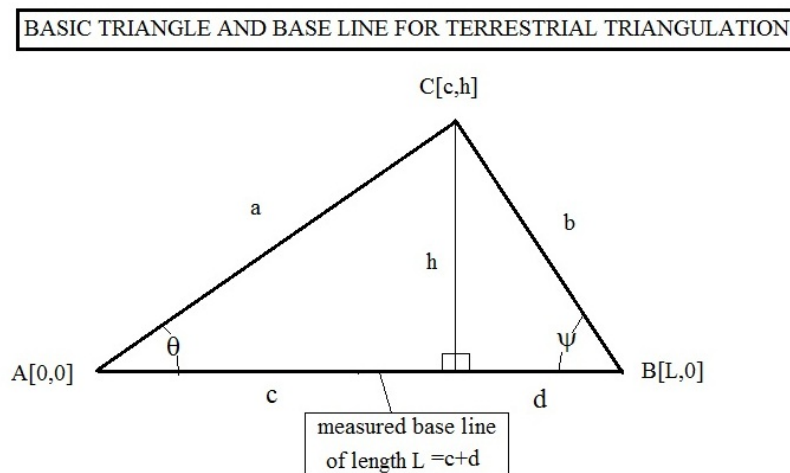


WHAT IS THE STRUVE GEODETIC ARC?

It has been known since ancient times that one can determine the height of any oblique triangle by knowing the exact length of one of its sides (the baseline) and two angles measured from its ends to the third vertex. This is essentially the key used in measuring precisely the circumference and departure from spherical shape of the earth. The most ambitious of such measurements involved the concatenation of some 258 triangles carried out under the direction of F.G.W. Struve between 1816 and 1855 along the 26E meridian extending from the Black Sea to the Norway. It is this measured arc length which is termed the Struve Geodetic Arc. Until the advent of space satellites it was the best way to determine precise distances between points starting with the known distance between two points on the earth's surface. We want here to look at the mathematical aspects for determining the third point of any triangle used in terrestrial triangulations and then show how a string of triangles are constructed to form a chain..

The main approach to terrestrial triangulation is to start with a precisely measured baseline of length L between two points A and B on the surface of the earth and then carefully determining the angles θ and ψ which a third point C makes with respect to the baseline as measured from the baseline ends. The usual instrument for determining the angles is a theodolite which has the capability to measure angles in both the vertical and horizontal plane. For the process to work it is necessary that the line of sight between the three vertices of the triangle A - B - C formed are unobstructed meaning that the elevation of the third vertex point C must be at an elevation of at least equal to $L^2/(2R)$, where R is the Earth radius. This restricts the length of the base line L to typical lengths of less than 50 kilometers. A schematic of the triangulation process for the first triangle is shown in the following figure-



We have there a carefully measured base-line length of L and two precisely determined angles θ and ψ obtained via a theodolite from the ends of the base line to a third point C

located at perpendicular distance h from the base line. Simple trigonometry then shows that –

$$\tan(\theta) = \frac{h}{c} \quad \text{and} \quad \tan(\psi) = \frac{h}{d} \quad \text{with} \quad L = c + d$$

Eliminating the triangle height h , and the lengths c and d from these equations produces the basic equation for terrestrial length measurements -

$$h = \frac{L}{\left[\frac{1}{\tan(\theta)} + \frac{1}{\tan(\psi)}\right]}$$

Since L , θ , and ψ are known measured quantities, the perpendicular distance from the base line to the third vertex C of the triangle can be determined precisely without needing to first measure the sides a and b . Once the coordinate positions of C have been determined, one places a second triangle $A-C-D$ tangent to side $AC='a'$ and picks out a local high elevation point as the third vertex D of the second triangle. Making angle measurements at A and C then allows one to precisely obtain the coordinates of D . Continuing this process along a particular chosen direction produces a chain of triangles whose vertices are determined exactly within the limits determined by the accuracy of the measuring instruments and atmospheric conditions. In most instances the third vertex C , D etc of these triangles will be at different elevations above sea-level. This however does not matter since one can always define a triangle by just three non-collinear points. The famous Struve Geodetic Arc is a chain of 258 triangles laid out from the Black Sea to Norway between 1816 and 1855 running along roughly the 26thE meridian. Its original base line was located at Tartu in Estonia and set up under the direction of Friedrich Georg Wilhelm von Struve, a German born mathematician and scientist working in Russia. He came from a very distinguished family of intellectuals and had a total of 18 children during his lifetime. Here is a picture of the Struve Geodetic Chain formed by its measurement triangles-

STRUVE'S GEODETIC ARC

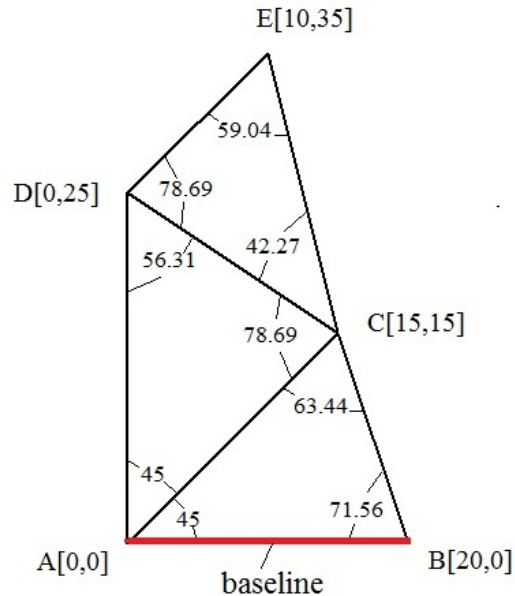


constructed from 1816-1855

Measurements of the earth's circumference and oblateness using Struve's Arc were surprisingly good considering the accuracy of the instruments and measuring techniques they were working with. It was not until the 1980s that satellite measurements led to more accurate results. A military satellite can today locate any point on earth to within a few inches or so.

Let us demonstrate the triangulation process by carrying out a simulation using three triangles starting with an east-west baseline where the coordinates of the end points are $A[0,0]$ and $B[L,0]$ with L set as 20km. To simplify the calculations we place the triangle vertices at integer value points. Carrying out the calculations we get the geodetic chain shown-

SIMULATION OF A 3 TRIANGLE GEODETIC CHAIN
USING A 20 KM LONG BASELINE



Note that the sum of the angles for each triangle add up to exactly 180° . The precise distance of point E from A is $\sqrt{1325}=36.40$ km. The famous mathematician C.F.Gauss was asked to set up one of these geodetic survey patterns for the province of Hanover. When he first accepted this job back in 1818 many of his colleagues thought it was beneath his intellectual integrity. However, he got the last laugh by coming up with some brilliant insights on curved surfaces and non-Euclidian geometry stemming directly from this work.

The above triangulation technique can also be used to measure the elevation of mountains. All that is required is that one have a carefully measured base line at known elevation above sea-level and then measure, via theodolites, the inclination of the plane of the triangle and angles to the top of the mountain from the two ends of the baseline. This is how the British first measured the height of Mount Everest as part of their Great Trigonometric Survey of India. Using giant theodolites they measured the height of the Mt. Everest as 29,002 ft above sea level. This is within 27 ft of the presently accepted value of 29,029ft. The announcement concerning the world's highest mountain was made by Andrew Waugh the Surveyor General of India in 1856. Waugh suggested that the mountain be named in honor of his predecessor Colonel Sir Georg Everest. Since Waugh was heavily involved in the final observations and calculations (and Everest was not) it would have been more appropriate for the Royal Geographic Society to have named the mountain Mt .Waugh.

These days we measure elevation of mountains either by earth satellites or radar reflections from airplanes flying at constant altitude. Just today it was announced that the latest radar measurements put the height of Mt. McKinley in Alaska at 20,237 ft. This is some 83 ft lower than the previous official value of 20,320 ft.