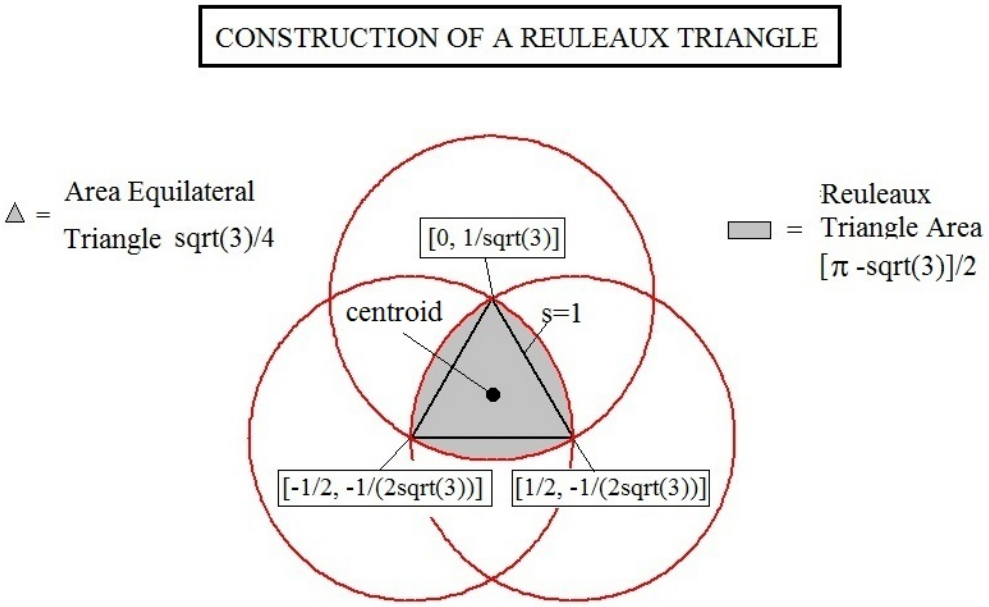
# REULEAUX TRIANGLE AND THE DRILLING OF SQUARE HOLES

In the 19thcentury the German engineer Franz Reuleaux discovered a new type of triangle now referred to as a Reuleaux Triangle. It is constructed by drawing an equilateral triangle of side-length s and then drawing three circles of radius s each centered at the three vertexes of the original triangle. The result is the Reuleaux Triangle as shown in gray in the accompanying figure-




The area of this triangle is determined by noting that the sector of any of the circles which are part of the equilateral triangle have area $\pi s^2/6$ while the equilateral triangle has area $\text{sqrt}(3)s^2/4$. Taking the difference yields the segment area of $(s^2/2)[\pi/3-\text{sqrt}(3)/2]$. So the total area of the Reuleaux Triangle becomes-

$$A_{Reuleaux} = \frac{s^2}{2}[\pi - \sqrt{3}]$$

The centroid of this triangle remains the same as the corresponding equilateral triangle . It is located at the origin of the coordinate system used.

What is now very interesting about this triangle is that the distance from anyone of the original equilateral triangle vertexes to the opposite border of the Reuleaux Triangle remains exactly r=s just like for a constant radius circle. If we now move the centroid of the Reuleaux Triangle along a certain closed curve and rotate the triangle continually, the Reuleraux Triangle will

sweep out a square with slightly rounded corners. I demonstrate such a moving and rotating triangle with the following photos of a wood model I constructed in my workshop-

WOOD MODEL OF MOVABLE REULEAUX TRIANGLE FOR DRILLING NEARLY SQUARE HOLES


The picture shows two different rotor positions plus a third image without the rotor to show the approximate centroid path.

The model is quite reminiscent of the type of configuration found for Wankel engines. I assume that Wankel was probably motivated in his work by first examining the rotation of Reuleaux Triangles. Also there are available commercial drills which can drill holes of the type shown in the above model's inner boundary. The invention of such drills goes back a hundred years to the English inventor Harry Watts(1914). A modification of the Reulaux triangle based on an isosceles triangle has also been developed to make perfect squares without the round corners. In the above case one can of course just use a chisel and /or file to make the corners perfectly square. There are also available much more expensive ultrasonic and laser driven hole makers which can make holes of any desired shape in most materials. The Reuleaux Triangle approach has the advantage that it is relatively inexpensive. Its main competitor is the router which can make excellent square and rectangular holes with very little effort although round corners need still be eliminated by other means. Routers are especially useful for making rectangular mortises in wooden doors when installing a dead-bolt.

Getting back to our model. The required closed curve along which the triangle centroid must move can be determined by sticking a sharpened pencil through a hole through the centroid and then marking points as the triangle is moved and rotated within the confines of the bounding walls. The closed path of the centroid around the rotation axis looks close to , but not exactly, a circle with the distance between the triangle centroid and the rotational axis being approximately $\varepsilon=s(1/\sqrt{3}-1/2)$. A more detailed model shows this closed path to be made up the segments of four long ellipses centered at the four vertexes of the bounding square and coming in diagonally (see- http://www.etudes.ru/en/etudes/drill/ ). For those cases where the round corners are minimized, the centroid path becomes more complicated than the near circle we are seeing with the above model. The four arrows around the model periphery intersect at the location of the rotations axis. In one of the photos we have removed the rotor to show the closed path our measured Reuleaux Triangle centroid takes about the rotational axis.

If one is interested in drilling holes of other than round or square cross-section, it is probably best to dispense with Reuleaux Triangle type drills and rather use metal guides in conjunction with routers. This approach is very fast and inexpensive in time and money to produce mortises for all sorts of wood and metal joining.