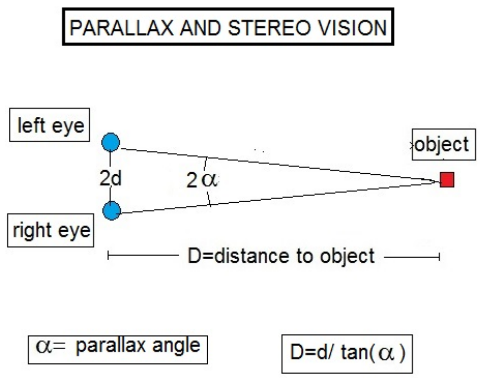
UNDERSTANDING PARALLAX AND STEREOPTIC VISION

If one looks at an object at a mean distance D away from one's eyes a stereoptic image of the object is formed on the retina which the brain interprets as a single image located at D. This is stereoptic vision involving two eyes separated by a distance of 2d from each other. The vertex angle of the isosceles triangle formed by connecting the two eyes and a point object is referred to as twice the parallax for the set-up. A schematic of the parallax angle α, is it applies to stereoptic vision, is given in the following sketch-



One sees that the parallax α equals half of the total angle between the object shown as a red square at D and the two eyes(in blue) separated by 2d. From the geometry we have

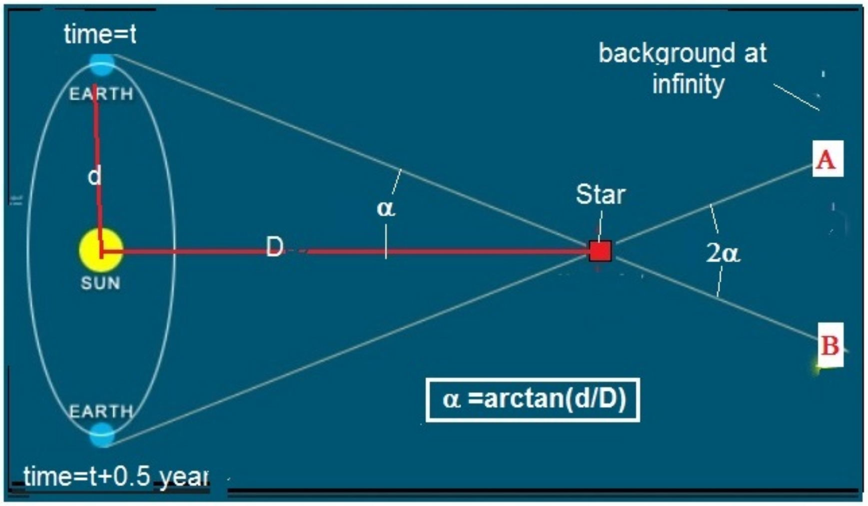
$$D=d/\tan(\alpha)$$

For humans 2d≈3inches so that an object at D=10 ft has a parallax of –

$$\alpha=\arctan(d/D)=\arctan(1.5/120)=0.012499..\text{ radians}$$

The eyes-brain combination can recognize this ten foot distance with ease meaning our stereoscopic vision clearly recognizes a 0.01 radian parallax. If only one eye is used all distance perception is lost. Artists use their thumb at the end of an outstretched arm to get the get the correct 3D perspective onto their 2D paintings.

The same parallax concept applies to measuring the distance of heavenly objects such as the Moon, Sun or nearby stars. The geometry for a star parallax measurement looks as follows-



Here the half-base distance is d=1AU=93 million miles and D is the distance to a nearby star. The parallax definition remains as α=arctan(d/D). The distance corresponding to a parallax of one second of arc(equal to 1/3600 deg) is called a parsec. The parsec equals about 3.26 light years. In a star distance measurement one takes a picture of it and its background of B and compares it with a second exposure on the same plate a half year later when the background will be A. The parallax is then simply determined by measuring the angle 2α between A and B.

Our nearest star after the Sun is Proxima Centauri. By parallax measurement using a double exposure separated by half a year shows a shift in image position relative to the star's background A-B corresponding of 4.2 light years. The nearby binary star system Alpha and Beta Centauri in the same constellation lie at a distance of 4.3 light years. These are our nearest neighbors. The separation from the solar system is a truly huge distances is seen by recalling that-

$$1 \text{ light year}=0.3066\text{parsecs}=9.46\text{x}10^{12}\text{km}=5.88\text{x}10^{12} \text{ miles}$$

To determine the distance from the Earth to the Sun one can use the transit of Venus across the face of the sun as seen from two different places separated by 2d on Earth. The parallax of arctan(d/D) here produces an earth-sun distance of-

$$D=148.597 \text{ million km}=92.96 \text{ million miles}$$

For the distance to the Moon one need not use the parallax approach to find D. It is simpler there to use the following circular orbit equation-

$$gR^2 = 4\pi^2 D^3 / \tau^2$$

, where R is the Earth radius, g the acceleration of gravity, and τ the orbit period of the Moon. Substituting in the values R=3960miles, g=9.8m/sec^2=32ft/sec^2, and τ=27days, produces-

$$D=384{,}400 \text{ km}=238{,}900\text{miles}$$

This number is easy to confirm these days by measuring the time it takes a radar or laser pulse sent from Earth to reflect off of the Moon and return to Earth. The round trip takes 2.6 light seconds, so the Moon is at a distance of $D=1.3x3x10^5=3.9x10^5$ km from Earth.

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February 14,2019
Gainesville, Florida