9955 A graph of f(N) in the immediate neighborhood looks as follows-



So we have shown that number sandwiches may be constructed in two different ways. In all cases the central super-composite must be divisible by six in order to have the possibility that f(N-1)= f(N+1)=0. There appear to be an infinite number of number sandwiches. The presence of N in the denominator of the f(N) definition appears to show that f(N) remains finite for all N. A proof that this is so needs to still be given. One of the largest f(N) values which we have been able to find using our home pc is f(N)=4.213 corresponding to the large composite number-

$$N=2^{72} \times 3^{57} \times 5^{34} \times 7^{23} \times 11^{11} \times 13^{7} \quad .$$

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