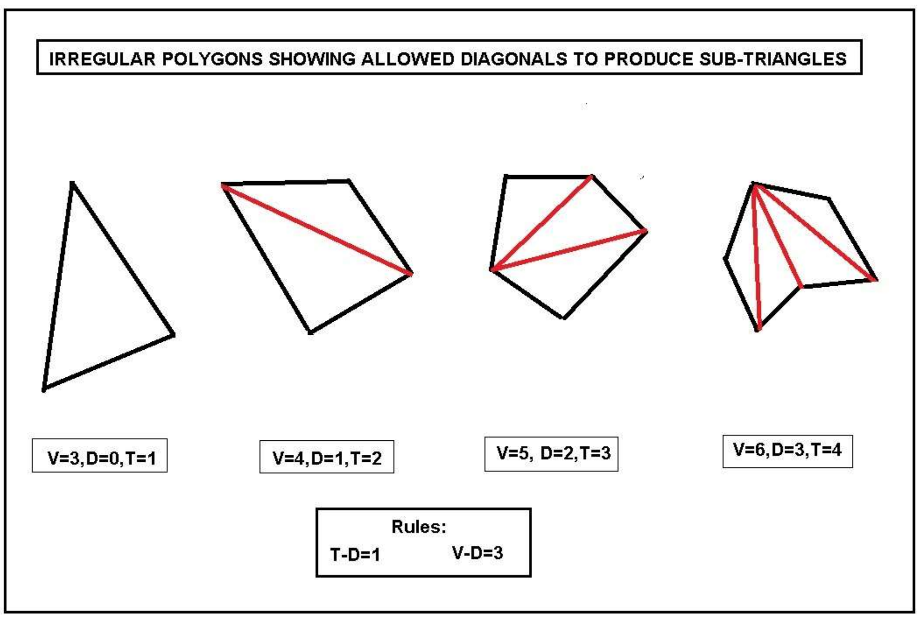
AREA OF N SIDED IRREGULAR POLYGONS

The area of any N sided polygon can be determined by adding up the area of sub-triangles formed by the appropriate placement of diagonal lines. The resultant area of these sub-triangles can be determined by use of either the Heron Formula or in many cases by the simpler approach of just using the formula (base x height)/2. We want in this note to find the numerical values of the number of vertexes V, the number of diagonals D, and the number of sub-triangles T needed to find the area of any N sided polygon. Also, once [V,D,T] are determined, the areas for several different irregular polygons will be derived.

We start by looking at the graphs of irregular of polygons with 3, 4, 5, and 6 sides. Here is a sketch of such polygons with drawn in diagonals shown in red-

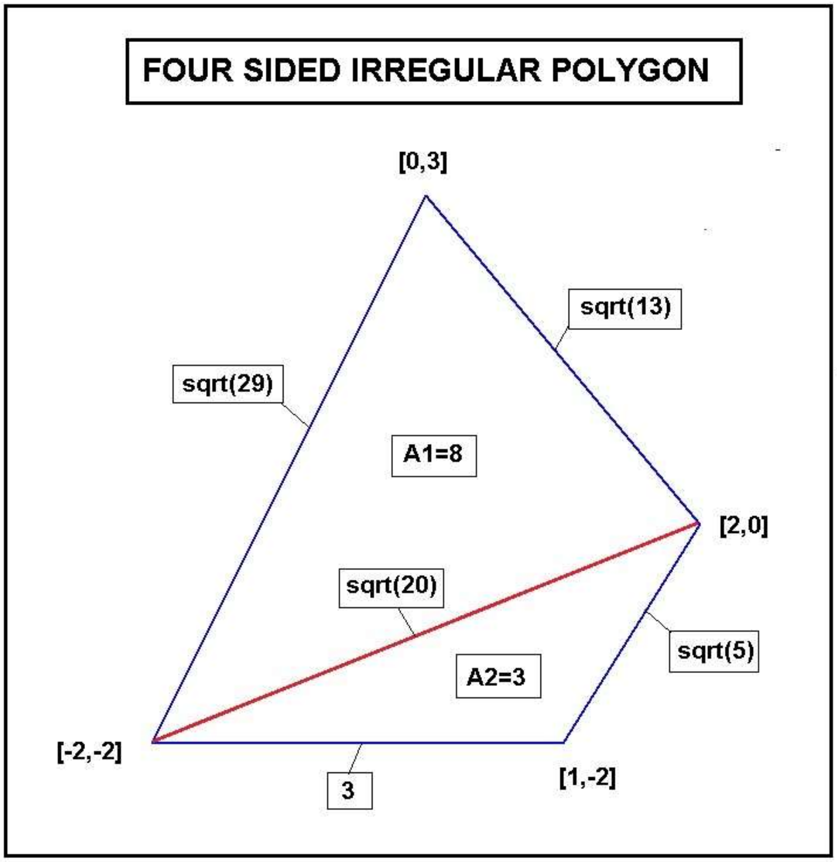


The first thing one notices is that the number of sides of a polygon equals the number of vertexes V. Also the number of diagonal lines D drawn from one of the vertexes to an opposite vertex equals D=V-3. Furthermore the number of sub-triangles T=D+1. So we have the rules-

$$T-D=1 \quad \text{and} \quad V-D=3$$

In some cases we may have one of the sub-areas be zero. This will be so if three vertex points fall along the same straight line. We will show an example of this below.

Let us begin some specific polynomial area calculations by starting with the quadrangle shown-



Here V=4, D=1 and T=2. We evaluate the two sub-triangles using the Hero Formula-

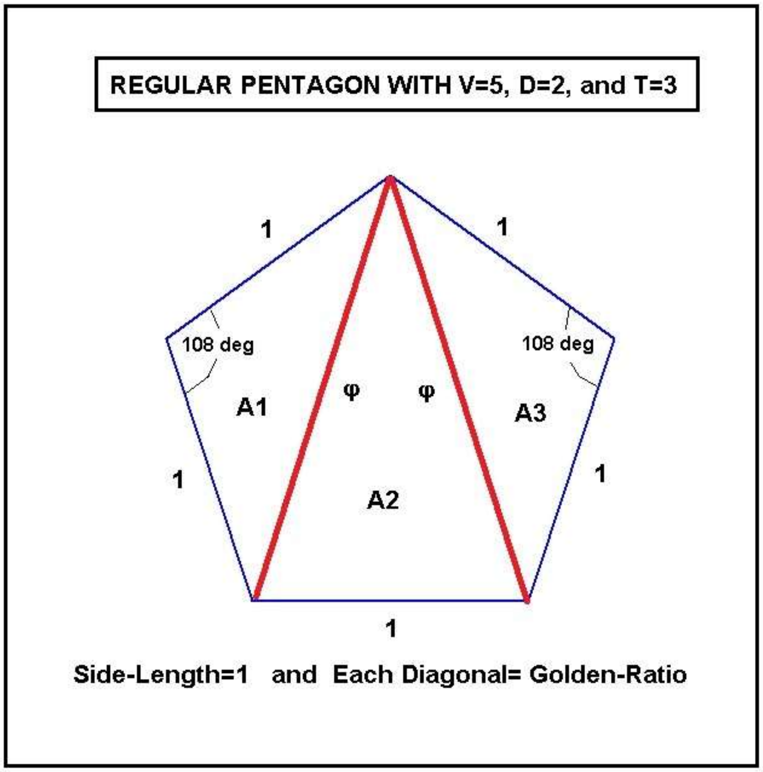
Sub-Area =sqrt[s(s-a)(s-b)(s-c)]

, with s=(a+b+c)/ the semi-parameter and a,b,and c being the side-lengths of a given sub-triangle. Here we are lucky in our choice of quadrangle dimensions since the Heron Formula yields the simple answer A1=8 and A2=3. So the total area of this four sided polygon becomes –

A=A1+A2=8+3=11

If we stretch the sides of this quadrangle by a factor of 100 so that the horizontal line of length 3 now becomes 300 ft, the area of the polygon increases in area by a factor of 10^4. The acreage of a piece of land of this shape would equal 110,000 ft^2 or 20.83 acres.

We consider next the area of a regular pentagon with sides of one unit length. Here we have V=5, D=2, and T=3. The two red diagonals originate at the top vertex and create three isosceles triangles as shown in the following-



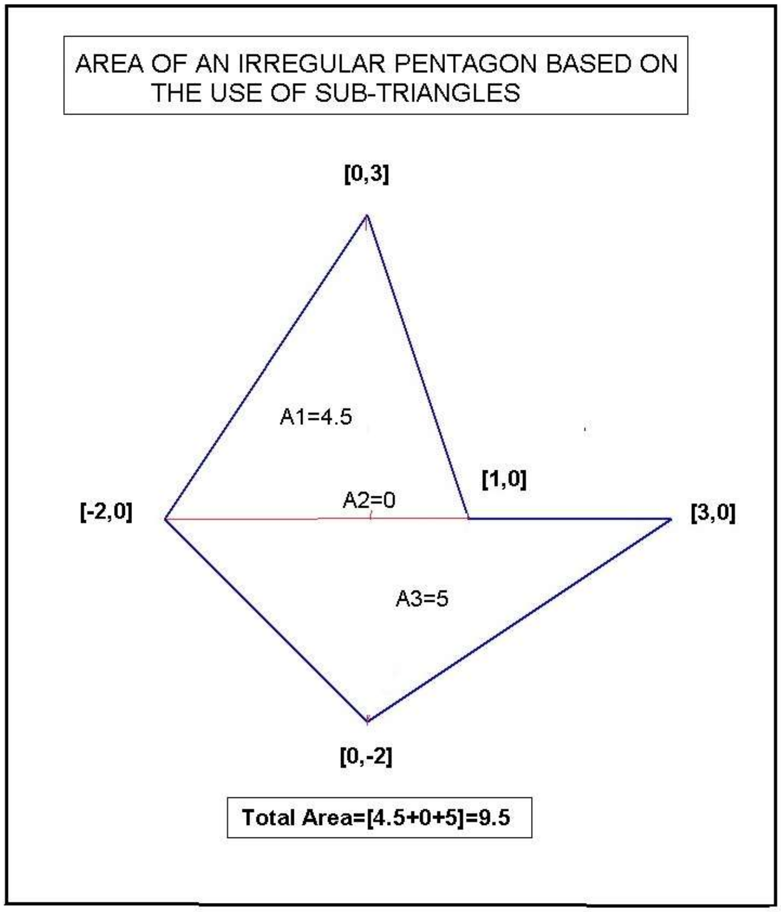
By the Pythagorean theorem one finds the diagonal length to be equal to the Golden Ratio φ =[1+sqrt(5)]/2=1.6180339… .This time we evaluate the sub-triangles without the use of the more complicated Hero's Formula. The height of the two equal side triangles equals h=sqrt(4-φ^2)/2 *and the* height of the middle isosceles triangle equals H=sqrt$[\varphi^2-(\frac{1}{4})]$.Thus-

A1=A3=$\left(\frac{\varphi}{4}\right)sqrt[4-\varphi^2]$ and A2=(1/2)sqrt$[\varphi^2-(\frac{1}{4})]$

Adding things together produces the total pentagon area of-

A=A1+A2+A3=$(\frac{\varphi}{2})sqrt[4-\varphi^2]+(1/2)sqrt[\varphi^2-(\frac{1}{4})]$= 1.720477… .

As a final polygon area to derive, consider the following irregular pentagon -



Here [V,D,T]=[5,2,3]. This time, because there are three vertexes falling along a single straight line, the middle area A2 is zero leaving use with just two non-vanishing triangles A1 and A3.Reading off the height of the remaining sub-triangles leads directly to the polygon area-

A=A1+A3=4.5+5=9.5 square units

We have shown via the above examples that the area of any polygon can be determined by first breaking up the figure via diagonals originating at one of the polygon vertexes, then evaluating the sub-triangle areas, and finally adding them together the get the final answer. One chooses the starting vertex for the diagonals in such a wary so as to minimize the evaluation process. Typically there will be only a single vertex which requires a minimum of work in finding the polygon area.

U.H.Kurzweg
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