A NEW TEST FOR PRIMES

In several recent notes we have looked at a type of prime number defined by-

$$Q = 6n \pm 1 \quad with \quad n = 1,2,3,\ldots$$

and found that these Qs can represent **all** primes greater than 3. Thus, we have the primes-

2574983 =6(42914)-1 , 63799137=6(10633190)-1 and 763035523=6(127172587)+1

However, there are also many Qs which do not correspond to prime numbers such as-

138941 =6(23157)-1 and 579331093= 6(96555182)+1

How does one distinguish between the two groups? The answer is that one needs to use the number fraction f_N (see our earlier note at http://www2.mae.ufl.edu/~uhk/NUMBER-FRACTION.pdf) . It will be different from zero for Ns which are composite but vanish when N is a prime. Thus we have the new prime number test -

Every prime number above 3 has N mod(6) equal to 1 or 5 and in addition it <u>must</u> satisfy the condition f[N]=[sum(divisors of N)-(N+1)]/N=0

This prime test tells us at once that the Fermat Number-

$$F = 2^{32} + 1 = 4294967297$$

is a composite. Here we have-

4294967297 mod(6)=5

so that the first part of the prime test is satisfied. Next carrying out a f_N calculation on F=4294967297, we find –

f_N:= evalf((add(i,i=divisors(N))-(N+1))/N) =0.001560211647

This differs from zero and hence F is composite and not a prime.

Fermat actually claimed that F was prime, but Leonard Euler first showed it wasn't. Euler spent several months over 200 years ago to actually factor F into the product F= (641)(6700417). Quite a feat for someone operating without the benefit of electronic computers.

Consider next the Mersenne Number-

$$M=2^{61}-1= 2305843009213693951$$

Is it prime or not? First we look at-

$$2305843009213693951 \bmod(6)=1$$

and so see that the number M is of the type 6n+1. Next we evaluate –

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fN:= evalf(( add(i,i=divisors(N))-(N+1))/N)
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It yields a zero value and hence M is prime. This fact was first established back in 1883 by Pervushin by a much more complicated and elaborate pre-computer approach.

As a third example, consider the odd number-

$$N=74074071$$

For this case we have-

$$74074071 \bmod(6)=3$$

This number is therefore not of the $6n\pm1$ type and hence it must be composite. As a final example consider the large number-

$$N:=342789032178923759$$

Here the computer commands-

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N mod(6);      f[N]:= evalf((add(i,i=divisors(N)))/N-(N+1)/N);
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yield-

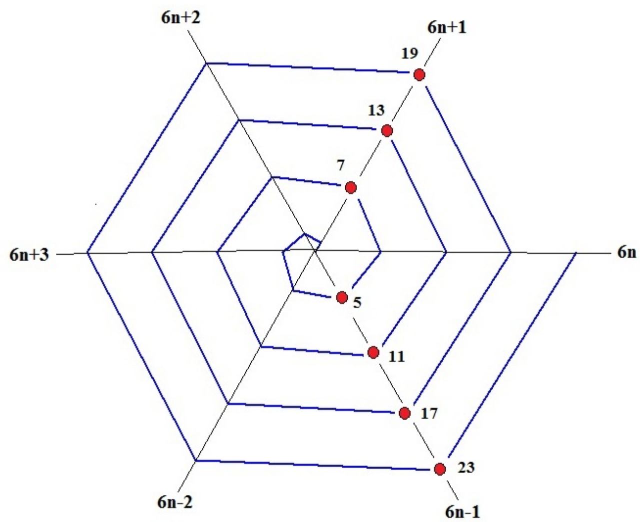
$$5 \quad \text{and} \quad f[N]= 0.2355078347$$

, respectively. That is, N is a composite number of the form 6n-1.

The above examples have shown that for a prime to exist it must not only be of the form $Q=6n\pm1$ but also have its number fraction f_N vanish. The number fraction calculation can become somewhat tedious as N gets very large.

Graphically we can represent all prime numbers via the following graph-

Integer Spiral r=n , $\theta = (\pi/3)\, n$ showing primes along 6n±1



Note that all primes(marked by red circles) must lie along the diagonal lines $\theta=\pm\pi/3$ and have values 6n±1. Not every number along these lines will be prime and so the f_N evaluation must still be carried out. The jump in integer values along any of the straight lines is always 6 when going from one turn of the hexagonal spiral to the next. The odd numbers 6n+3 =9, 15, 21, etc can never be prime. The numbers 6n, 6n+2, and 6n-2 are always even and hence composite (2 excepted).

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