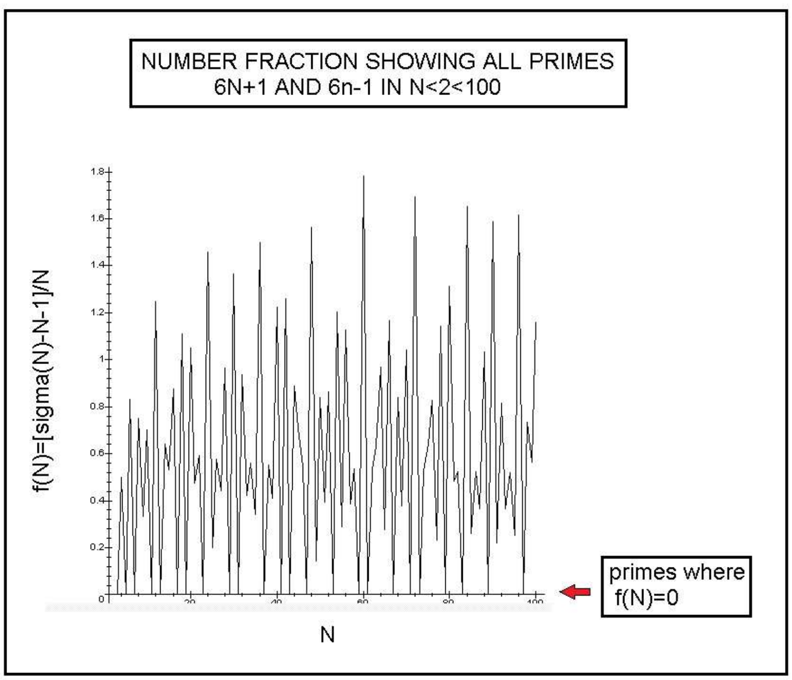
RELATION BETWEEN THE SIGMA FUNCTION AND

THE NUMBER FRACTION

Several years ago we came up with a new fraction defined as-

f(N)=[(Sum of all Divisors of N)-(N+1)]/N=[σ(N)-1-N]/N

We have called this function the ***number fraction*** with σ(N) being the sigma function of number theory. This function f(N) has the interesting property that it vanishes when N is a prime. Also , because of the N term in the denominator, its average value as N gets large tends to remain relatively small. Looking over the range of 2 to 100, one arrive at the following list plot-



What s quite clear from this graph is that –

All primes five or greater must have the form 6n±1 without exception.

Writing out the number fraction for a few powers of two yields-

f(2^2)=7/8, f(2^3=3/4, f(2^4)=7/8, f(2^5)=15/16

From these one can generalize things to get-

$$f(2^n)=\frac{2^{n-1}-1}{2^{n-1}} = 1-\frac{1}{2^{n-1}}$$

This means the number fraction approaches 1 as 2^(n-1) goes to infinity. One can also show that-

$$f(3^n)=(1/2)[1-\frac{1}{3^{(n-1)}}] \quad \text{and} \quad f(4^n)=1-\frac{1}{4^{(n-1)}}$$

Larger values for f(N) occur when N contains many divisors suggesting those Ns consist of products of small integers including their powers. A good example is the number N=60 in the above graph. It has-

$$N=60=3x4x5 \quad \text{which yields } f(N) = \frac{2+3+4+5+6+10+12+15+20+30}{60} = \frac{107}{60} = 1.7833\ldots$$

When a number has a number fraction f(N) greater than unity, we call it a super-composites. There are an infinite numbers of these. We can easily construct super-composites by writing them as the product of increasing lower primes taken to progressively lower powers.Thus the number-

N=(2^16)(3^8)(5^4)=268,738,566,000 has f(N)=2.74858…

This is still a relatively low value for such a large number. It is not known whether any super-composites have number values approaching infinity. So far we have not found any f(N)s above about seven.

Going back to the definition of the number fraction given above, we can rewrite things as-

σ(N)=1+N+Nf(N)

Since my math program(MAPLE) give values of σ(N) up to about 40 digits length for N , we can easily determine the sigma value for the above 268 billion digit long number. It is-

σ(N)=1007388244291

From this we also have the same result as earlier-

f(N)=[σ(N)-N-1]/N=2.7485…

Note that if N is a prime p, then f(p)=0 and σ(p)=1+N. On replacing p by p^2 allows us to state that-

Any number is a prime if Nf(N^2)=1 and σ(N^2)-N(1+N)=1

To check this statement consider the three digit long number N=373 where σ(373^2)=139503 and f(373^2)= 0.0026809. Here Nf(N^2)=1 and $\sigma(N^2) = N(1 + N = 1$. So $\mathrm{N} = 373$ is a prime. Note here that f for N=373^2 lies just slightly above zero. This means that N^2 is likely to be a semi-prime which indeed it is.

Let us next look at semi-primes N=pq, where p and q are the prime components. We have-

f(pq)=(p +q)/pq and σ(pq)=1+p+q+pq

or combining to get the semi-prime relation-

σ(N)-Nf(N)=1+N

We see from this last equality why σ(N) is only slightly larger than N when N gets large.

For the semi-prime of N=455839, we find at once that f(N)=0.0029835… and σ(N)=457200.

Note the near zero value of f(N) and the fact that σ(N) is only slightly larger than N.

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