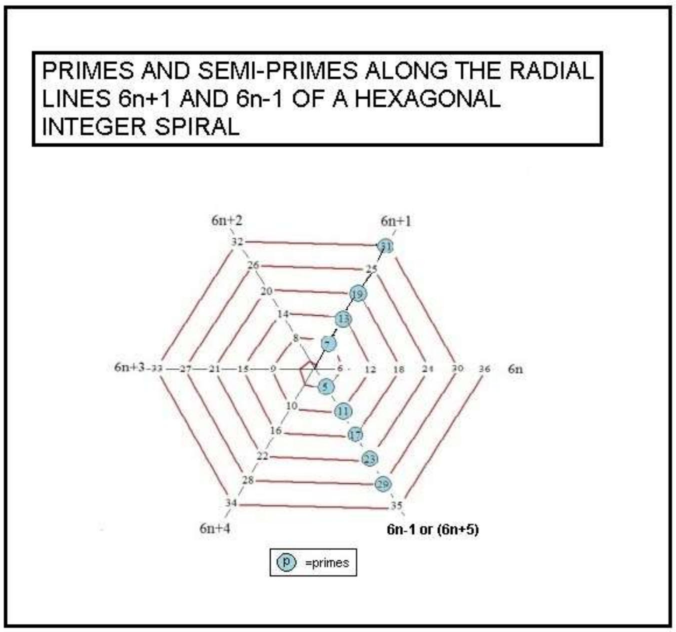
NUMBER TRIPLETS

When looking at all positive integers there are certain contiguous triplets such as –

[5-6-7], [17-18-19], [71-72-73], [239-240-241

where the first and last integer in each square bracket is a prime (and known as a twin prime) and the middle number tends to be a large composite number always devisable by six. We wish in this note to look at the properties of such number triplets and to evaluate the number fraction f(x) associated with each component.

We start our discussion by first looking at the earlier found hexagonal spiral integer picture which plots all positive integers and shows the primes as blue circles-



One sees at once that twin primes, corresponding to the first and last term in each square bracket, are designated by 6n+1 and 6n-1 provided n=1,2,3,…That is , all primes five or greater must have the form $6n\pm1$. This statement however is not sufficient since there are also some composites lying along $6n\pm1$. What is clear is that all Number Triplets must have the form-

$$T[n]= [6n-1, 6n, 6n+1]$$

Here both 6n+1 and 6n-1 must be primes. Also all twin primes must have an average value of 6n. The values of n for which this T(n) formula holds requires a solution of the one-line program-

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for n from 1 to 25 do({n,isprime(6*n-1),isprime(6*n+1)})od;
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It produces the affirmative answers

n= [1, 2, 3, 5, 7, 10, 12, 17, 18, 23, 25]

from which T[n] can be found.. As an example, consider n=23. It produces the triplet-

T[23]=[137,138,139]

Three additional number triplets, from an infinite number of additional ones, are generated by n=137, n=1248, and n=31245. They read –

T[137]=[821,822,823] , T[1248]=[7487,7488,7489] , T[31245]=[187469,187470,187471]

Notice that the middle term in the square bracket for all of the above Ts are divisible by 6. The middle term in each of the triplets is a large composite number as will be further discussed below.

Using the number fraction f(x), discovered by us about a decade ago, one can quickly determine the size of the three contiguous integers in a T[n]. The number fraction is defined as-

$$f(x)=[\sigma(x)-x-1)/x]$$

, where σ(x) is the sigma function from number theory. It represents the sum all integer divisors of the number x including 1 and x. The first important new observation concerning f(x) is that it vanishes when x is a prime. Thus in our integer triplets evaluations we have the first and third element in each triplet have f(x)=0. The middle term however can become quite large since there f(x)>1.

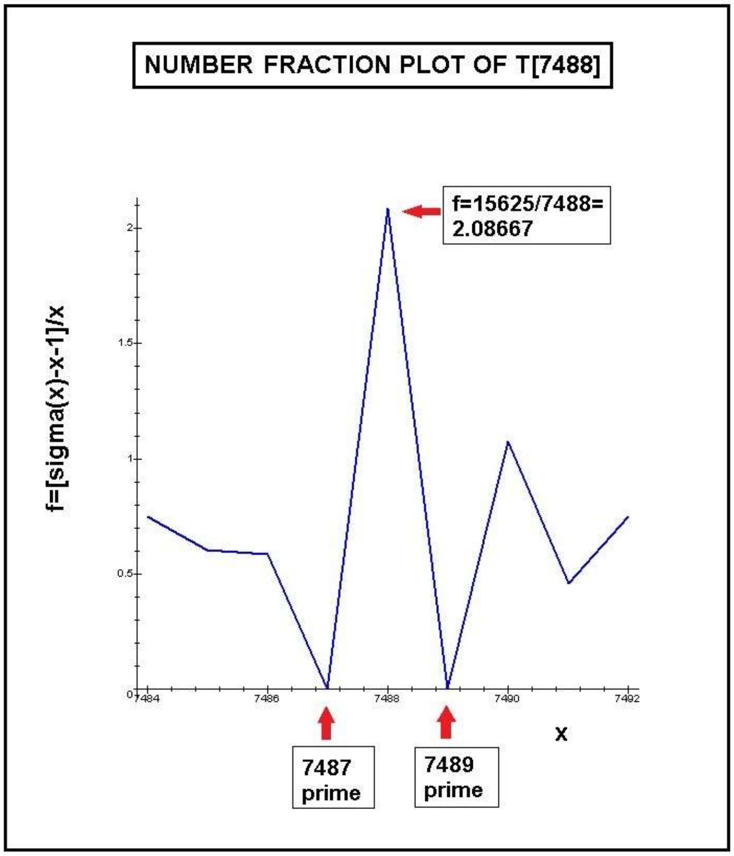
Let us work out the f(x) values for the middle term in T. For the middle term we have x=6n , so that-

$$f(6n)= [sigma(6n)-6n-1]/6n$$

Taking the special case of n=1248 we find-

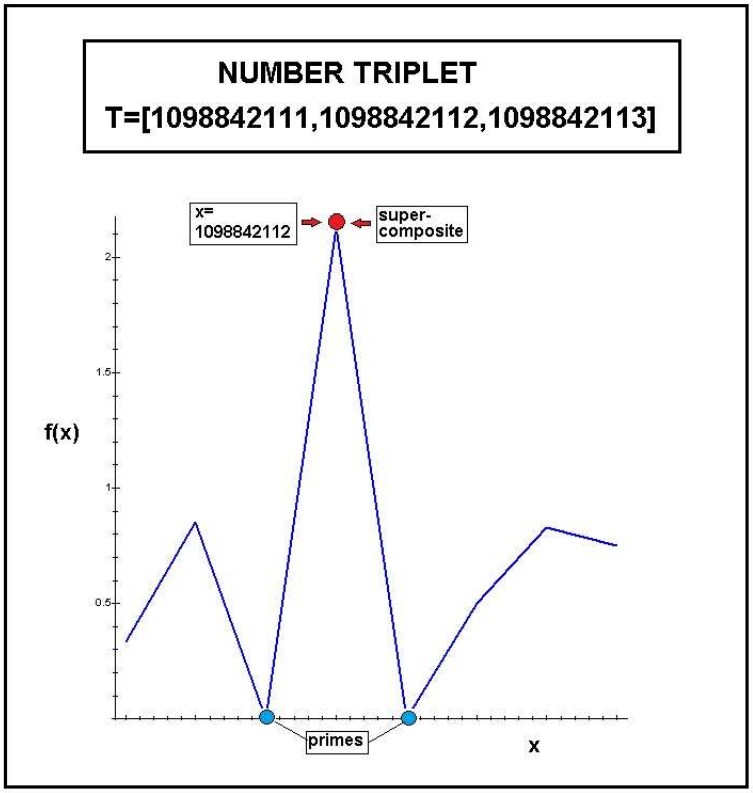
$$f(6n)=f(7488)=15625/7488=2.08667$$

So in terms of f(x) we have the triplet [0,2.08667,0]. Here the middle number is what we call a <u>super-composite</u>. It can be written in exponential form as $7488=2^6*3^2*13$. A picture of T in f(x) coordinates looks as follows-



We see in the graph a super-composite value of f=2.08667 bounded by the primes at 7487 and 7489. One notes the magnitude f(7488) exceeds the value of all of its near neighbors. We reserve the name super-composite for all those numbers where f(x)>1.

In looking at the exponential expansion of x= 7488 we have the equivalent form 2^6*3^2*13. From this form we know 7488 is divisible by six because of the 2*3 term. It suggests we look at another number triplet having a central element, say, x=2^16*3^6*23=1098842112. In this choice we put the power of 2 higher than that of 3 etc. This guaranties more divisors and hence sharper central peaks in an f(x) plot. For this last x we get the following f(x) versus x graph-



This graph looks very similar to the earlier case discussed above. Here we clearly have the number triplet-

T=[1098842111, 1098842112, 1098842113]

The peak of this number triplet lies at f(x)=2.1289… Note that the value of the central part of the number triplet grows only very slowly with increasing x. Nevertheless this central element is expected to approach infinity as x goes to infinity. The points $x\pm1$ of course remain as $f(x\pm1)=0$.

We have shown above how number triplets are derived. The first point is that the number triplet has its central element divisible by six. This means that in exponential expansion of x=6n contains a term 2*3. Next one tests if 6n-1 and 6n+1 yield primes. The one line program shown above in red will do this. The graphs of the Number Triplets, when plotted as f(x) versus x, all look quite similar with a large central peak followed by zero values for f in the immediate neighborhood. If the twin prime is known beforehand then the central term in a Number Triplet is just the average of the two primes.

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