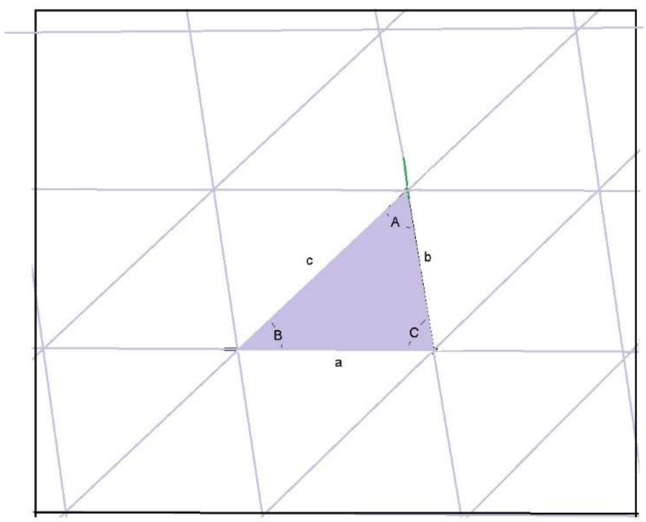
TESSELLATION PATTERNS PRODUCED BY AN ARRAY OF OBLIQUE TRIANGLES

It is well known that squares , rectangles, and regular hexagons represent surfaces which may be tiled in the x-y plane without leaving gaps or overlaps. Mathematically any arrangement of such figures by themselves or a combination thereof are referred to as tessellations. Often overlooked are tessellations involving oblique triangles which can also be arranged to form gapless periodic arrays within an infinite plane. It is our purpose here to show how such arrays are constructed.

We start with drawing a single oblique triangle of sidelength a, b, and c and vertex angles A, B, and C together with a three part grid parallel to the three sides. Here is the picture-

FLOOR TILING WITH AN OBLIQUE TRIANGLE



The oblique triangle, shown in blue, has an area-

$$A=(ab/2)\sin(C)$$

as is easily established by taking half of the absolute value of the cross product of v_1=-ia and

$v_{2.}$ =ibcos(C)+jbsin(C). Next using the law of cosines we have cos(C)=(a^2+b^2-c^2)/2ab. So eliminating the angle C, we get the equivalent area-

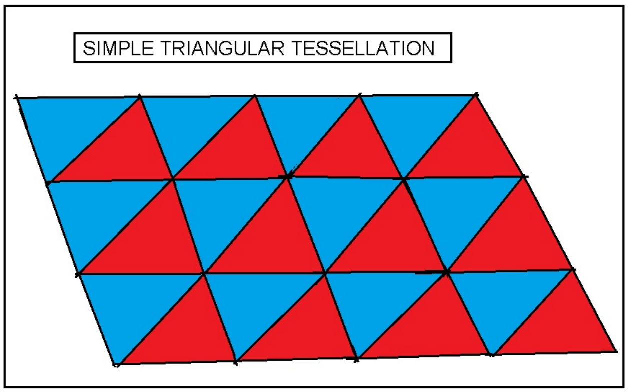
A=[(ab)/2]sqrt{1-[(a^2+b^2-c^2)/(2ab)]^2}

This last result is identical with Hero's formula involving the semi-perimeter s=(a+b+c)/2. What the result clearly shows is that for the case of a right triangle, where C=90deg, the area is simply the product of a and b divided by two.

Note that the three grid structure contains two flipped oblique triangles within each rhomboid of area-

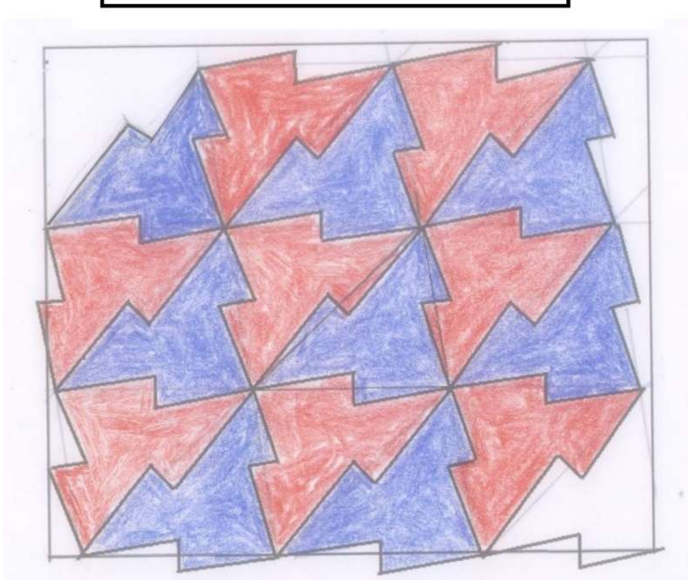
$$A_{RHOM}=ab\sqrt{1-\left[\frac{c^2-a^2-b^2}{2ab}\right]^2}$$

The fact that the rhomboids formed by the three grids fills the entire x-y plane without gaps or overlaps implies that the same is true for any oblique triangle. Here is a simple tessellation of blue and red tiles using one oblique triangle-



More complicatd tessellations follow by distorting the edges of a given oblique triangle. To keep the area of the new configuration the same as the undistorted triangle one needs to restrict the distortions so that their net value along any side equals zero and to make sure there is no overlap with neighboring sides. Here is such an example-

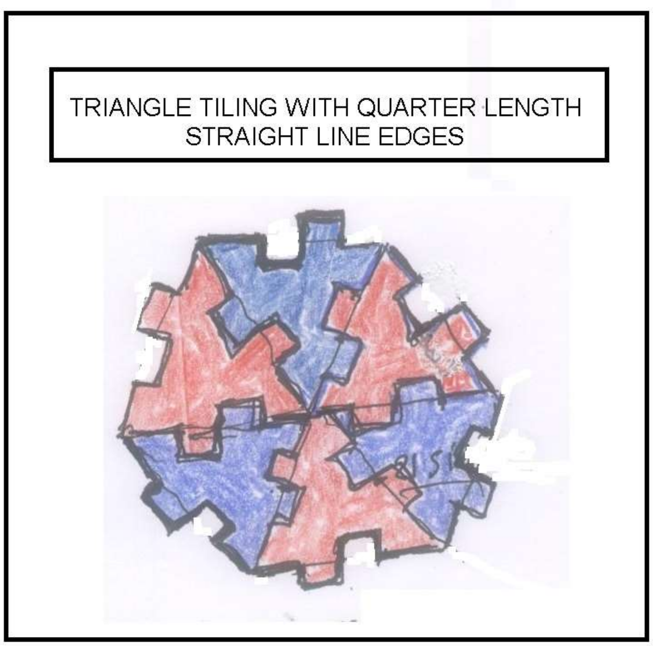
STRAIGHT LINE TESSELLATION PATTERN



An even more intricate tessellation occurs if one replaces the oblique triangle sides by a sine pattern. The resultant tessellation follows-

These last two patterns were drawn by hand since it was faster to obtain their shape than trying a Microsoft paint program approach. Note that the area of each of the shapes retain the same area as the original triangle.

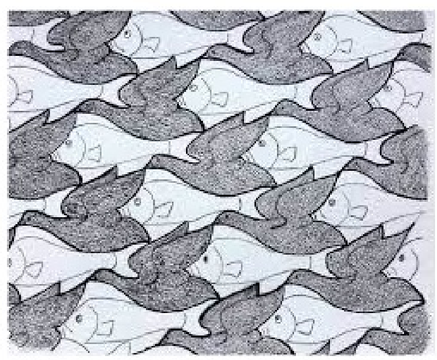
As a final tessellation consider distorting the triangle sides by a straight line fence pattern. Here one finds the result-



One finds the overall pattern to resemble a typical 2D version of a virus shape. Or, one could also envision the sub-pattern to resemble a walking dog with big ears.

What is clear from the above results is that very intricate periodic floor and wall patterns can be created using an oblique triangle as the starting point. With an appropriate chosen triangle edge distortion one can come up with continuous and periodic floor tiling resembling a variety of different entities. This is essentially what the Dutch artist M.C. Escher did after studying some islamic art in the 1930s. Here is an image of one of his paintings-

FISH-BIRD TESSELLATION
BY M.C.ESCHER



U.H.Kurzweg
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